Dark Matter (WS 2018/19) - Problem sheet 9

Tutorials: Florian Jörg (florian.joerg@mpi-hd.mpg.de)

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Sterile neutrinos as dark matter

Scott Dodelson and Lawrence M. Widrow proposed in 1993 (accessible on arXiv:hep-ph/9303287) that dark matter could be explained by sterile neutrinos. They argue: "By far the simplest dark matter candidate, at least from the point of view of particle physics is the neutrino. Massive neutrinos require only the addition of right-handed or sterile neutrino fields to the standard model". At tree level, the most efficient way to produce sterile neutrinos is via oscillations: $\nu_L \to \nu_R$. The main task of this problem sheet is to go through a calculation of oscillation probabilities, assuming only one generation of neutrinos for reasons of simplicity.

9.1 Neutrinos in general *4 Points*

Answer the following questions:

a) What is the chirality of a particle?

b) What is the difference between Majorana and Dirac particles? Which are the possible consequences if neutrinos have Majorana mass terms?

c) What is the difference between neutrino mass eigenstates and flavor eigenstates?

d) Why are sterile neutrino called "sterile"?

9.2 Oscillation probability *2 Points*

The time evolution of a neutrino state in the mass basis, which at $t = 0$ is in the flavor state $|\nu_a\rangle$ for some flavor a, can be written as a plane wave:

$$|\nu_a(t)\rangle = \sum_j U^*_{a\to j}\, e^{-iE_j t}\, |\nu_j\rangle \tag{1}$$

with $U_{a\to j}$ being the mixing matrix, E_j the energy of the mass eigenstate j and t the time. The oscillation probability P is given by the absolute square of the transition amplitude A:

$$P(\nu_a \to \nu_b;\, t) = |A(\nu_a \to \nu_b;\, t)|^2 = |\langle \nu_b|\nu_a(t)\rangle|^2, \tag{2}$$

with a and b denoting two different flavors. Compute a general expression for P.

9.3 Oscillation of active to sterile neutrinos *3 Points*

If you consider only a left and right-handed neutrino, then the mixing matrix can be parametrized as:

$$\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} = \begin{pmatrix} \cos(\theta_0) & \sin(\theta_0) \\ -\sin(\theta_0) & \cos(\theta_0) \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}. \tag{3}$$

Approximate the mass eigenstate energies as $E_i \approx p + \frac{m_i^2}{2p}$ (ultrarelativistic limit, equal momenta) and calculate $P(\nu_L \to \nu_R;\, t)$. You should arrive, after having used appropriate trigonometric identities, at:

$$P(\nu_L \to \nu_R;\, t) = \sin^2(2\theta_0)\sin^2\left(\frac{\Delta m^2}{4p}t\right), \tag{4}$$

with $\Delta m^2 = m_2^2 - m_1^2$.

9.4 Detecting sterile neutrinos *1 Points*

How could sterile neutrinos be detected?