Exercises for Experimental methods in Astroparticle Physics

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Sheet 3

1. **Luminosity of Sco X-1**

   In 1962, Riccardo Giacconi and his team designed and launched an experiment with the goal of detecting the X-ray fluorescence emission from the Moon surface due to solar X-rays. The scientific payload consisted of a couple of Geiger counters as detectors. The experiment scanned the sky for 5 minutes and 50 seconds, corresponding to the time the rocket was more than 80 km above the Earth's surface. Surprisingly, the main X-ray source was not the Moon, but instead originated from the constellation of Scorpius. This source is labeled Sco X-1. Today, we known it is a stellar binary system in our galaxy.

   Scorpius X-1 delivers roughly $10^6\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}$ x-ray photons with an average energy of $5\,\mathrm{keV}$ to the Earth. Given the distance of the binary system to the Earth to be $D = 2800\,\mathrm{pc}$, calculate the luminosity (emitted power) in terms of the sun luminosity.

2. **Synchrotron Emission I**

   The power radiated by a single electron through synchrotron emission is given by:

   $$\frac{dE}{dt} = \frac{4}{3}\sigma_{\mathrm{T}} c \gamma^2 U_{\mathrm{B}} \tag{1}$$

   An electron is losing energy via synchrotron radiation in a constant magnetic field B. Write a differential equation for γ as a function of time for an initial $\gamma\,(t = 0) = \gamma_0$. Solve the equation and find an expression for how long it takes for the electron to lose half its energy.

3. **Synchrotron Emission II**

   The Crab Nebula has an average magnetic field of $B = 5 \times 10^{-4}$ G. Estimate the gyroradii of electrons in the Crab Nebula that emit synchrotron radiation in the radio frequency ($10^8\,\mathrm{Hz}$) and in the gamma-ray range ($10^{22}\,\mathrm{Hz}$). Are these gyroradii smaller than the radius of the nebula ($5 \times 10^{18}\,\mathrm{cm}$)?

4. Black Body Emission

Consider a plasma sphere of radius R with mass M composed of ionized hydrogen (electrons and protons). The plasma is optically thick and has a uniform temperature of $T = 10^7$ K. In the following, solar parameters are denoted by $\odot$.

(a) Find an expression for the energy of this system. Make use of the fact that #electrons = #protons = $\frac{M}{m_\mathrm{p}}$.

(b) Define the surface luminosity (power emitted by the total surface) due to black body radiation. Demonstrate that this can be written as

$$L = 3.5 \times 10^{39}\,\mathrm{W\,s^{-1}} \times \left(\frac{R}{R_\odot}\right)^2 \times \left(\frac{T}{10^7\,\mathrm{K}}\right)^3 . \tag{2}$$

(c) Determine the cooling time and demonstrate that this can be written as

$$\tau = 140\,\mathrm{s} \times \frac{M}{M_\odot} \times \left(\frac{R}{R_\odot}\right)^{-2} \times \left(\frac{T}{10^7\,\mathrm{K}}\right)^{-3} . \tag{3}$$

(d) Let us consider a system with a mass equal to the solar mass and a temperature of 10^7 K. Calculate the luminosity and the cooling time for systems with:

i. $R = R_\odot$

ii. $R = 3 \times 10^{-3}\,\mathrm{R}_\odot$

iii. $R = 10^{-5}\,\mathrm{R}_\odot$