Dark Matter (WS 2018/19) - Problem sheet 2

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Deadline for this sheet: 07.11.2018

Gravitational lensing

General relativity predicts light to be deflected by the presence of massive objects which can act as lenses. Estimating the frequency of how often such a lens passes the line of sight between a light source and an observer allows to extract information about the distribution of massive compact halo objects (MACHOs) in our galaxy.

2.1 Einstein rings *3 Points*

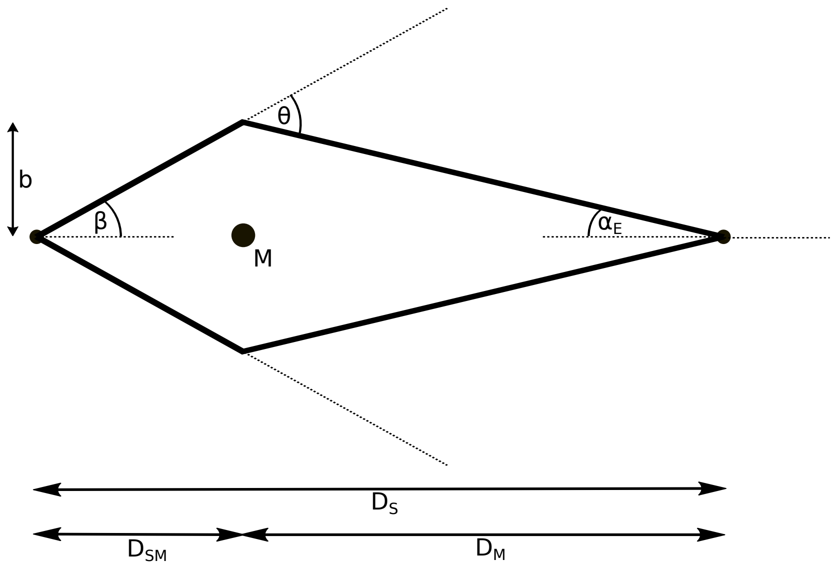

Figure 1: Sketch for task 2.1

In the case where a massive object is directly between a light source and an observer, light from the source is deflected such, that the observer will see a luminous ring, called *Einstein ring*, around the massive object. Given figure 1 and the equation for the deflection angle θ:

$$\theta = \frac{4GM}{bc^2}, \tag{1}$$

Where G is the gravitational constant and c is the speed of light. Derive an expression for the opening angle α_E of the Einstein ring which only depends on the objects mass M as well as the distances D_{SM}, D_M and D_S as given in figure 1. Assume α_E and β to be small in order to approximate trigonometric functions *(Hint: $\alpha_E \propto \sqrt{M}$ in the final result)*.

2.2 Microlensing *2 Points*

While Einstein rings caused by MACHOs are usually too small to be resolved in telescope observations, one can still measure an apparent amplification of a light source if such an object passes the line of sight between source and observer, acting as a lens. If the object moves at a velocity v perpendicular to the line of sight, the typical time within which lensing happens is given by:

$$T = \frac{D_M \alpha_E}{v} \tag{2}$$

Calculate T for an object with $M = 0.2M_\odot$, $v = 200km/s$ which lies halfway between an observer and a light source at a distance of $D_s = 60kpc$ to the observer *(Hint: use a_E from the previous exercise).*

The cosmic microwave background

Temperatures and densities in the early universe were so high, that photons were not able to propagate freely. Instead, they were coupled to matter via Compton scattering and other interactions. After the universe had expanded and cooled down sufficiently, photons were able to propagate freely. These photons from the early universe now make up the so-called cosmic microwave background (CMB). Fluctuations of the CMB spectrum across the sky provide a way to probe physics during the early stages of the universe.

2.3 General properties *3 Points*

a) The temperature of the universe at which photons decoupled from matter is estimated to be $T_{dec} \approx 3000K$. Use Wiens law to estimate the peak wavelength of the photon spectrum at the time of decoupling. Would the CMB back then have been visible to the human eye? If so: What color would it have had?

b) The average temperature of the CMB measured today is $T \approx 2.73K$. What is the corresponding peak wavelength?

c) Assuming the past evolution of the universe to be matter-dominated yields for the time evolution of the scale factor $a(t) \propto t^{2/3}$. Use the fact, that the energy density of a photon gas ρ_r is proportional to T^4 while it also depends on the scale factor as $\rho_r \propto a^{-4}$ to give an estimate for the time of decoupling (in years after the Big Bang). You can assume the age of the universe being 13.8 billion years.

2.4 Parameters *2 Points*

Go to `https://lambda.gsfc.nasa.gov/education/cmb_plotter/` and play around with the CMB plotter. Observe, how the power spectrum changes when modifying the sliders, understand the meaning of the parameters and try to find the best fit values without clicking on the answer button. Why does the flatness of the universe inferred from the fit, which is also shown in the plotter, only depend on the first three parameters, but not on todays value of the Hubble constant, the re-ionization redshift or the spectral index?

Is Dark Matter the only explanation?

Modified Newtonian Dynamics (MOND) has been suggested to explain the rotation curves of disk galaxies without the need for dark matter.

2.5 Modified Newtonian Dynamics *5 Points*

NGC2998 is a spiral galaxy in Ursa Major. You can download its rotation curve data from `http://astroweb.case.edu/ssm/620f03/n2998.dat` (by Stacy McGaugh). The first column gives the radius, the second the observed/inferred circular velocity with its 1σ uncertainty in the third column. The next few columns provide rotation curves that would arise from the stellar disk, gaseous disk, and the bulge alone.

a) Plot the observed circular velocity as a function of the radius (use e.g. ROOT or Python). The plot should also contain the uncertainties on the circular velocity. If you do not have access to a tool for plotting, you can also draw the curve by hand.

b) MOND postulates that:

$$\mu\left(\frac{a}{a_0}\right) a = \frac{MG}{R^2}, \qquad \text{with} \begin{cases} \mu\left(\frac{a}{a_0}\right) = \frac{a}{a_0}, & a \ll a_0 \\ \mu\left(\frac{a}{a_0}\right) = 1, & a \gg a_0 \end{cases} \tag{3}$$

where a is the gravitational acceleration, a_0 is a constant introduced by MOND and μ is an interpolation function. One possible way to test MOND is to check to which extent kinematic data from different systems really can be fitted with the same value of a_0. Use the rotation curve data of NGC2998 to test if this object can be reconciled with $a_0 \approx 1.2 \cdot 10^{-10} ms^{-2}$, which is the value for a_0 suggested by most previous attempts to fit galaxy data with MOND. Treating the galaxy as a point-mass is OK as long as you use the data in a way that minimizes the errors introduced by this approximation (Use: $M_{visible} = 24.4 \cdot 10^{40} kg$, $G = 6.67 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$). Please visualize the expectation of MOND in the plot of the rotation curve.

c) Look up the *Pioneer anomaly*. Explain its significance in the context of MOND. Try to find possible other explanations for this observation.