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# High Frequency GW Sources


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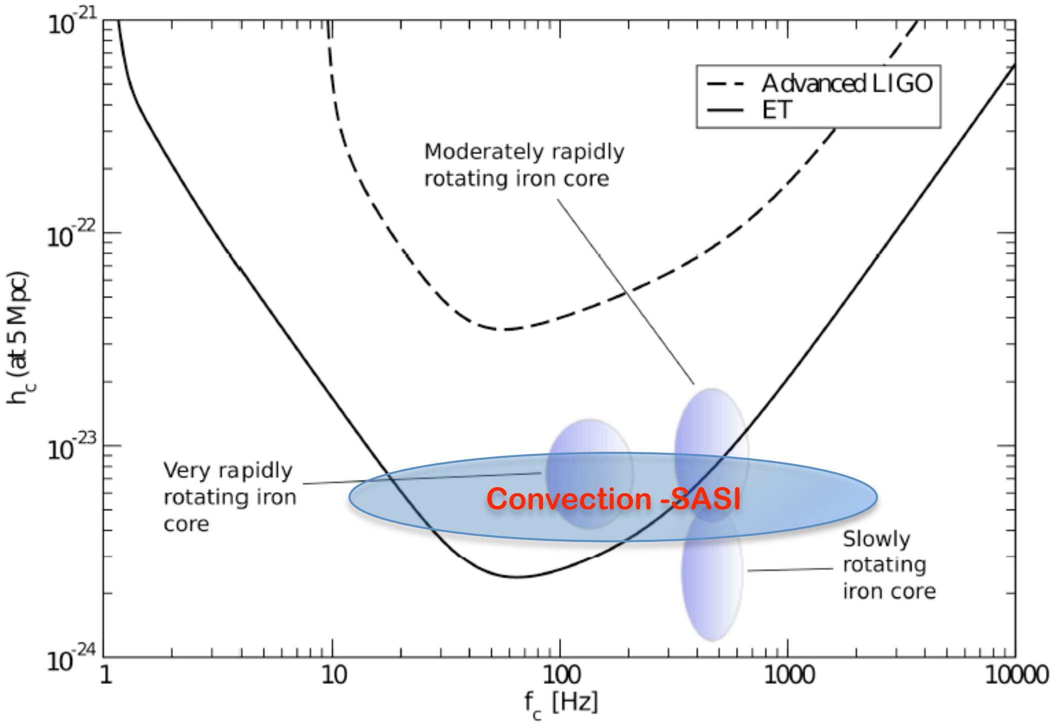
# The High Frequency Window

- **Supernova Core Collapse**
  - The violent dynamics associated with a supernova core collapse is expected to lead to GW emission through a number of channels
- **Rotating Deformed Neutron Stars**
  - Asymmetries, generated either by strains in the star's crust or by the magnetic field, are expected to slowly leak rotational energy away from spinning neutron stars.
- **Oscillations and Instabilities of NS**
  - Neutron stars have rich oscillation spectra which, if detected, could allow us to probe the internal composition "GW Asteroseismology"
- **Magnetars**
  - Magnetar flares emit huge amounts of EM radiation, if a small percentage is emitted in GW they can be a promising source.

# Core Collapse

**Leads to GW emission through a number of channels related to:**

- ✓ The dynamics of the PNS and its immediate environment
- ✓ The convective zone behind the shock front



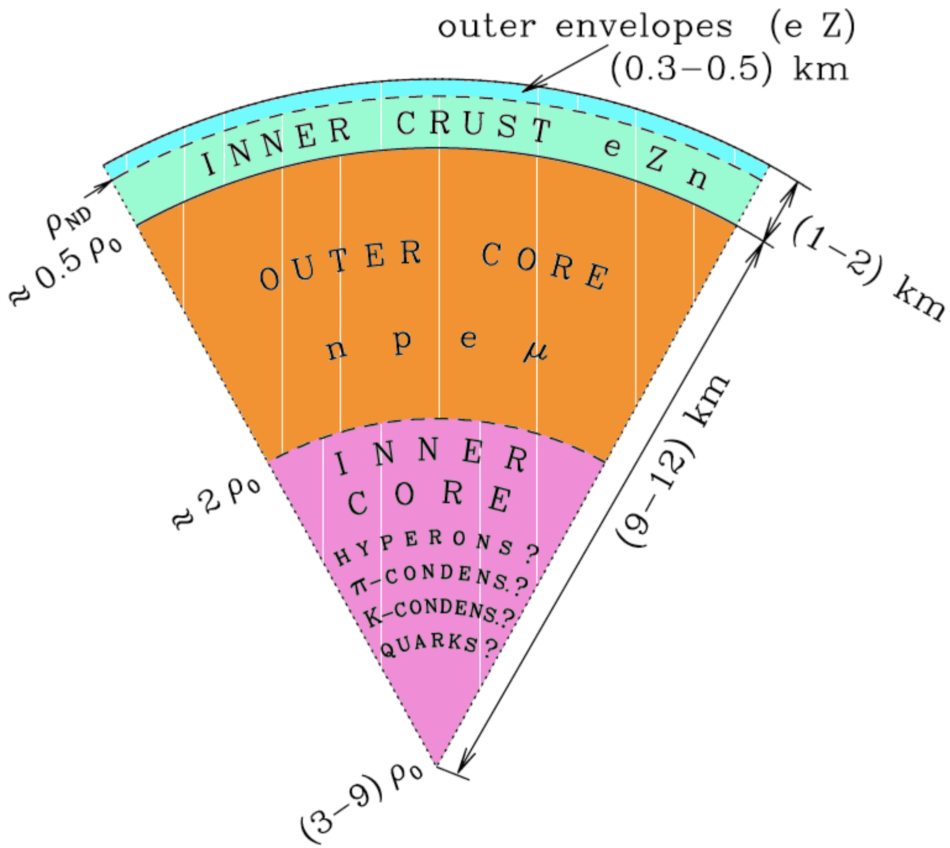

1. **Slowly rotating iron cores** : bounce and initial ringdown (**700-900Hz**)
2. **Faster rotation** amplifies the bounce signal (**400-800 Hz**)
3. **Very rapid rotation** leads to bounce at subnuclear densities (**100-200 Hz**)
4. **Prompt convection** shortly after core bounce due to negative lepton gradients (**50-1000Hz**)
5. **Neutrino-driven convection** and SASI (Standing Accretion Shock Instability) (**100-800 Hz**)

- ✓ A major uncertainty connected with supernova models is the initial state, in particular the angular momentum distribution in the iron core.
- ✓ Current expectations from stellar evolution calculations imply a slowly rotating core as a canonical case



Reviews: Ott 2009, Andersson etal 2010

# Neutron Stars in Microphysics

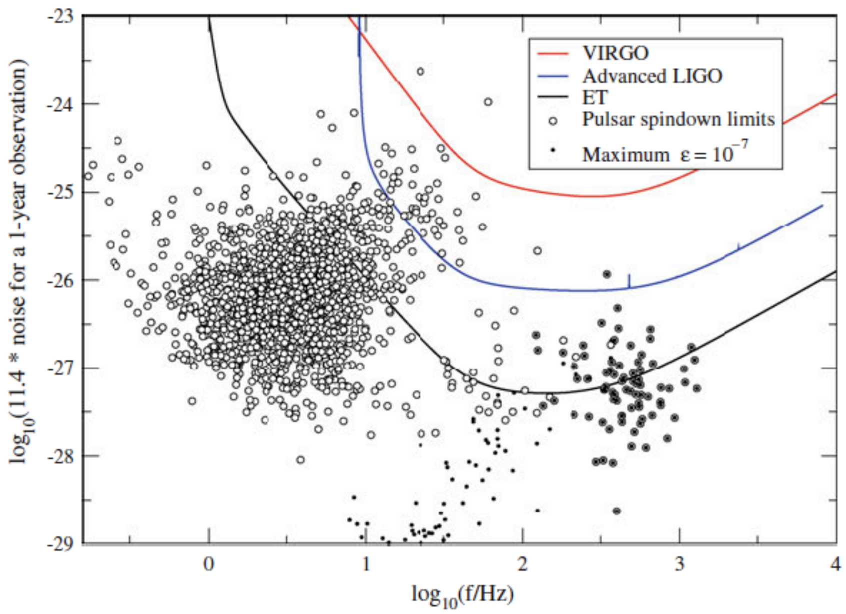



- ✓ Neutron star EoS is known for the **outer star**, but not in the high-density **inner core**.
- ✓ Thus, EoS models depend upon assumptions about matter phase of inner core (hadronic matter, pion/ kaon condensates, quark matter…).
- ✓ Each new phase increases compressibility, affecting M-R relation
- ✓ The different Equations of State (EoS) predict up to 7 times higher pressure for the same density

**GWs can provide a unique tool to study NS interior**

# Rotating deformed neutron stars

- ✓ The radio pulsar and the accreting LMXB binary systems, are prime candidates for GW detection via targeted searches.
- ✓ There may be a population of neutron stars currently invisible to electromagnetic observations, spinning down by GW emission.




$u_{break}$: crustal breaking strain

**Complicated story:**
different physical assumptions lead to very different possible maximum mountain sizes

**Ellipticity** $\varepsilon < 2\times 10^{-5}\left(\frac{u_{break}}{10^{-1}}\right)$

**Magnetic fields** $\varepsilon \approx 10^{-12}\left(\frac{B}{10^{12}G}\right)^2$

**Solid quark star** $\varepsilon < 6\times 10^{-4}\left(\frac{u_{break}}{10^{-2}}\right)$

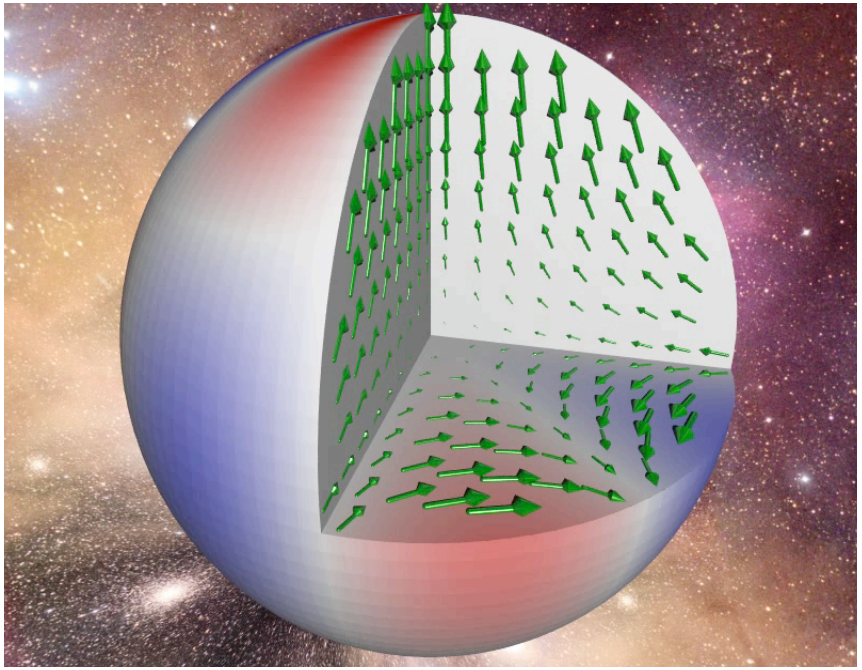
**Supercontacting Core** $\varepsilon \approx 10^{-9}\left(\frac{B}{10^{12}G}\right)\left(\frac{H_{crit}}{10^{15}G}\right)$

1. S5 data for Grab : no more than 2% of the spin-down energy was being emitted in the GW channel, corresponding to an ellipticity bound of approximately $\varepsilon < 10^{-4}$
2. If nature supplies millisecond pulsars deformed at the level of one part in $10^7$, ET may provide the key to detecting them



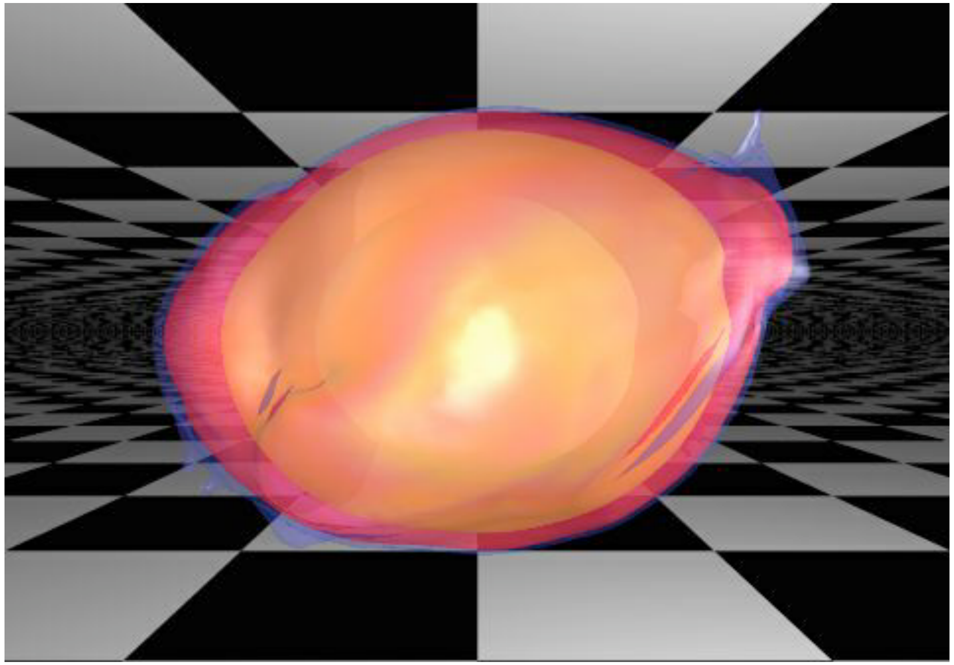
Ushomirsky et al '00, Cutler '02, Owen '05, Haskell et all '06,…

# Gravitational Wave Asteroseismology



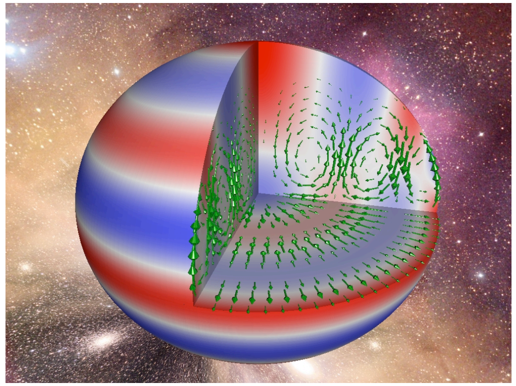
Neutron Stars oscillate wildly during the very first seconds of their life

We can potentially estimate their **masses, radii, equations of state** by analysing the seismic data via the emitted gravitational waves



**Rotation** is responsible for a number of instabilities which emit copious amounts of GWs

# Neutron Star "ringing"



**p-modes**: main restoring force is the **pressure (f-mode)** *(>1.5 kHz)*

$$\sigma \approx \sqrt{\frac{M}{R^3}}$$

**Inertial modes: (r-modes)** main restoring force is the **Coriolis force**

$$\sigma \approx \Omega$$

**w-modes:** pure **space-time modes** (only in GR) *(>5kHz)*

$$\sigma \approx \frac{1}{R}\left(\frac{M}{R}\right)$$

**Torsional modes** (t-modes) *(>20 Hz)* shear deformations. Restoring force, the weak Coulomb force of the crystal ions.

$$\sigma \approx \sqrt{\frac{v_S}{R}}$$

**… and many more**

# Effect of Rotation & Magnetic Fields

## ROTATION

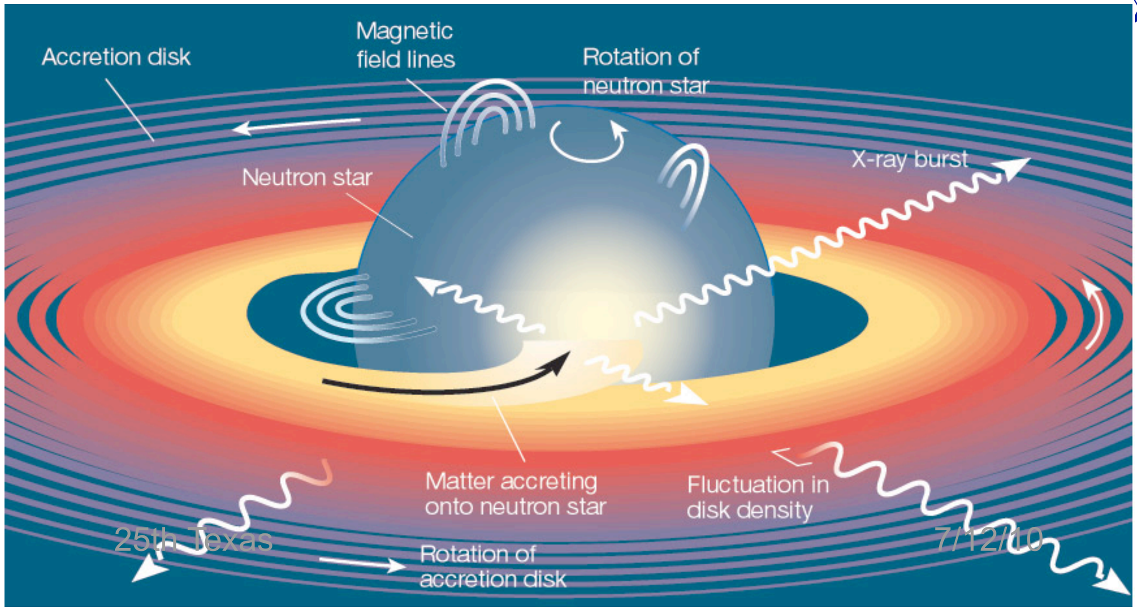
- **Frame dragging**
- **Quadrupole deformation**
- **Rotational instabilities**

## MAGNETIC FIELD

- **No significant effect** in the fluid frequencies and damping/growth times

$$\frac{\text{magnetic energy}}{\text{gravitational energy}} \sim \frac{B^2 R^3}{GM^2/R} \sim 10^{-4}\left(\frac{B}{10^{16}G}\right)^2$$

- For **magnetars** we may observe **Alfvén oscillations**



# Stability of Rotating Stars

## Non-Axisymmetric Perturbations

A general criterion is:

$$\beta = \frac{T}{|W|} \approx \frac{2}{15} e^2 + \ldots$$

**T** : rot. kinetic energy
**W** : grav. binding energy

### Dynamical Instabilities

- Driven by hydrodynamical forces (bar-mode instability)
- Develop at a time scale of about one rotation period $\sim (G\rho)^{-1/2}$

$$\beta_N \geq 0.27$$
$$\beta_{GR} \geq 0.25$$

### Secular Instabilities

- Driven by dissipative forces (*viscosity, gravitational radiation*)
- Develop at a time scale of several rotation periods.
- Viscosity driven instability causes a Maclaurin spheroid to evolve into a non-axisymmetric Jacobi ellipsoid.
- Gravitational radiation driven instability causes a Maclaurin spheroid to evolve into a stationary but non-axisymmetric Dedekind ellipsoid.

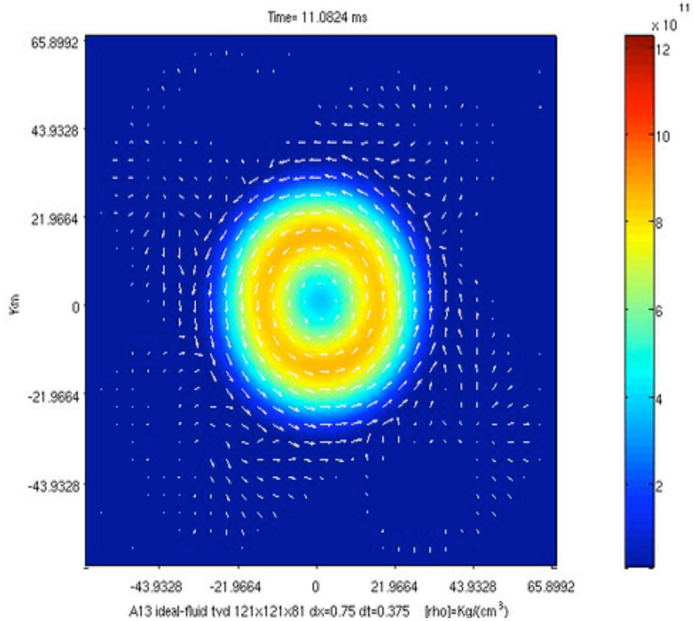
Chandrasekhar-Friedman-Schutz (CFS)

$$\beta_N \geq 0.14$$
$$\beta_{GR} \geq 0.07$$

**GR and/or differential rotation suggest considerably lower $\beta$ for the onset of the instabilities**

# Bar-mode dynamical instability

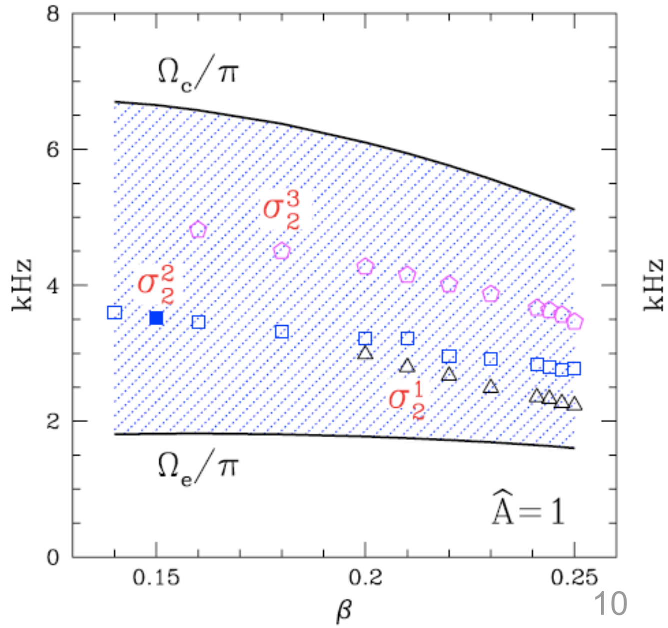
- ✓ For rapidly (differentially!) rotating stars with: $\beta > 0.27$.
- ✓ **Typical Frequencies ~1.5-3.5kHz**
- ✓ The "bar-mode" grows on a dynamical timescale.
- ✓ Once it is active the instability **does not persist for long** due to nonlinear mode-mode coupling




## LOW T/|W| Instability (Shibata et al 2007)

- ✓ These are "**Shear instabilities**" associated with the existence of a corotation band.
- ✓ They develop for any value of the instability parameter $\beta$ when sufficient amounts of differential rotation are present.

(Watts etal 2002, Corvino etal 2010)



# The CFS instability

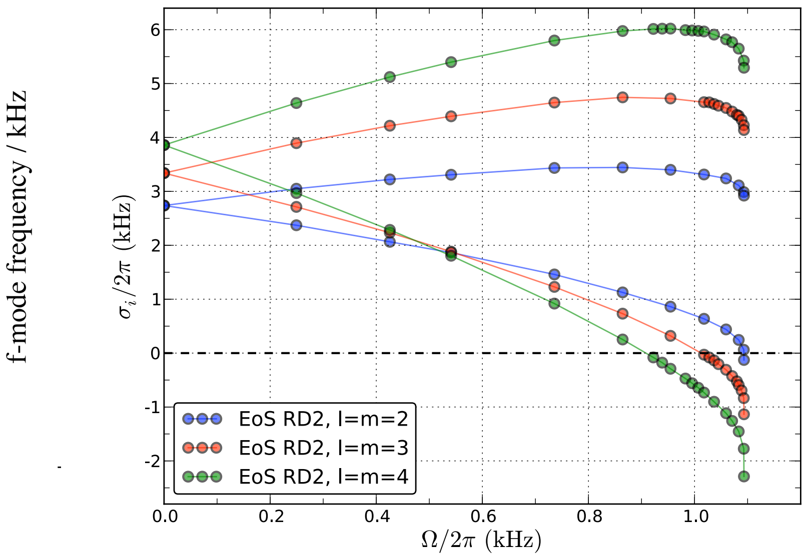
**C**handrasekhar 1970: Gravitational waves lead to a secular instability

**F**riedman & **S**chutz 1978: The instability is generic, modes with sufficiently large *m* are unstable.

**A neutral mode of oscillation signals the onset of CFS instability**

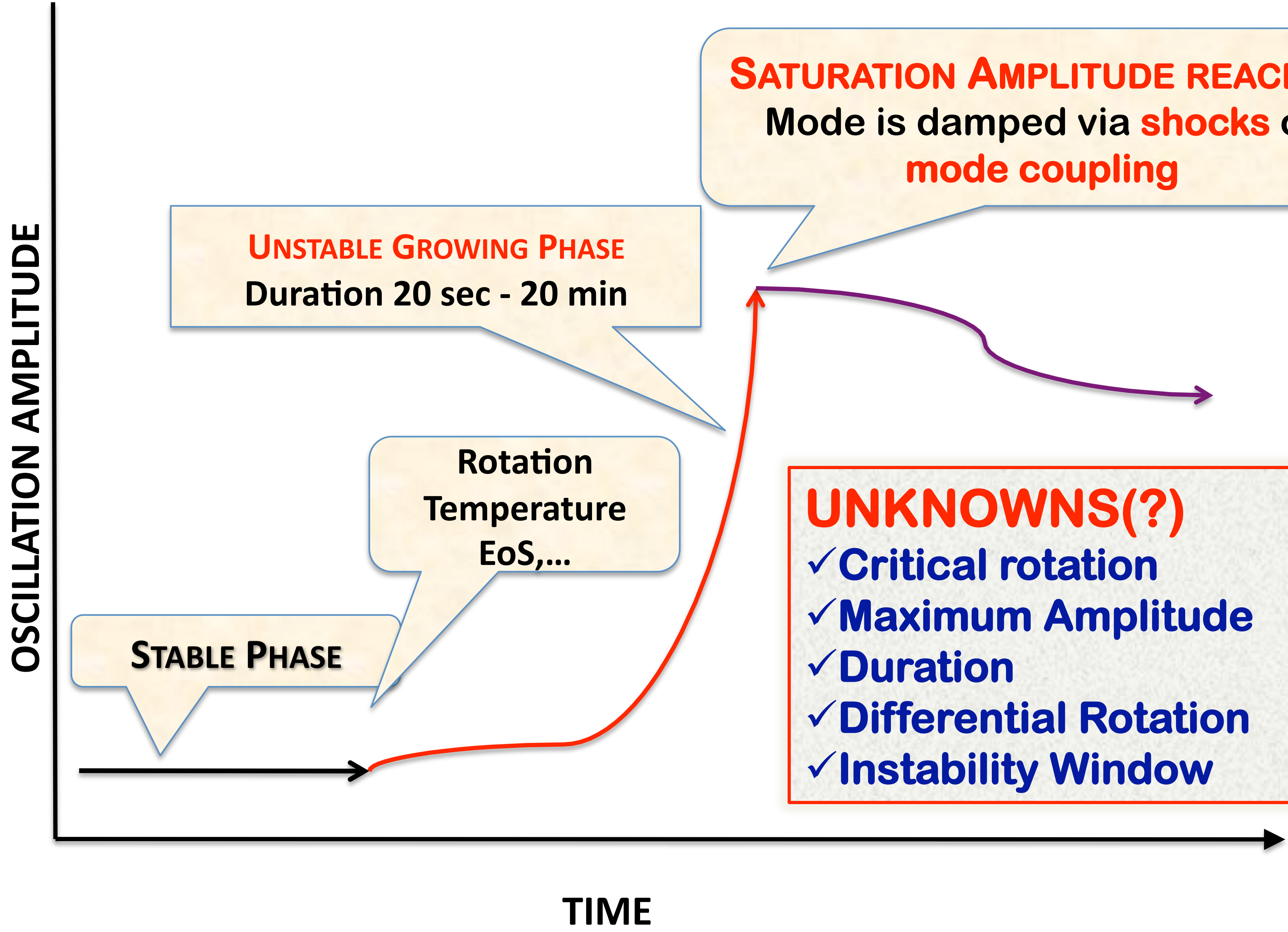
Gaertig+KK 2008

- ✓ Radiation drives a mode unstable if the mode pattern moves backwards according to an observer on the star *($J_{rot}$<0)*, but forwards according to someone far away *($J_{rot}$>0).*
- ✓ They radiate positive angular momentum, thus in the rotating frame the angular momentum of the mode increases leading to an increase in mode's amplitude.




$$\frac{\omega_{in}}{m} = -\frac{\omega_{rot}}{m} + \Omega$$

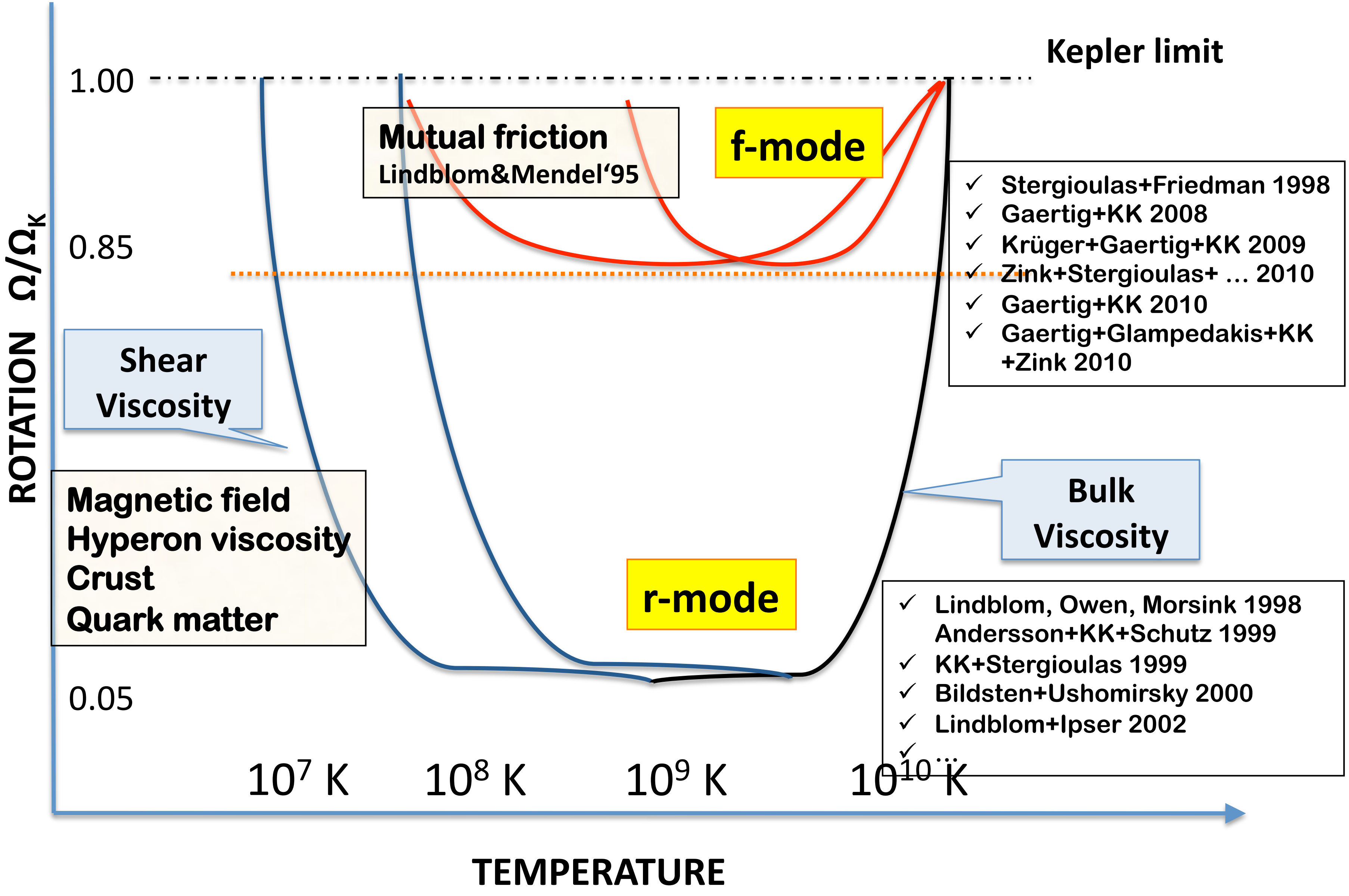
# The Excitation of Secular Instabilities



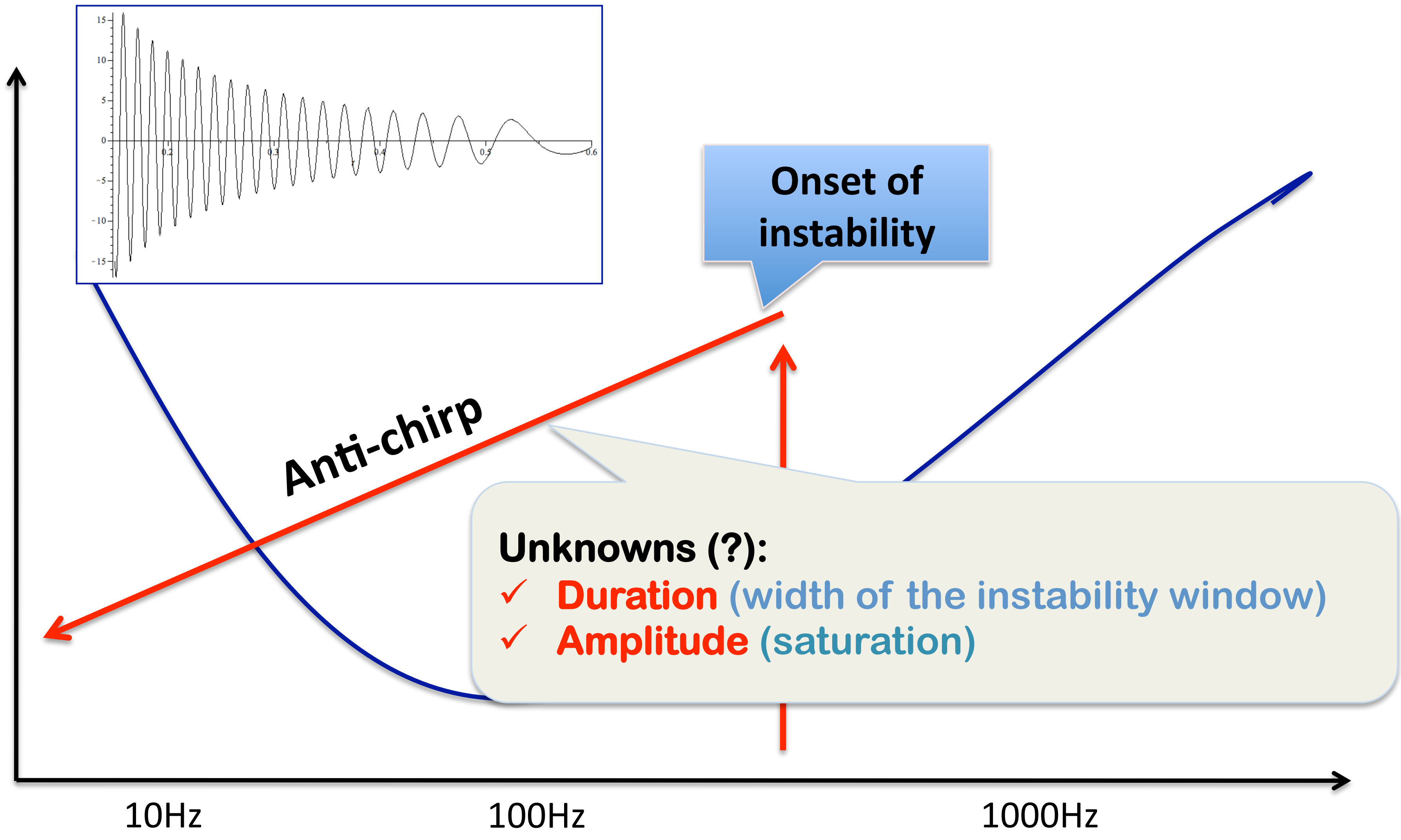

**UNKNOWNS(?)**

- ✓Critical rotation
- ✓Maximum Amplitude
- ✓Duration
- ✓Differential Rotation
- ✓Instability Window

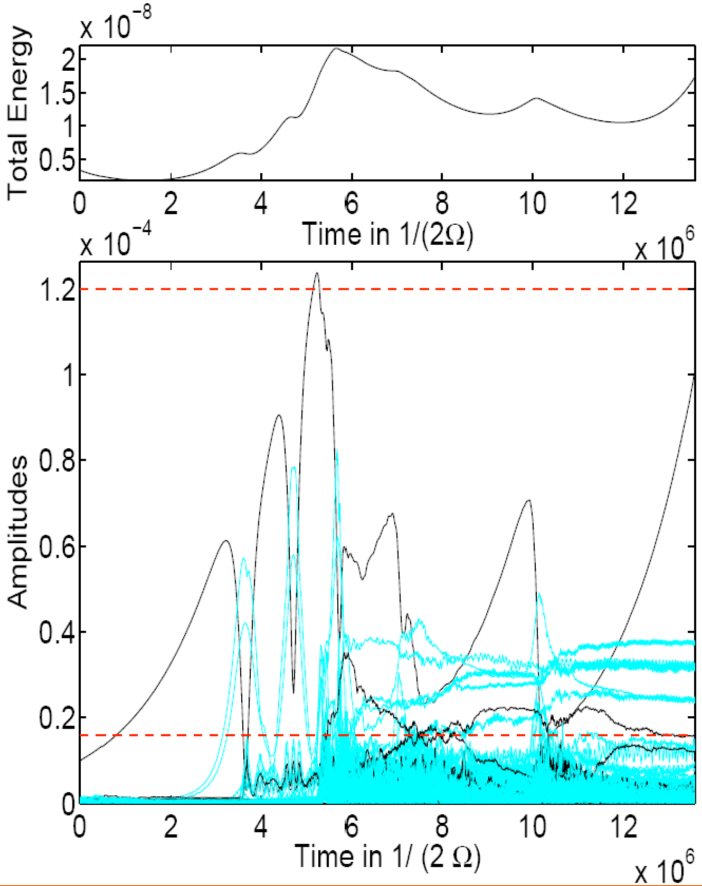
# INSTABILITY WINDOW



# f-mode Instability



# R-modes




$$h(t) \approx 10^{-20} \alpha \left( \frac{\Omega}{1 \text{ kHz}} \right) \left( \frac{10\text{Kpc}}{d} \right)$$

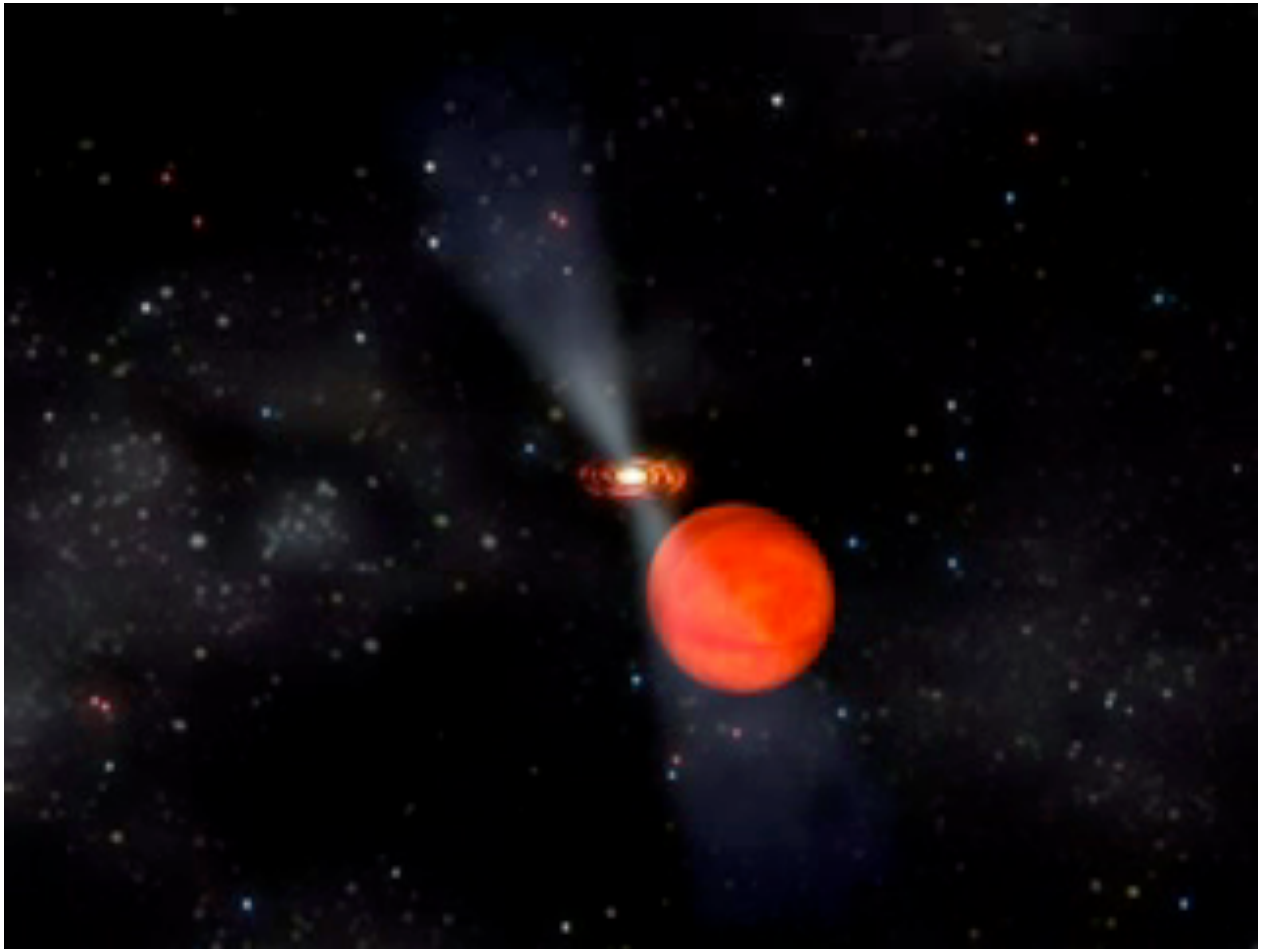
$$\alpha \simeq 10^{-3} - 10^{-4}$$

## GW amplitude depends on the saturation amplitude

- ✓ The existence of *crust*, hyperons in the core, magnetic fields, affects the efficiency of the instability.
- ✓ **Mode coupling** might not allow the growth of instability to high amplitudes (Cornell group \`04-\`08)
- ✓ R-mode instability for **newly born** neutron stars might be quite weak ; unless we have the creation of a strange star
- ✓ **Old accreting neutron** (or strange) stars, probably the best source!

# R-modes

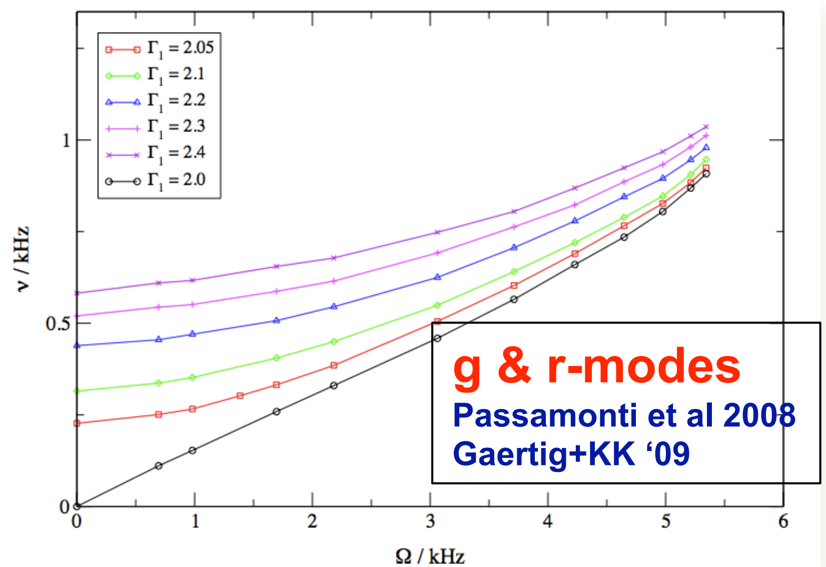
## A successful application (!)



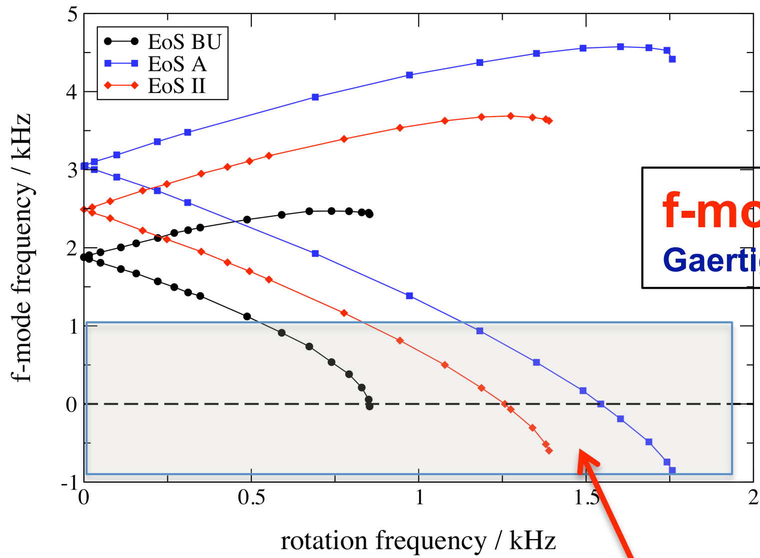
# Fast Rotating NS in GR: f, g, r-modes

We can evolve the linear & non-linear form of Einstein's equations and simulate the dynamics of fast rotating and magnetized neutron stars

- ✓ In GR the *m=2* mode becomes unstable for $\Omega > 0.85\,\Omega_{Kepler}$
- ✓ Differential rotation affects the onset of the instability




**g & r-modes**
Passamonti et al 2008
Gaertig+KK '09




**f-modes**
Gaertig+KK 2008

LIGO/Virgo/GEO-HF band

Gaertig+KK 2008,09,10, Krüger, Gaertig, KK 2009, Zink etal 2010

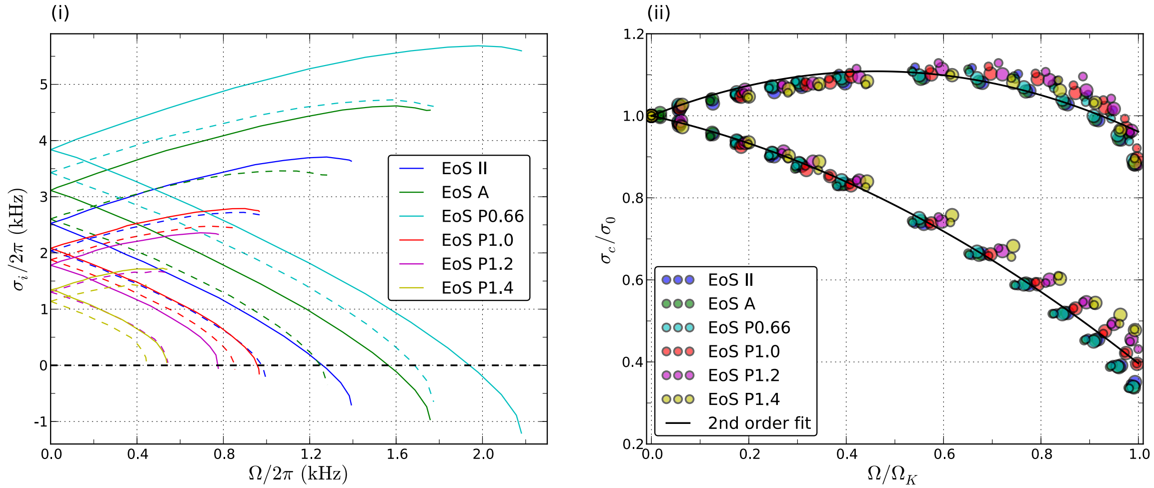
# f-modes: Asteroseismology

**We can produce empirical relation relating the parameters of the neutron stars to the observed frequencies.**

## Frequency

$$\frac{\sigma}{\sigma_0} \approx 1 + 0.63\left(\frac{\Omega}{\Omega_K}\right) - 0.32\left(\frac{\Omega}{\Omega_K}\right)^2 + \ldots \quad (\mathrm{m}=2)$$

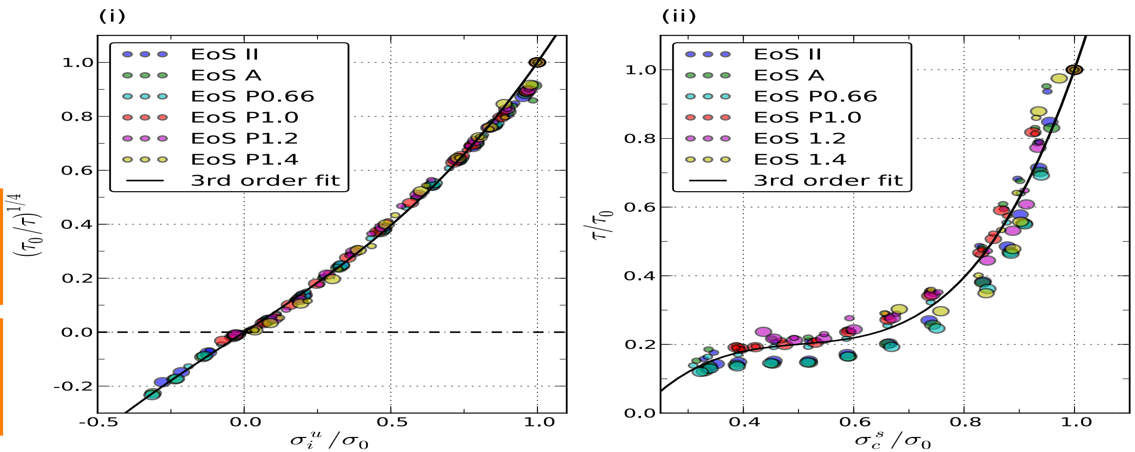
$$\frac{\sigma}{\sigma_0} \approx 1 - 0.41\left(\frac{\Omega}{\Omega_K}\right) - 0.53\left(\frac{\Omega}{\Omega_K}\right)^2 + \ldots \quad (\mathrm{m}=\text{-}2)$$




## Damping/Growth time

$$\left[\frac{\tau_0}{\tau}\right]^{1/4} \approx \mathrm{sgn}(\sigma_i) 0.71\left(\frac{\sigma_i}{\sigma_0}\right)\left[1 + 0.048\left(\frac{\sigma_i}{\sigma_0}\right) + 0.35\left(\frac{\sigma_i}{\sigma_0}\right)^2\right]$$

$$\left[\frac{\tau_0}{\tau}\right] \approx -0.66\left[1 - 7.33\left(\frac{\sigma_c}{\sigma_0}\right) + 15.06\left(\frac{\sigma_c}{\sigma_0}\right)^2 - 9.26\left(\frac{\sigma_c}{\sigma_0}\right)^3\right]$$




**Gaertig-KK 2008, 2010**

# f-mode: Instability window

$$E = \frac{1}{2}\int\left[\rho\delta u^a \delta u_a^* + \frac{\delta p}{\rho}\delta\rho^*\right]d^3x \quad \Rightarrow \quad E \approx \sigma^2$$

$$\frac{dE}{dt} = -\sigma_i(\sigma_i + m\Omega)N_\ell\left|\delta D_{\ell m}\right|\sigma_i^4 \quad \Rightarrow \quad \frac{dE}{dt} \approx \sigma_i^6$$

$$\frac{1}{\tau_{GR}} = -\frac{1}{2E}\left(\frac{dE}{dt}\right) \approx \sigma_i^3(\sigma_i + m\Omega)$$

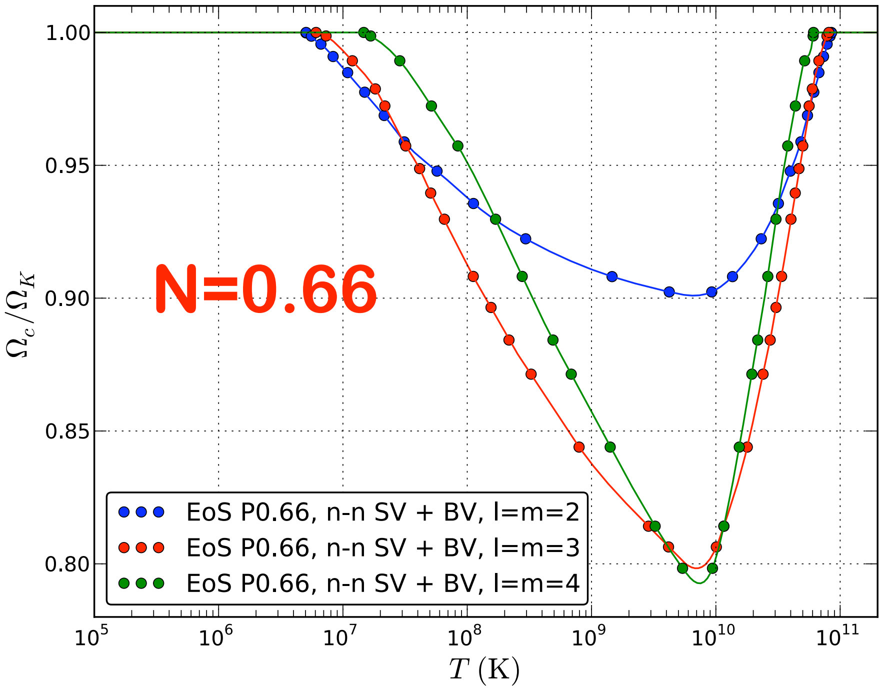
Ipser-Lindblom 1991

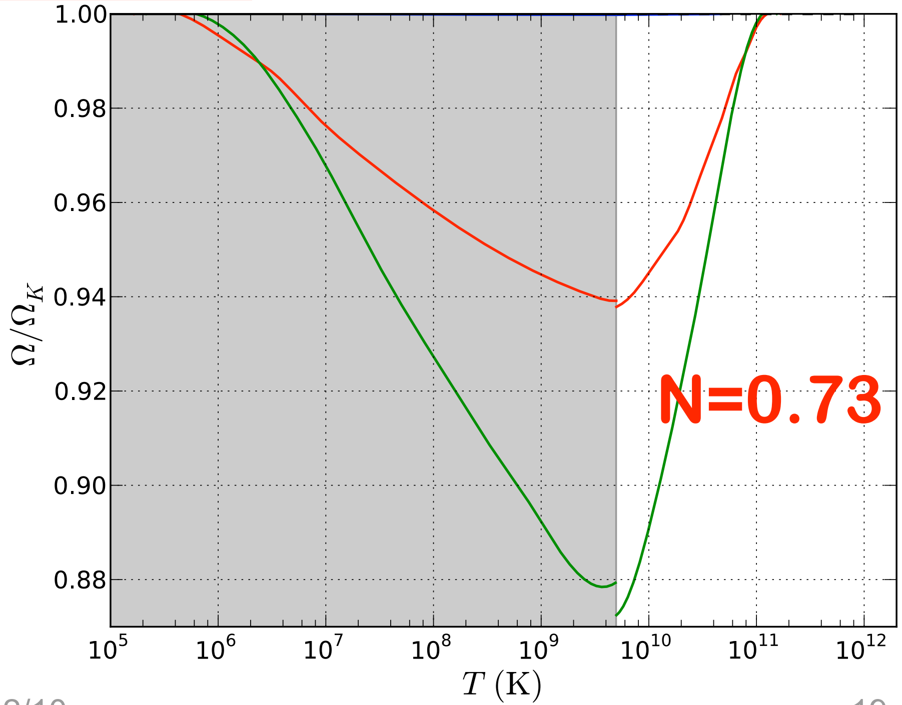
$$\frac{1}{\tau_{BV}} = \frac{1}{2E}\int \zeta\,\delta\sigma\,\delta\sigma^* d^3x$$

$$\frac{1}{\tau_{SV}} = \frac{1}{2E}\int \eta\,\delta\sigma^{ab}\,\delta\sigma_{ab}^* d^3x$$

$$\frac{1}{\tau_{GR}} = \frac{1}{\tau_{SV}} + \frac{1}{\tau_{BV}}$$

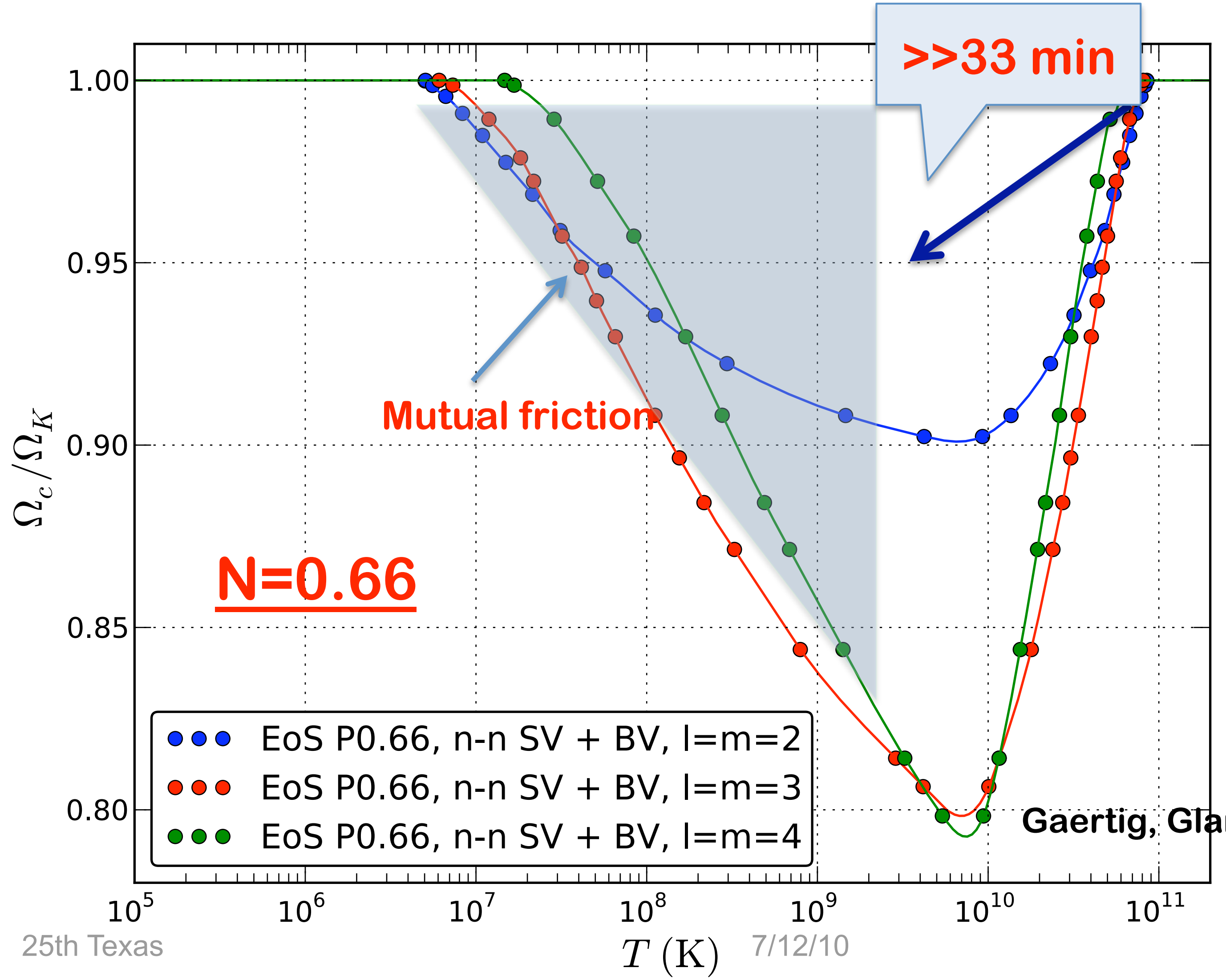
Gaertig-Glampedakis-KK-Zink 2010



# Instability Window

- ✓ For the first time we have the window of f-mode instability in GR
- ✓ Newtonian: (l=m=4) Ipser-Lindblom (1991)



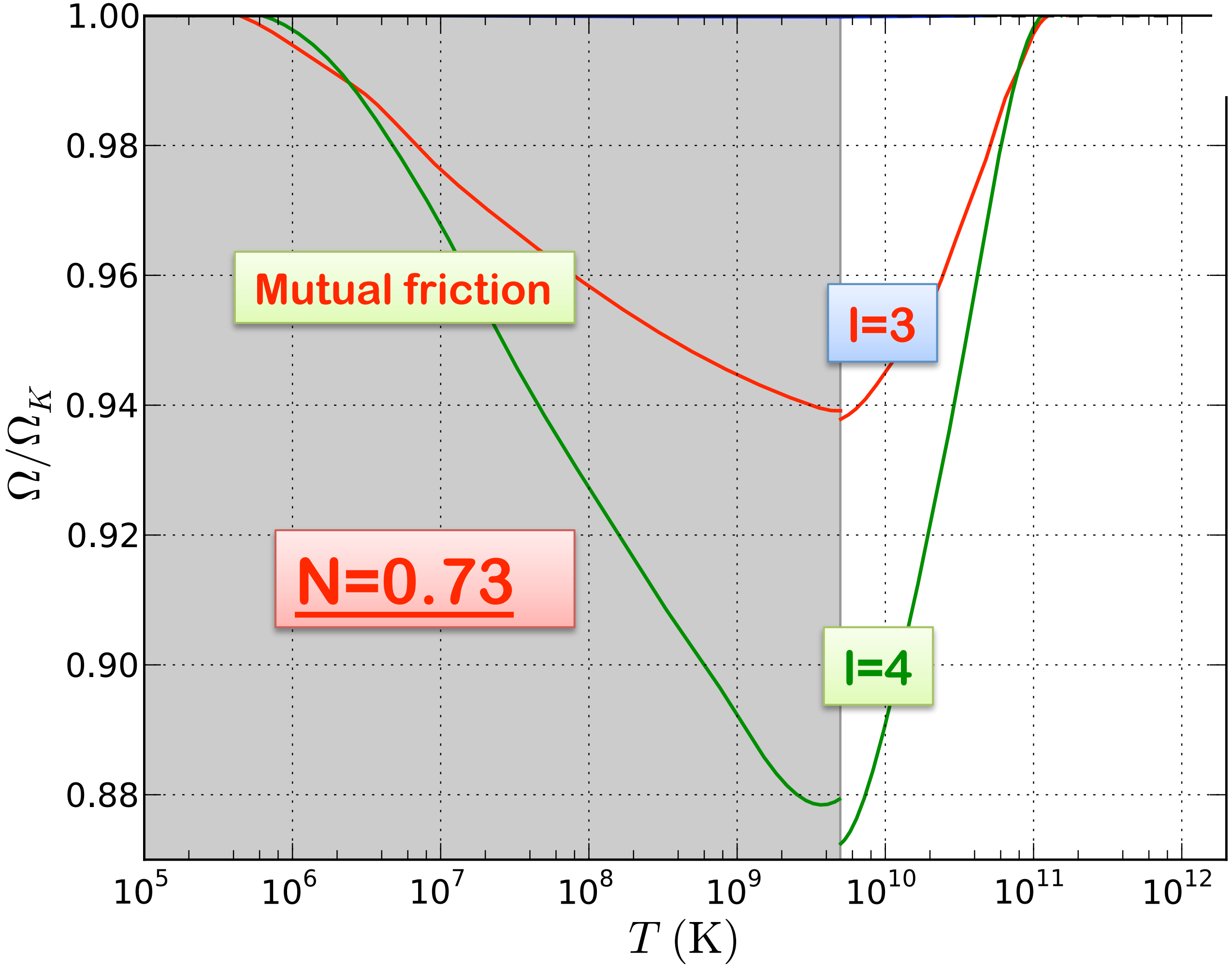

...typical Growth Time

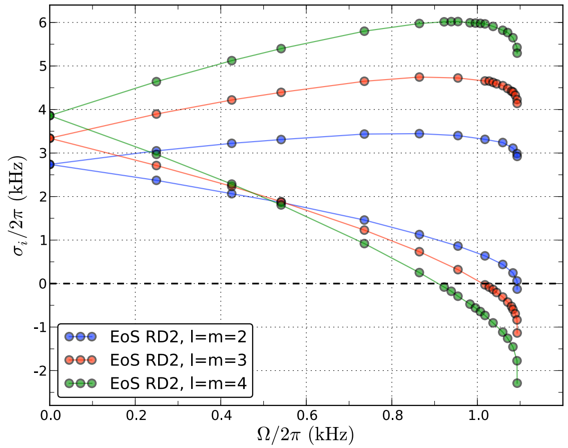
~38 sec l=2
~56 sec l=3
~264 sec l=4

Gaertig, Glampedakis, KK, Zink (2010)

# Instability Window

Gaertig, Glampedakis, KK, Zink (2010)



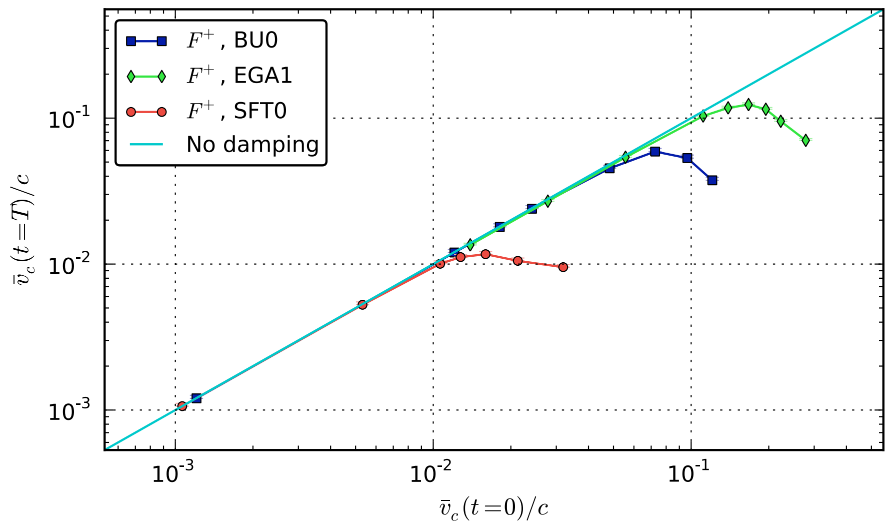

…typical growth times

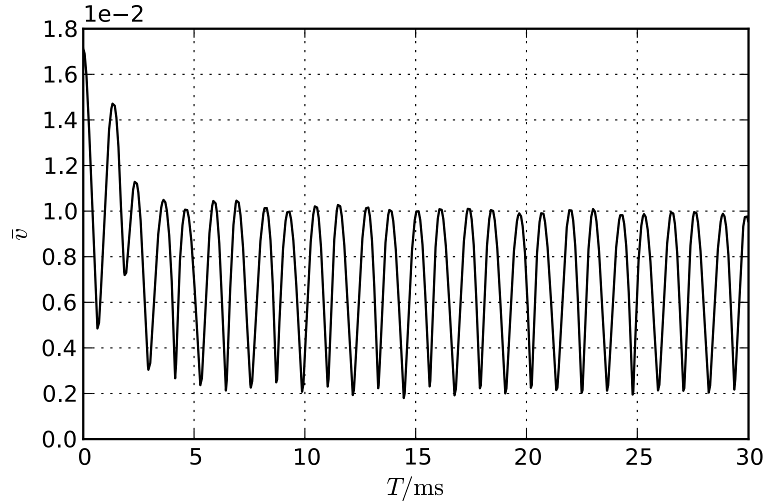
~47 h l=2
~30 min l=3
~40 min l=4

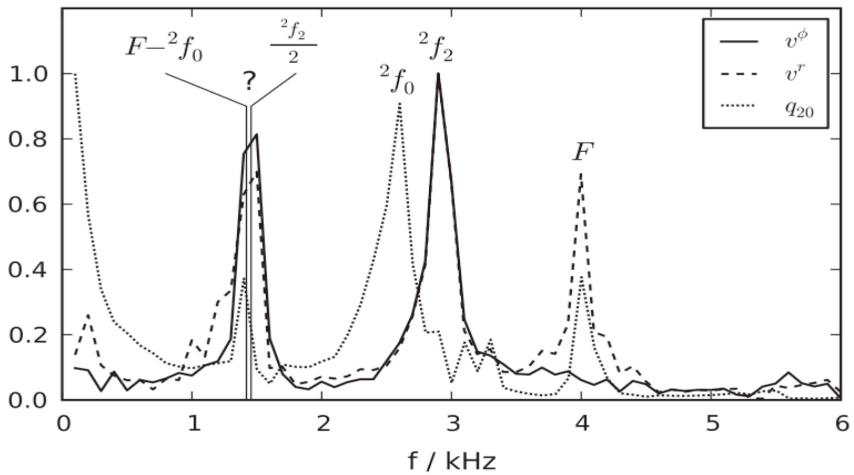
…typical cooling times

~30 min $T_c$~5x10$^9$K
~10 h $T_c$~3x10$^9$K
~5 d $T_c$~2x10$^9$K
~1 y $T_c$~1x10$^9$K

# Saturation Amplitudes



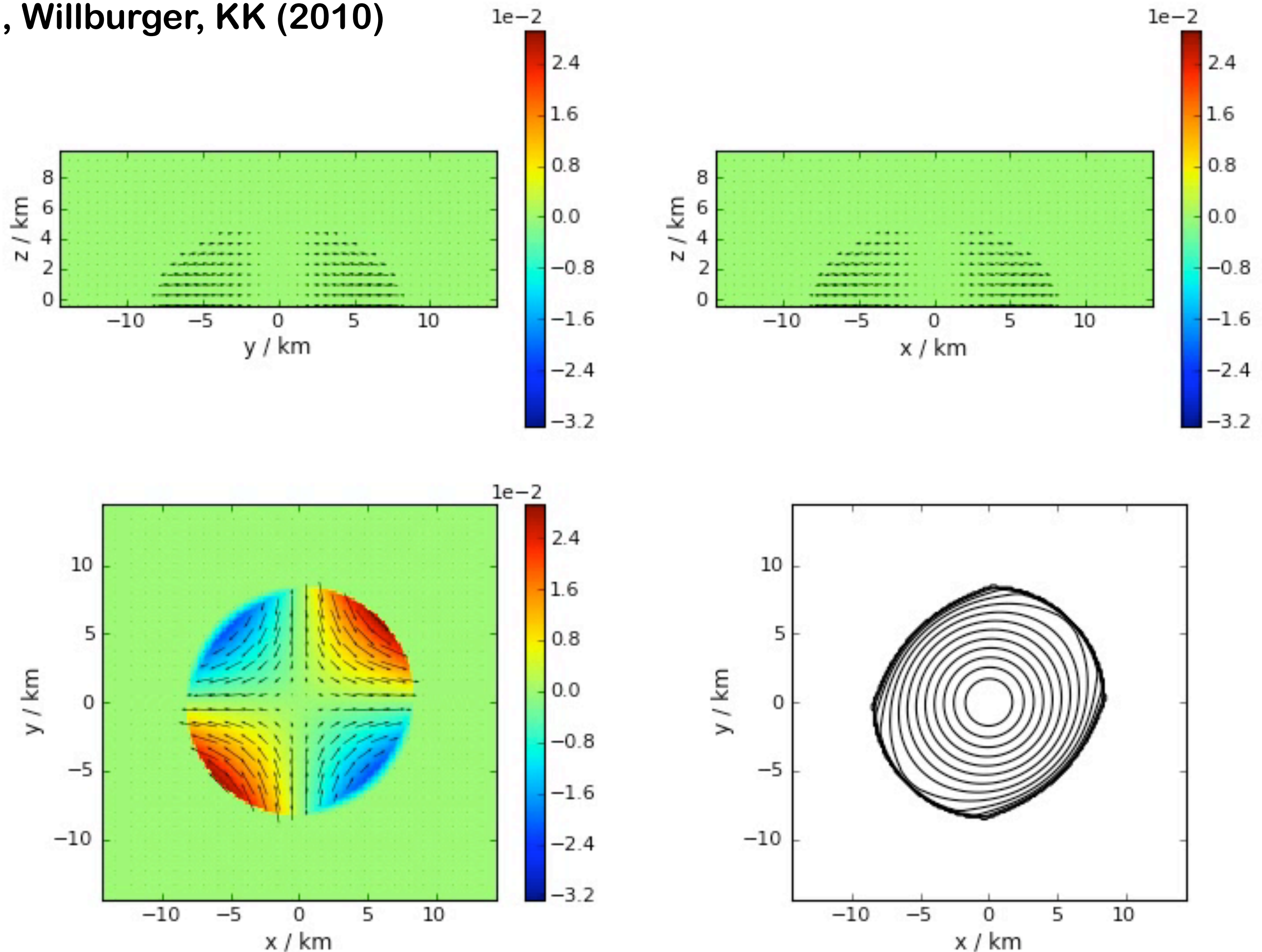




**Possible mode coupling with inertial modes**

**Kastaun, Willburger, KK (2010)**

# Animation of the l=m=2 f-mode

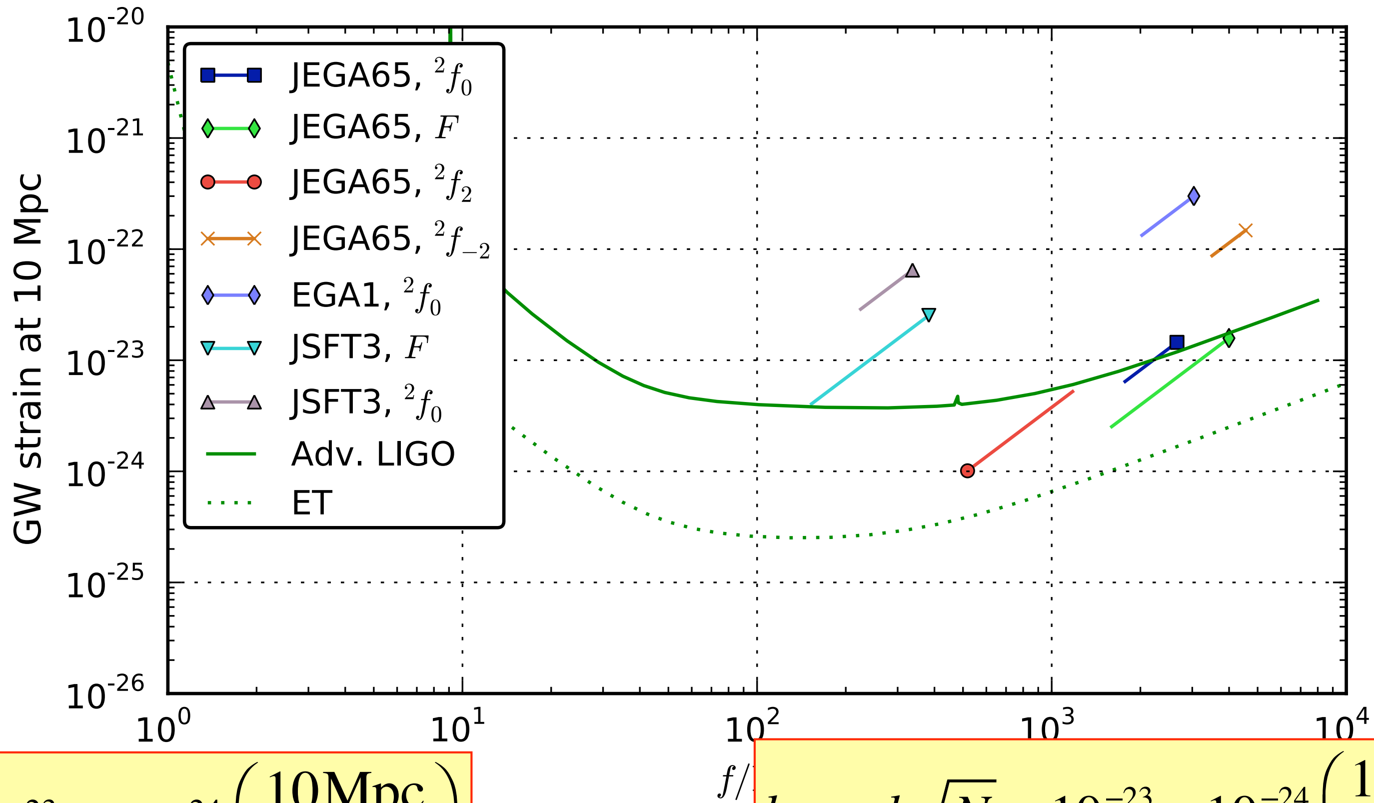
Kastaun, Willburger, KK (2010)




- ✓ **Quasi-Radial &Axisymmetric:** **damped due to shock formation**
- ✓ **Non-axisymmetric:** **damped due to wave breaking on the surface**

# Detectability (10Mpc)

Kastaun, Willburger, KK (2010)




$$h \approx 10^{-23} - 10^{-24}\left(\frac{10\text{Mpc}}{r}\right)$$

$$h_{\text{eff}} = h\sqrt{N} \approx 10^{-23} - 10^{-24}\left(\frac{10\text{Mpc}}{r}\right) \times \sqrt{N}$$

$$N \Rightarrow 10 - 10^{8}$$

**Efficiency depends on the value of the f-mode frequency in the inertial frame, thus it depends strongly on the stiffness of the EoS**

# Magnetars

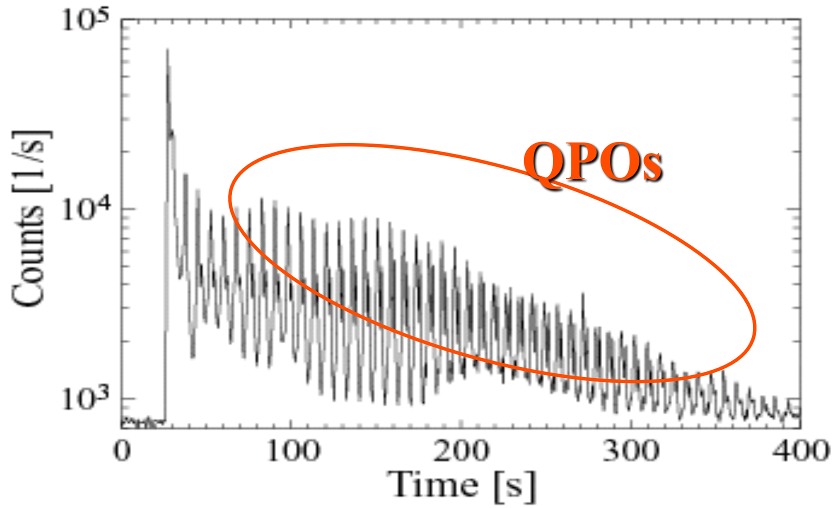
## $10^{15}$-$10^{16}$ Gauss !

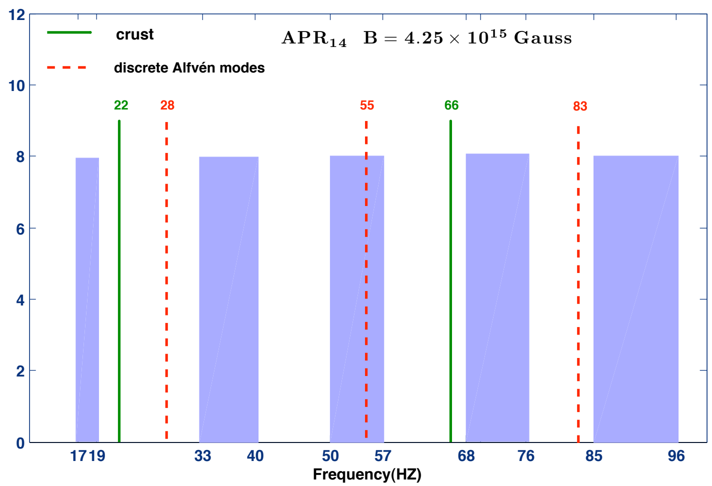
### Giant flares

- Up to now, 3 giant flares have been detected.
  - *SGR 0526-66 in 1979,*
  - *SGR 1900+14 in 1998,*
  - *SGR 1806-20 in 2004*
- **Peak luminosities :$10^{44} – 10^{46}$ erg/s**
- A decaying tail for several hundred seconds follows the flare.

### QPOs in decaying tail (Israel *et al.* 2005; Watts & Strohmayer 2005, 2006)

- **SGR 1900+14** : 28, 54, 84, & 155 Hz
- **SGR 1806-20** : **18, 26,** 29, 92.5, 150, 626.5, 720, 1837 & 2384 Hz
  - A few more : 17, 21, 36, 59, 116 Hz (Habaryan, Neuhauser, KK 2010)



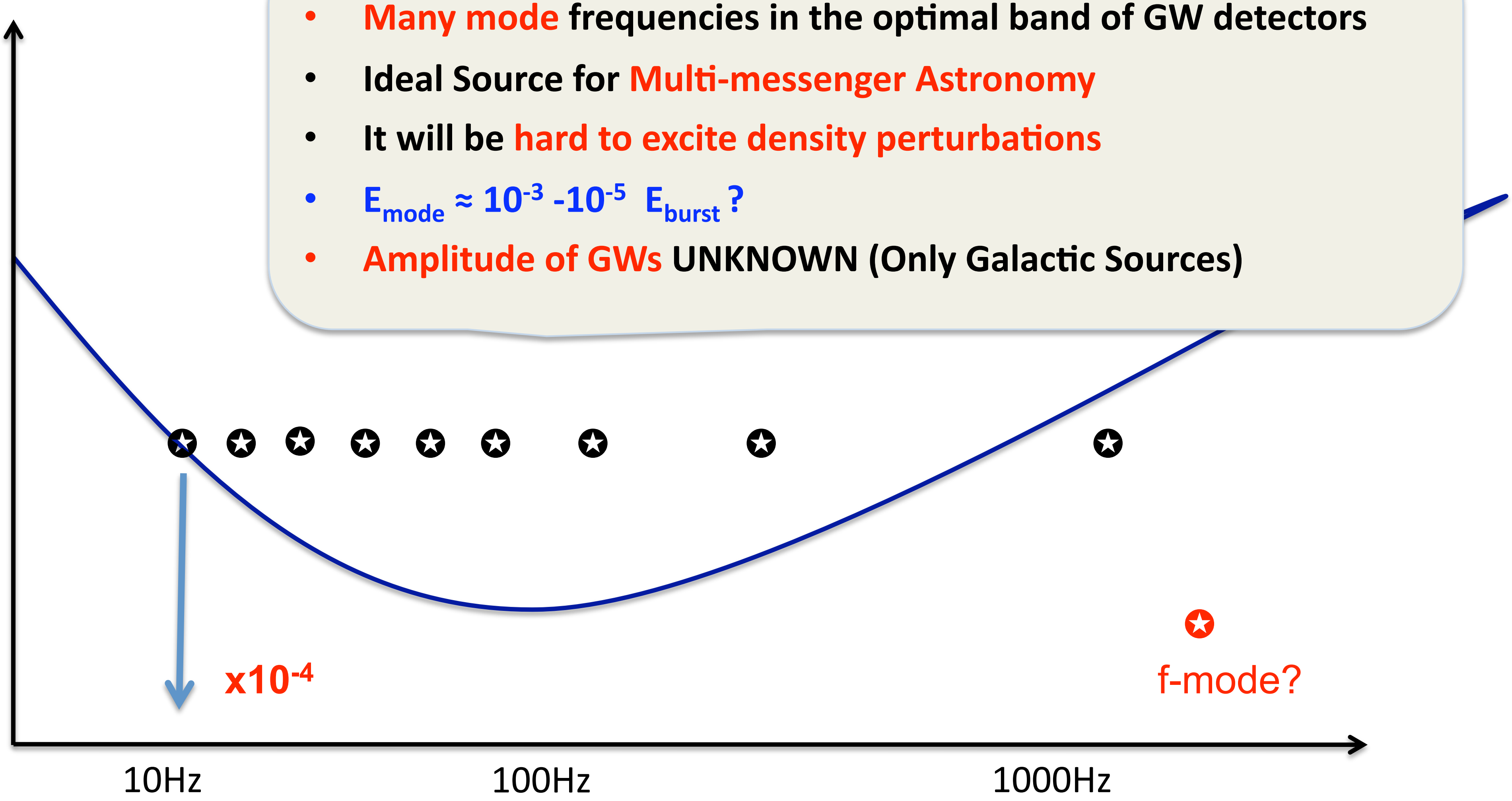

For **SGR 1806-20** (Colaiuda+KK '09,'10)

- We show that crust and Alfvén modes can explain all observed QPOs.
- The magnetar has EoS **APR,** mass **$1.4M_{\odot}$** and **11.6km** radius.

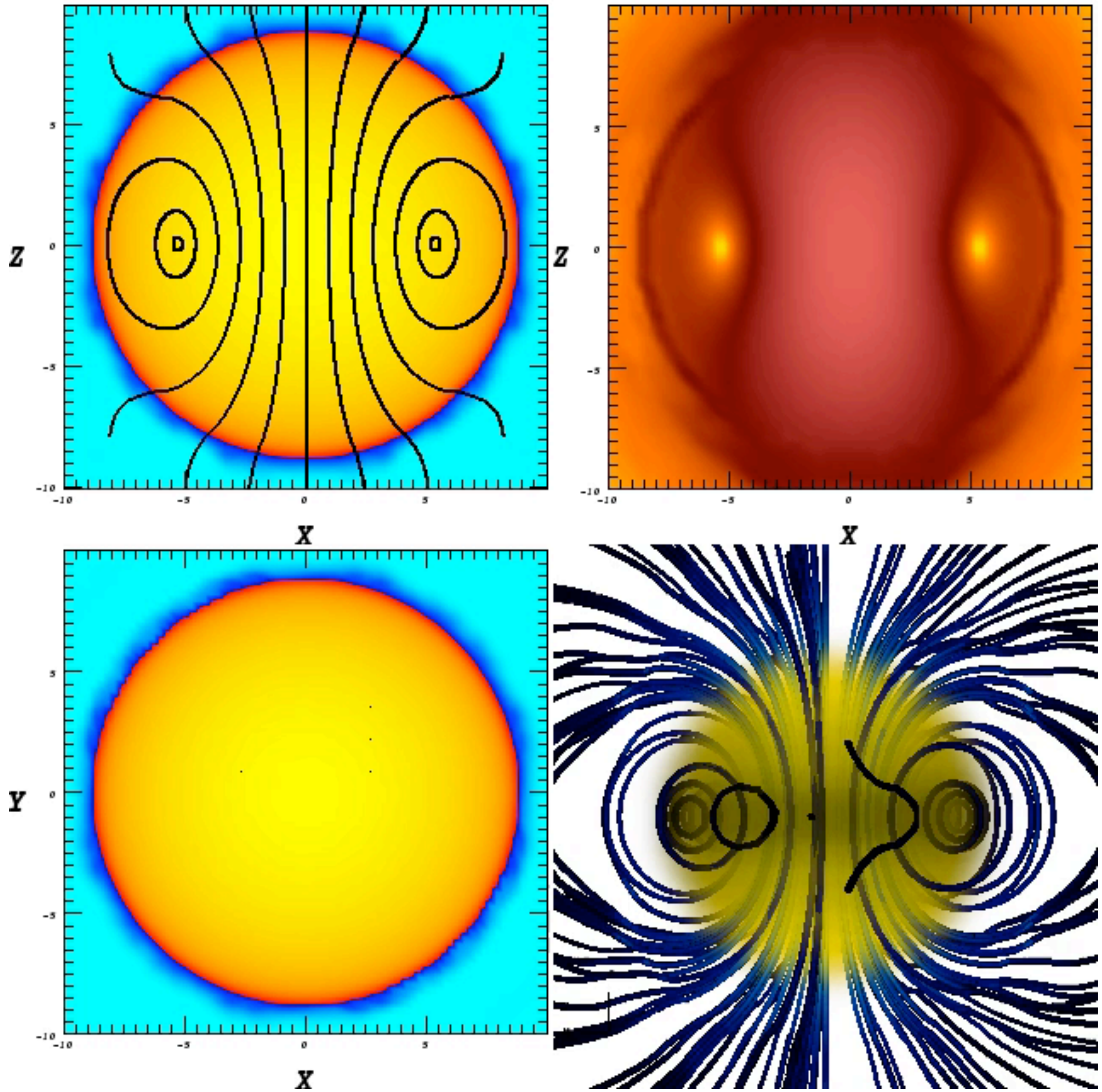
Sotani etal '07,`08,'09
Levin `07,`08,`10, Glampedakis etal '07
Cerda-Duran etal '09,'10, Samuelsson etal'07

# Magnetars & GWs

- Many mode frequencies in the optimal band of GW detectors
- Ideal Source for Multi-messenger Astronomy
- It will be hard to excite density perturbations
- $E_{mode} \approx 10^{-3}$ -$10^{-5}$ $E_{burst}$ ?
- Amplitude of GWs UNKNOWN (Only Galactic Sources)



# Magnetar Oscillations




**Full 3D GR-MHD code**

- ✓ Tayler Instability (~10ms growth time)
- ✓ Toroidal component for the B-field
- ✓ No GWs yet

**Simulation**

- ✓ Pure poloidal field
- ✓ Initial data from Lorene
- ✓ B~$10^{16}$ G
- ✓ Stable for a few hundred ms

Lasky, KK, Zink 2011

# Conclusions

- **Supernova Core Collapse**
  - ✓ The event rate may still not be overwhelmingly impressive,
  - ✓ It appears that different suggested supernova explosion mechanisms may lead to rather different GW signals.
  - ✓ The different emission mechanisms have quite characteristic signatures, so GW measurements would provide an unusually direct (probably the only besides neutrinos) way of probing the conditions inside core collapse supernovae
  - ✓ The investigation of the core collapse supernova mechanism with GWs absolutely requires a 3G detector to obtain meaningful statistics
- **Rotational Instabilities of Neutron Stars**
  - ✓ Are potential sources for GW beyond our galaxy
  - ✓ **Many open issues** (growth time, EoS, non-linear coupling,…) have already or soon will be resolved.
- **Magnetars**
  - ✓ Offers the possibility to understand their structure
  - ✓ Most probably a **weak source** for GW with the present generation detectors