Gravitational Wave Emission From Rotating Deformed Stars

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Two Main Theoretical Sources

- Binary Stars
- Isolated stars
  - Need asymmetries or deformations

Models for Isolated Stars

- Fluid models
  - Cannot support deformations
- Solid Models
  - Can support deformations
  - Not a realistic model for neutron stars
- Crustal models, with fluid cores
  - Can support deformations
  - Good candidate for modelling neutron stars

History

- Carter & Quintana - 1970's
  - First comprehensive study of relativistic elasticity
  - Became a standard reference
- Karlovini & Samuelsson - 2000's
  - Extended the work by Carter and Quintana
  - More rigorous treatment of the nonlinear shear scalar

No axisymmetric numerical treatments to date

General Stress-Energy

$$T^{\mu\nu} = (\check{\rho} + \check{P})u^{\mu}u^{\nu} + \check{P}g^{\mu\nu} - \check{\mu}s^{\mu\nu} \quad (1)$$

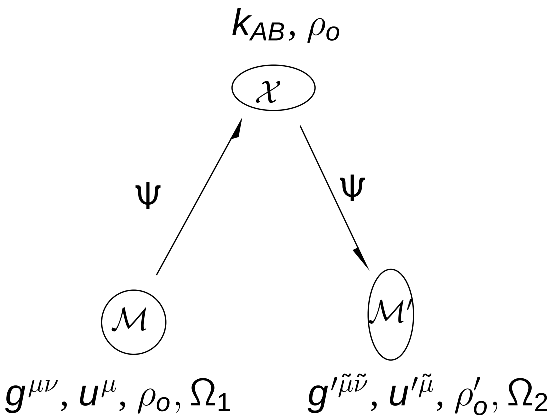
- First 2 terms on right come directly from a fluid model
- Last term is a shearing term
  - $\check{\mu} \equiv \check{\mu}(\rho_0)$
  - ρ_0 - baryon number density
  - $s_{\mu\nu}u^{\mu} = 0$ - flowline orthogonal

Linear Elasticity

$$s_{\mu\nu} = (h_{\mu\nu} - \eta_{\mu\nu}) \tag{2}$$

- $h_{\mu\nu}$ - projection operator
- $\eta_{\mu\nu}$ - reference tensor
  - $\eta_{\mu\nu} = \Psi^{A}{}_{,\mu}\Psi^{B}{}_{,\nu}\eta_{AB}$
  - η_{AB} - matter space metric
  - Ψ - map from matter space to spacetime
- $\check{\mu} = \kappa P$ - shear is proportional to the pressure

Matter Space ↔ Spacetime



A schematic representation of the relativistic elastic system.

Equation of state used by Carter and Karlovini,

$$\rho = \check{\rho} + \check{\mu}(\rho_o)s^2 \tag{3}$$

- s - shear scalar
- $\check{\rho} = \rho_o + \frac{\check{P}}{\Gamma - 1}$
- $\check{P} = P(\rho_o)$

Equations

- $G_{\mu\nu} = 8\pi T_{\mu\nu}$ – Einstein equations
- $\nabla_\mu T^{\mu\nu} = 0$ – Conservation of stress-energy
  - Bernoulli equation
  - $\nabla_\mu(\check{\mu} s^{\mu\nu}) = f^\nu$
- Equation of state

ADM 3+1 split using formalism from Bonazzola *et al.* (1993) to reduce these to a set of coupled partial differential equations

Numerical Routine

LORENE

- Iterative spectral elliptic PDE solver
  - Solves Einstein equations
  - Solves mapping equations using conservation of stress-energy
  - Checks solution against Bernoulli equation
- http://www.lorene.obspm.fr/

Summary

- Devised a numerical non-perturbative relastic neutron star model
- Numerical treatment is general enough to lead to more interesting cases

Outlook

- Investigate interesting external forces to strain the star
- Add a non-symmetric perturbation to define mountains

Future Directions

Research Improvements/Modifications:

- Fluid Core
- Magnetic Fields
- Nonaxisymmetric calculations
- Different types of equations of state

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ADM/3 + 1 Variables

slice an axisymmetric spacetime in spacelike hypersurfaces in order to re-express the spacetime metric to make time a parameter

$$ds^2 = -\alpha^2 dt^2 + B^2 r^2 \sin^2\theta \left(d\phi - \beta^\phi dt\right)^2 + A^2(dr^2 + r^2 d\theta^2)$$