# Dark Matter (WS 2018/19) - Problem sheet 1

**Tutorials**: Florian Jörg (florian.joerg@mpi-hd.mpg.de)

**Deadline for this sheet**: 24.10.2018

## Dark matter in galaxies and the Coma cluster

### 1.1 The virial theorem *4 Points*

The first indication for dark matter is given in a paper written by F. Zwicky in 1933 [1,2]. He observed the radial velocities of galaxies in the Coma cluster and used the virial theorem to calculate its mass. The state of the system is often quoted in terms of the virial ratio:

$$Q_{vir} = \frac{\langle\, T \,\rangle}{-\langle\, \Omega \,\rangle}. \tag{1}$$

A system which is in virial equilibrium has $Q_{vir} = 0.5$, with $\Omega$ being the potential energy and T the kinetic energy.

**a)** Derive an expression for the total mass of a gravitationally bound system as a function of the dispersion velocity $\langle v^2 \rangle$.

**b)** Zwicky measured that the diameter of the Coma cluster is around $2 \cdot 10^{22} m$. He observed a velocity dispersion of about $\langle v^2 \rangle^{1/2} = 2000 \frac{km}{s}$. Assume that the Coma cluster contains approximately 800 galaxies with an average luminosity of $10^9$ solar luminosities ($L_\odot$) per galaxy. Calculate the mass to light ratio $\Upsilon = \frac{M_\odot}{L_\odot}$ for the Coma cluster.

**c)** Give a short interpretation of this result.

### 1.2 Rotation curves: Prediction vs. measurement *4 Points*

Measurements of galactic rotation curves imply an apparent mass deficit in galaxies (see publications by V. Rubin et al. from the 1970s/80s [3]). Under the assumption of spherical symmetry of a rotating galaxy one can calculate the mass inside a sphere of a given radius from the circular velocity of the stars at its surface and compare it to an estimate from visible stars.

**a)** Give a formula which expresses the circular velocity in terms of the enclosed mass and the distance to the galactic center.

[1] ZWICKY, Fritz. Die rotverschiebung von extragalaktischen nebeln. Helvetica Physica Acta, 1933, 6. Jg., S. 110-127.

[2] ANDERNACH, Heinz; ZWICKY, Fritz. English and Spanish Translation of Zwicky's (1933) The Redshift of Extragalactic Nebulae. arXiv preprint arXiv:1711.01693, 2017.(English translation)

[3] RUBIN, Vera C.; FORD JR, W. Kent. Rotation of the Andromeda nebula from a spectroscopic survey of emission regions. The Astrophysical Journal, 1970, 159. Jg., S. 379.

**b)** First let us consider the case that most of the mass of the galaxy is located in a central region extending up to a radius $r_0$. For simplicity we assume that the mass density is constant there ($\rho_0$) and vanishes outside of $r_0$. Draw a sketch of how the rotation curve looks like inside and outside of $r_0$.

**c)** Now consider a more realistic distribution in the form of:

$$\rho(r) = \frac{\rho_0}{(1 + r/r_0)^\alpha}. \tag{2}$$

Determine a value for $\alpha$ which gives a flat rotation curve at $r \gg r_0$ , as indicated by measurements, and derive the rotation curve $v(r)$ for this value.

### 1.3 Estimation of dark matter *2 Points*

At $r = 10^5$ light years a star velocity measurement yields $v_{meas} = 225 km/s$. The expected velocity as it is calculated based on the visible mass of this galaxy is $v_{calc} = 15 km/s$. Calculate the visible as well as the true galaxy mass. What is the percentage of dark matter in the galaxy? How large is the average dark matter mass density? (Use: $G = 6.67 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$).