Exp. Methods in Astroparticle Physics (SS 2020) - Problem sheet 2

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Explaining observations: Is Dark Matter the only solution?

2.1 Microlensing: Detecting MACHOs *2 Points*

Massive astrophysical compact halo objects (MACHOs) are astronomical bodies such as e.g. black holes, brown dwarfs or Jupiter-like planets (i.e. baryonic) that cannot be detected by the emission or absorption of light. Their possible presence in large abundances in a galactic halo could be an explanation for the observations seen in galactic rotation curves. One can extract information about the MACHO distribution in our galaxy from their gravitational lensing effect. While full Einstein rings caused by MACHOs are usually too small to be resolved in telescope observations, one can still measure an apparent increase in the magnitude of a light source if such an object passes the line of sight between source and observer (microlensing).
If an object moves at a velocity v perpendicular to the line of sight, the typical time within which lensing happens is given by:

$$T = \frac{D_M \alpha_E}{v} \tag{1}$$

Calculate T for an object with mass $M = 0.2 M_\odot$ and $v = 200\,\mathrm{km/s}$ which lies halfway between an observer on earth and a light source located in the Large Magellanic Cloud ($D_S = 50\,\mathrm{kpc}$).

(Hint: use $\alpha_E = \sqrt{\frac{4GM}{c^2}\frac{D_{SM}}{D_M D_S}}$ *from the previous exercise sheet).*

2.2 Modified Newtonian Dynamics *4 Points*

Modified Newtonian Dynamics (MOND) theory has been suggested to explain the rotation curves of galaxies without the need for additional dark matter. You can download rotation curve data of NGC2998 from `http://astroweb.case.edu/ssm/620f03/n2998.dat` (by Stacy McGaugh), which is a spiral galaxy in Ursa Major. The first column gives the radius, the second the observed/inferred circular velocity with its 1σ uncertainty in the third column. The next few columns provide rotation curves that would arise from the stellar disk, gaseous disk, and the bulge alone.

a) Plot the observed circular velocity as a function of the radius (use e.g. ROOT or Python). The plot should also contain the uncertainties on the circular velocity. If you do not have access to a tool for plotting, you can also draw the curve by hand.

b) In MOND theory, the gravitational acceleration a is scaled by postulating a modified version of Newton's first law:

$$\mu\left(\frac{a}{a_0}\right)a = \frac{MG}{R^2}, \qquad \text{with} \begin{cases} \mu\left(\frac{a}{a_0}\right) = \frac{a}{a_0}, & a \ll a_0 \\ \mu\left(\frac{a}{a_0}\right) = 1, & a \gg a_0 \end{cases} \tag{2}$$

where a_0 is a constant introduced by MOND and μ is an interpolation function. One possible way to test MOND is to check to which extent kinematic data from different systems really can be fitted with the same value of a_0.

Use the rotation curve data of NGC2998 to test if this object can be reconciled with $a_0 \approx 1.2 \cdot 10^{-10}\,\mathrm{m\,s^{-2}}$, which is the value for a_0 suggested by most previous attempts to fit galaxy data with MOND. You can treat the galaxy as a point-mass as long as you use the data in a way that minimizes the errors introduced by this approximation. Use $M_{visible} = 24.4 \cdot 10^{40}\,\mathrm{kg}$ and $G = 6.67 \cdot 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$). Visualize the expectation of MOND in the plot of the rotation curve.

2.3 Particle Dark Matter: The Tremaine-Gunn bound *4 Points*

a) List the requirements on fundamental particle properties for viable particle dark matter candidates and give a short reason for each of them.

b) Presume neutrinos to be massive enough for them to be non-relativistic today. A gas of such neutrinos would not be homogeneous, but clustered around galaxies instead. Assume, that such a neutrino gas almost makes up the entire mass of a galaxy (i.e. other components are negligible). Measuring how fast a star rotates at a certain radius r relative to the galactic center allows to calculate the total mass contained within this radius (see problem sheet 1). The number of neutrinos which makes up this mass is limited by the amount of available states in phase space, as they obey the Pauli exclusion principle. Use this fact, while assuming, for simplicity, that there exists only one kind of neutrino, to derive a lower limit on the neutrino mass m_ν (Tremaine-Gunn limit). You can postulate that all possible states are populated, with the number of states per unit phase space being given by:

$$n = \frac{g}{(2\pi\hbar)^3}, \tag{3}$$

where g ($= 2$ in this case) denotes the number of relativistic degrees of freedom. Note that neutrinos can only be gravitationally bound if they are below escape velocity. Assume, in addition, spherical symmetry, that the escape velocity at r smaller than some radius R is the same as for $r = R$, and that a measurement of the galactic rotation curve at $r = 12\,\mathrm{kpc}$ yields $v(r) = 220\,\mathrm{km/s}$.

c) Compare your result for the neutrino mass limit to the currently known limits on neutrino masses. What is your conclusion on SM neutrinos as dark matter particles?