Exercises for Experimental methods in Astroparticle Physics

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Sheet 9

1. **Decoupling temperature**

   Temperatures and densities in the early universe were so high, that photons were not able to propagate freely. Instead, they were "coupled" to matter via Compton scattering and other interactions. After the universe had expanded and cooled down sufficiently, photons were able to propagate freely. These photons from the early universe now make up the so-called cosmic microwave background (CMB). Fluctuations of the CMB spectrum across the sky provide a way to probe physics during the early stages of the universe. The temperature of the last scattering depends only weakly on cosmological parameters, but mainly on the ionization energy of hydrogen and on the baryon to photon ratio. The ionization energy of hydrogen is $B = 13.6\,\mathrm{eV}$. The ratio between the baryon number density and the photon number density is

   $$\eta = \frac{n_b}{n_\gamma} \approx 5 \times 10^{-10} \tag{1}$$

   The Saha's ionization equilibrium equation can be used to derive the recombination temperature:

   $$\frac{x_e^2}{1 - x_e} = \left(\frac{n_\gamma}{n_{tot}}\right) e^{-B/kT} \frac{1}{3.84} \left(\frac{m_e c^2}{kT}\right)^{3/2} \tag{2}$$

   n_{tot} is the total number of baryons defined as $n_{tot} = n_p + m_H$ and $x_e = n_p/n_{tot}$ is the ionization fraction. Therefore, the following equation holds

   $$\frac{n_p n_e}{n_H n_{tot}} = \frac{x_e^2}{1 - x_e}. \tag{3}$$

   Due to the exponential dominant factor the ionization drops quickly as kT drops below B. Where the sharp transition occurs depends on the density ratio of the baryon and photon densities. A low baryon density medium is easy to keep ionized with the high energy photons in the tail of the black body.

   (a) Consider that recombination occurs when $x_e = \frac{1}{2}$ and calculate the corresponding temperature.

   (b) Calculate the temperature of the universe at which photons decoupled from matter.

   (c) Use Wien's law to estimate the peak wavelength of the photon spectrum at the time of decoupling. Would the CMB back then have been visible to the human eye? If so: What color would it have had?

   (d) Derive the redshift parameter z at the time of recombination.

2. Intrinsic photon noise

In radio astronomy (frequency observation lower than the one in CMB) the following expression gives the fractional error on an observation:

$$\frac{\delta I}{I} = \left[\frac{T_{sky} + T_{sys}}{T_{sky}}\right]^2 \frac{1}{\sqrt{\Delta B t_{obs}}}, \tag{4}$$

where t_{obs} is the time of observation given in seconds,
δB is the acquired bandwidth given in Hertz,
T_{sky} is the sky brightness temperature given in Kelvin, and
T_{sys} is the additional noise introduced within the receiver given in Kelvin.

(a) Calculate how long a measurement should last in order to reach a fractional error of 10^{-5} for the detection of CMB in the frequency range $30\,\mathrm{GHz}$ – $35\,\mathrm{GHz}$ in case the intrinsic receiver noise has $T_{sys} = 100\,\mathrm{K}$.

The formula (4) is valid in the Rayleigh-Jeans part of the black body spectrum where the photon occupation number is larger than 1. This is the case for $T_{source} > h\nu/k_B$.

(b) For which frequencies formula (4) is valid?

3. WMAP and Lagrangian point

The WMAP experiment was launched in 2001 with destination the Lagrangian point L2. Lagrangian points are locations where the gravitational forces and the orbital motion of a body balance with each other. L2 is located at $1.5 \times 10^6\,\mathrm{km}$ behind the Earth with respect to the Sun. L2 is at about 4 times the distance Earth – Moon and orbits the Sun at the same rate as the Earth.

(a) List which could be the advantages to have experiments measuring CMB at L2.

(b) At which angle is the Earth seen by an experiment at L2?