Exp. Methods in Astroparticle Physics (SS 2020) - Problem sheet 1

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Dark matter indications: Coma cluster, galactic rotation curves and strong lensing

1.1 Galaxy clusters: The virial theorem *4 Points*

The first indication for dark matter is given in a paper written by F. Zwicky in 1933 [1]. He observed the radial velocities of the galaxies in the Coma galaxy cluster and used the virial theorem to calculate its mass. The state of the system is often quoted in terms of the virial ratio:

$$Q_{vir} = \frac{\langle\, T \,\rangle}{-\,\langle\, \Omega \,\rangle}, \tag{1}$$

with Ω being the potential energy and T the kinetic energy. Stable, self-gravitating, spherical distributions of equal mass objects such as (approximately) galaxy clusters are in virial equilibrium and have $Q_{vir} = 1/2$.

a) Derive an expression for the total mass of a gravitationally bound system as a function of the dispersion velocity $\langle v^2 \rangle$.

b) Zwicky measured that the diameter of the Coma cluster is around $2 \cdot 10^{22} m$ and observed a velocity dispersion of about $\langle v^2 \rangle^{1/2} = 2000 \frac{km}{s}$. Using these observational values, calculate the total mass of the Coma cluster as derived from the virial theorem.

c) The Coma cluster contains approximately 800 galaxies with an average luminosity of 10^9 solar luminosities ($L_\odot$) per galaxy. Calculate the mass to light ratio $\Upsilon = \frac{M_\odot}{L_\odot}$ for the Coma cluster. Considering the typical mass to light ratio of stars, give a short interpretation of this result.

1.2 Galactic rotation curves: Prediction vs. measurement *4 Points*

Measurements of galactic rotation curves imply an apparent mass deficit in galaxies (e.g. in publications by V. Rubin et al. from the 1970s/80s). Under the assumption of spherical symmetry of a rotating galaxy one can calculate the mass inside a sphere of a given radius from the circular velocity of the stars at its surface and compare it to an estimate from visible stars.

a) Give a formula which expresses the circular velocity in terms of the enclosed mass and the distance to the galactic center.

[1] ZWICKY, Fritz. Die Rotverschiebung von Extragalaktischen Nebeln. Helvetica Physica Acta, 1933, 6. Jg., S. 110-127.

b) First let us consider the case that most of the mass of the galaxy is located in a central region extending up to a radius r_0. For simplicity we assume that the mass density is constant there (ρ_0) and vanishes outside of r_0. Draw a sketch of how the rotation curve looks like inside and outside of r_0.

c) Now consider a more realistic mass density distribution in the form of:

$$\rho(r) = \frac{\rho_0}{(1 + r/r_0)^\alpha}. \tag{2}$$

Determine a value for α which gives a flat rotation curve at $r \gg r_0$, as indicated by measurements, and derive the rotation curve $v(r)$ for this value.

1.3 Gravitational lensing: Einstein rings *2 Points*

General relativity predicts light to be deflected by the presence of massive objects acting as lenses. In the case where a massive object is directly between a light source and an observer, light from the source is deflected such that the observer will see a luminous ring around the massive object, called *Einstein ring*, as sketched in Figure 1.

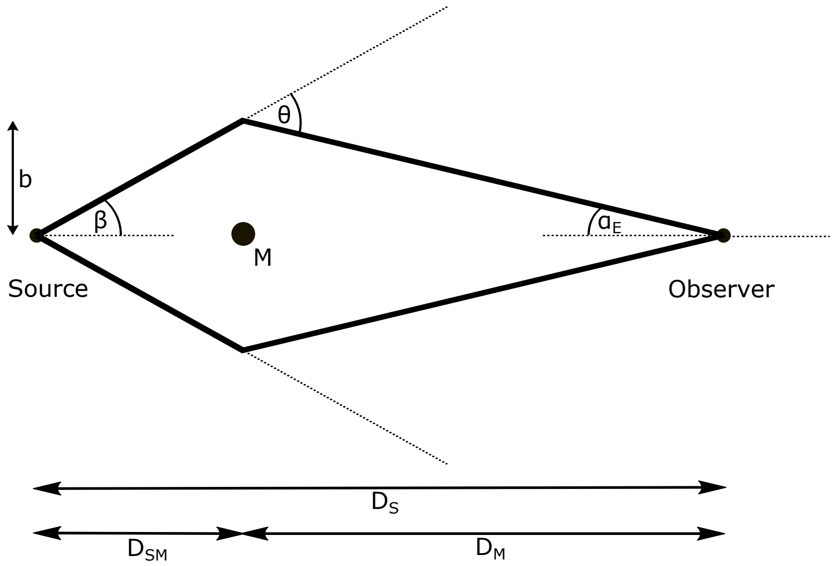

Figure 1: Sketch for task 1.3

Given the equation for the deflection angle

$$\theta = \frac{4GM}{bc^2}, \tag{3}$$

derive an expression for the opening angle α_E of the Einstein ring which only depends on the objects mass M and the distances D_{SM}, D_M and D_S. G is the gravitational constant and c the speed of light. Assume α_E and β to be small in order to approximate trigonometric functions.
(Hint: $\alpha_E \propto \sqrt{M}$ in the final result.)