Exp. Methods in Astroparticle Physics (SS 2020) - Problem sheet 11

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Deadline for this sheet: Thu, 09.07.2020

Neutrinos: See-saw mechanism, hierarchy, free-streaming scale

11.1 See-saw mechanism *3 Points*

In the lecture, we discussed the See-saw mechanism. Try to answer these rapid-fire questions:

a) How does a mass term for neutrinos look like if we introduce right-handed neutrinos ν_R?

b) Is it a Dirac or Majorana mass term?

c) Which hypercharge does ν_R have?

d) Which other term in the Lagrangian is thus allowed, and what type of mass term is it?

e) How can the two mass terms be combined?

f) How can this matrix be diagonalized?

g) How do the entries of the diagonal matrix look like, and what is the limit if $M \gg m$?

11.2 Neutrino mass hierarchy *3 Points*

Neutrino oscillations have provided precise measurements of $\Delta m^2_{12} = 7.39 \times 10^{-5}\,\mathrm{eV}^2$ and $\Delta m^2_{\mathrm{atm}} = 2.523 \times 10^{-3}\,\mathrm{eV}^2$ (https://arxiv.org/abs/1811.05487).

a) Assuming the smallest mass eigenstate to be vanishing, calculate the value of the other two mass eigenstates for normal and inverted ordering.

b) Which processes were keeping the neutrinos in thermal equilibrium in the primordial plasma, and at which temperature did the neutrinos then decouple? Were the neutrinos relativistic at the time of decoupling, and are they still at the present time?

11.3 Neutrino free-streaming scale *4 Points*

We will now calculate the neutrino free-streaming scale k_{ks}. First, determine the typical momentum of the neutrinos with temperature T_ν/a, where a is the scaling factor. Then, calculate the typical distance x_{fs} that a neutrino travels (with this momentum and mass m_ν) within a time interval $\Delta t = 1/H$, where H is the Hubble constant.

a) Determine the free-streaming scale given by $k_{\mathrm{fs}} = 1/x_{\mathrm{fs}}$ as a function of the neutrino mass.

b) Calculate the free-streaming length for $\sum_i m_i = 0.2\,\mathrm{eV}$ and discuss what this means for the structure formation and the CMB.
Hint: If you are unsure on certain aspects of the evolution of the universe, have a look in the book "Introduction to the Physics of Massive and Mixed Neutrinos" by Samoil Bilenky, available also in HEIDI (the catalog for the library of Heidelberg University).