Dark Matter (WS 2018/19) - Problem sheet 5

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WIMP Dark Matter

A thermal relic is a dark matter candidate which in the early phases of the universe was in thermal equilibrium with the Standard Model particles. Weakly Interacting Massive Particles (WIMPs) are thermal relic dark matter candidates with masses of the order of $(m_{DM} \sim \mathcal{O}(100)\,\mathrm{GeV})$ and couplings typical for electroweak physics.

5.1 The "WIMP miracle" *6 Points*

The fact that the observed relic density

$$\left(\frac{\Omega_{DM}}{0.2}\right) \approx \frac{10^{-8}\,\mathrm{GeV}^{-2}}{\sigma} \tag{1}$$

can be easily explained by a weak interaction is referred to as the *WIMP miracle*.

a) Dark matter is generally believed to be cold, meaning that the temperature at which it thermally decoupled from the Standard Model particles is much lower than its mass. In this case, the dark matter number density η_{DM} is given by

$$\eta_{DM} \propto (m_{DM}T)^{3/2} \exp\left(-\frac{m_{DM}}{T}\right) \qquad \text{for} \quad m_{DM}/T \gg 1. \tag{2}$$

When the dark matter interaction rate Γ becomes comparable to the Hubble expansion rate H, dark matter freezes out. In terms of the number density, one can write $\Gamma = \eta_{DM} \cdot \sigma$ with σ being an interaction cross section. In a radiation dominated Universe, the Friedmann equation gives $H \propto T^2/M_{Pl}$ with the Planck Scale $M_{Pl} = 10^{19}\,\mathrm{GeV/c^2}$. Use the freeze-out condition $\Gamma = H$ and show that this implies $m_{DM} \cdot M_{Pl} \cdot \sigma > 1$.

b) Use the result from part a) and the cross section suggested by the relic density (eq. 1) $\sigma = 10^{-8}\,\mathrm{GeV}^{-2}$ to derive a lower bound on the dark matter mass.

c) Unitarity constraints provide an upper bound on the annihilation cross section. For a velocity of $v/c = 0.3$, one has

$$\sigma \lesssim \frac{4\pi}{m_{DM}^2 v^2}. \tag{3}$$

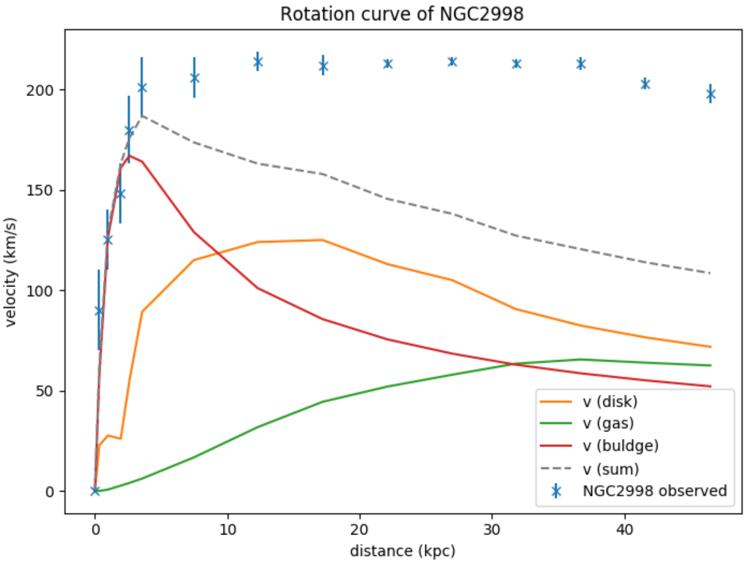
Use this constraint to derive an upper bound on the dark matter mass.

d) The currently best limit from direct detection states that $\sigma < 4.1 \times 10^{-47}\,\mathrm{cm}^2$ for a dark matter mass of $30\,\mathrm{GeV}/\mathrm{c}^2$. How does the cross section from eq. 1 compare to this result and what does this imply for the so called *miracle*? (*You can use the following conversion from natural units to SI units:* $1\,\mathrm{cm} \mathrel{\hat{=}} 5.07 \times 10^{13}\,\mathrm{GeV}^{-1}$)

Dark matter distribution

5.2 Revisiting NGC2998 *4 Points*

The below plot shows the rotation curve of the spiral galaxy NGC2998 that we already encountered on exercise sheet 2. This time the expected rotation curves from the different components of the galaxy are drawn as well.



a) Explain qualitatively the shape of the different contributions.

b) What is the disk-halo conspiracy? What is the difference between the Einasto, NFW and Burkert profile? Explain the difference between cusped and cored profiles.

5.3 Dark matter in the solar system *2 Bonus Points*

We have observed the significant effect of dark matter on the rotation velocity of stars in galaxies. From the previous exercise we know, that dark matter is also present within our solar system. Why is there no impact from this dark matter on the movement of the planets and on our space probes? (*Hint: Take the Earth as an example and compare the gravitational force due to dark matter with the one that is exerted by the Sun. You can also assume that the dark matter density in the solar system is* $\rho_{DM} = 6 \times 10^{-28}\,\mathrm{kg}/\mathrm{cm}^3$.)