Exercises for Experimental methods in Astroparticle Physics

Universität Heidelberg | 14. November 2019 | **Hand in:** 20. November 2019

Sheet 4

1. **Detector for X-Ray Astronomy**

   A proportional counter consists of a sealed volume filled with inert gas at high pressure. When X-rays enter through the thin window, they are absorbed in the gas volume, producing a photo-electron that then ionizes atoms in the gas. The resulting cloud of charged particles is accelerated towards a high voltage anode which amplifies the current to make it measurable. The current pulse is then proportional to the deposited energy of the X-ray photon.

   (a) Let us consider a $40\,\mu\mathrm{m}$ thick Be window. For what X-ray energy is this window transparent?

   (b) The gas inside the counter is Ar at a pressure of $3\,\mathrm{atm}$ and has a depth of $5\,\mathrm{cm}$. For which X-ray energy range is the counter opaque (at room temperature)?

   (c) What is the energy range at which the counter can be used?

   (d) Considering the mean Ar excitation energy $\epsilon = 26\,\mathrm{eV}$, estimate the energy resolution at $10\,\mathrm{keV}$.

   (e) Low temperature microcalorimeters based on transition-edge sensors which are developed for the X-IFU instrument for the future Athena mission feature an X-ray absorber consisting of a CuBi absorber with $1\,\mu\mathrm{m}$ Cu and $2.6\,\mu\mathrm{m}$ Bi. Estimate the quantum efficiency for $10\,\mathrm{keV}$ photons.

   (f) Wherein lies the advantage of using low temperature detectors?

   You can use `http://henke.lbl.gov/optical_constants/atten2.html` to find the attenuation length of the absorber materials.

2. **X-Ray Optics**

   Conventional optics does not work for X-rays due to their very short wavelengths. A typical method for focusing X-rays is an arrangement of confocal paraboloid and hyperboloid metal foils, a so-called Wolter telescope type I, as shown in Figure 1. Parallel beams coming from the front within a given angle range are totally reflected due to their grazing incidence on the metal surfaces. In the CHANDRA mission, the optics consist of 4 nested layers in order to increase the reflection surface (see Figure 2). In the relevant energy region, the dielectric constant of metals ϵ is given by the relation

$$\epsilon = 1 - \frac{\omega_\mathrm{p}^2}{\omega^2} \tag{1}$$

   where ω_p is the plasma frequency. For copper, ω_p is about $6 \times 10^{14}\,\mathrm{Hz}$. Discuss how the critical angle for total reflection on copper is connected to the energy of the incident X-rays. How large is the angle at $2\,\mathrm{keV}$?

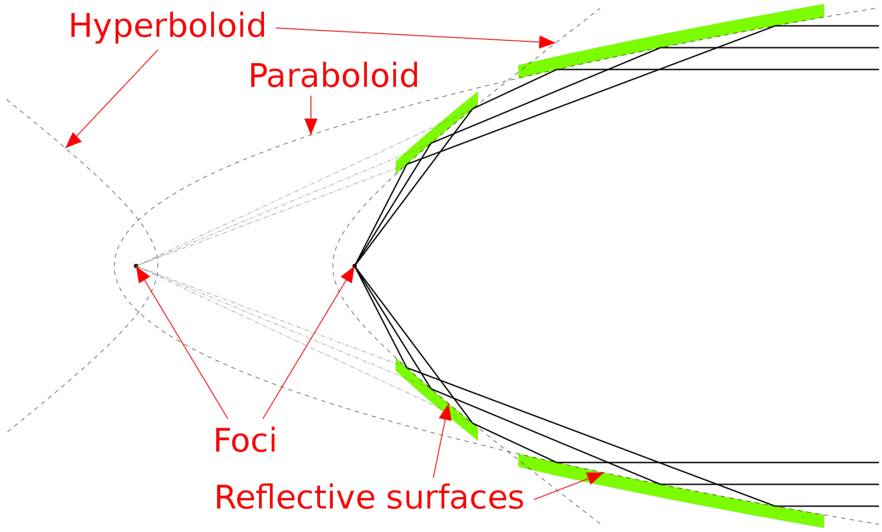

Figure 1: Wolter telescope of type I,
from `https://en.wikipedia.org/wiki/Wolter_telescope`

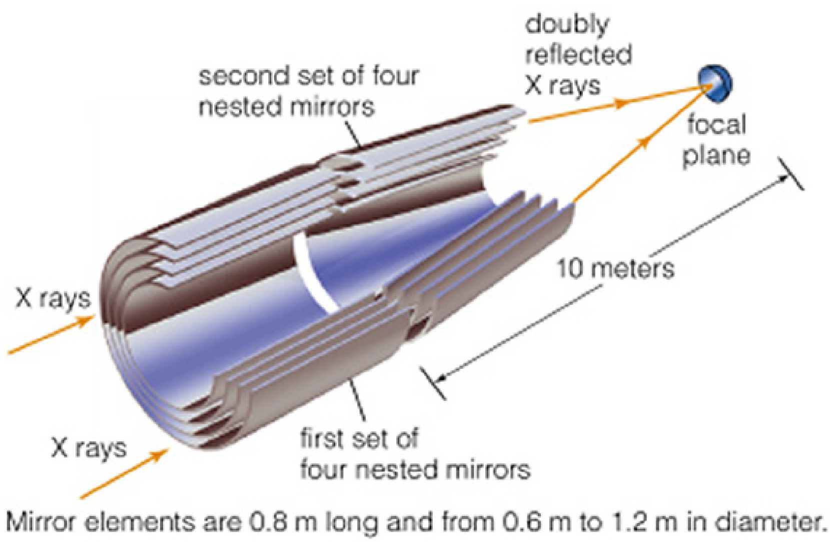

Figure 2: Wolter Telescope aboard the CHANDRA satellite,
from `https://www.physast.uga.edu/~rls/astro1020/ch7/ovhd.html`

3. Elemental Composition of Cosmic Rays

As seen during the lecture, if we compare the composition of the cosmic rays to the one of the solar system, we recognize that the abundances deviate for several elements. In the case we want to discuss, Li, Be, and B are more abundant in the cosmic rays because these are produced in spallation process of C, N, and O. Several different experiments measure the abundances of primary elements (C, N, O) and of secondary elements (Li, Be, B). The ratio of these abundances can be used to gain information on the escape time, the time scale on which cosmic rays diffuse in the galactic disc. Given n_{p} and n_{s} as the number density of primaries and secondaries, one can write a system of 2 coupled differential equations for the variation of these densities:

$$\frac{dn_{\mathrm{p}}}{dl} = -\frac{n_{\mathrm{p}}}{\lambda_{\mathrm{p}}} \tag{2a}$$

$$\frac{dn_{\mathrm{s}}}{dl} = -\frac{n_{\mathrm{s}}}{\lambda_{\mathrm{s}}} + \frac{p_{\mathrm{sp}} n_{\mathrm{p}}}{\lambda_{\mathrm{p}}} \tag{2b}$$

where $l = \int \rho\,(x)\,dx$ is the traversed column density that measures the amount of traversed matter, $\lambda = \frac{m}{\sigma}$ is the interaction length, and $p_{\mathrm{sp}} = \frac{\sigma_{\mathrm{sp}}}{\sigma_{\mathrm{tot}}}$ is the spallation probability.

(a) Solve the system for $m_s\,(0) = 0$ and find an expression for the abundance ratio $\frac{n_{\mathrm{s}}}{n_{\mathrm{p}}}$.

(b) Given $\lambda_{\mathrm{CNO}} = 6.7\,\mathrm{g\,cm^{-2}}$, $\lambda_{\mathrm{LiBeB}} = 10\,\mathrm{g\,cm^{-2}}$, $p_{\mathrm{sp}} = 0.35$ and an experimentally observed abundance ratio of $\frac{n_{\mathrm{s}}}{n_{\mathrm{p}}} = 0.25$, calculate the traversed column density l.

(c) Assuming a galactic disc with a thickness of $d = 300\,\mathrm{pc}$ composed of H with a density of $n_{\mathrm{H}} = 1\,\mathrm{cm^{-3}}$, calculate the column density of the disc via $l_{\mathrm{MW}} = m_{\mathrm{H}} n_{\mathrm{H}} d$. Then, estimate the escape time of the cosmic rays. To facilitate your calculations, assume that the cosmic rays travel at the speed of light.

4. East-West Effect

Read the initial paper on the East-West effect by B. Rossi:
`https://journals.aps.org/pr/pdf/10.1103/PhysRev.36.606`.

(a) The paper discusses the effects for cosmic electron radiation. How is the situation for protons?

(b) A proton moving towards the Earth from the east side with trajectory on the earth equatorial plane experiences a force due to the Earth magnetic field. Calculate the minimal proton energy so that the particle can reach the Earth surface (neglect the fact the the proton would interact with the atmosphere). Consider the magnetic field of the Earth as the field of a dipole with a magnetic moment $M = 8 \times 10^{22}\,\mathrm{A\,m}$. The Earth radius is $R = 6371\,\mathrm{km}$.

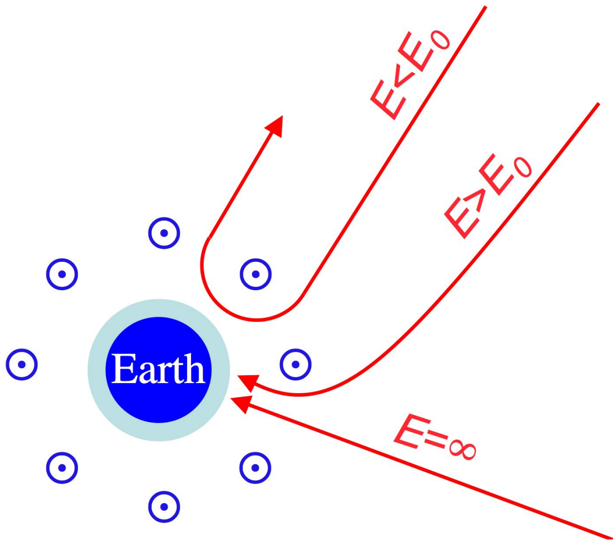

Figure 3: East-west effect for positively charged particles at different energies, *from* `http://www.nmdb.eu/public_outreach/en/03/`