



General properties Non-thermal – far from thermodynamic equilibrium with surroundings e.g., cosmic rays, relativistic electrons in supernova remnants, AGN, jets... High energy, very low density, e.g., cosmic rays: particle energy 10¹⁰ eV up to 10²⁰ eV (≈ 16 J) number density 10⁻¹⁰ × interstellar medium.

troduction: nonthermal particles and electromagnetic fields - Trajectories - Non-stochastic (single-shot) acceleratio

 Interactions with background almost exclusively via electromagnetic fields







les and electromagnetic fields Trajectories

- inhomogeneities and slow (quasi-neutral) variations lead to: additional drifts (grad-*B*, curvature...)
- and an electric field ($\vec{\nabla} \times \vec{E} = -\partial B / \partial t \Rightarrow E \sim B L_{\rm var} / t_{\rm var}$)
- but still no acceleration, $\oint dt (\gamma mc^2 + qA^0) = constant$

Solutions:

- Ideal MHD not valid *everywhere* (boundary conditions require dissipation)
- Adiabatic invariance violated by high-frequency (compared to eB/γmc) and/or short-wavelength fluctuations ⇒ scattering.





- If $v_{int} > c$ transform to perpendicular shock frame:
 - \vec{B} lies in shock plane
 - Acceleration can occur on crossing the shock
 - All particles transmitted



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Magnetic pumpingLet $\vec{B} = B(t)\vec{2}$ (e.g., very long wavelength (compressional)
magnetosonic wave) $\frac{\dot{p}_{\perp}}{p_{\perp}} = \frac{\dot{B}}{2B}$ $\dot{p}_z = 0$ The distribution function satisfies Liouville's eq:
 $\frac{\partial f}{\partial t} + \dot{p}_{\perp} \frac{\partial f}{\partial p_{\perp}} + \dot{p}_z \frac{\partial f}{\partial p_z} = 0$
so that
 $\frac{\partial f}{\partial t} + \frac{\dot{B}}{2B} p_{\perp} \frac{\partial f}{\partial p_{\perp}} = 0$

The pressure is $P = \frac{1}{3} \int \frac{d^3p}{E} p^2 f$ If $\langle p_z^2 \rangle \ll \langle p_\perp^2 \rangle$, and assuming relativistic particles $\Rightarrow PB^{-3/2} = \text{constant}$ Adiabat for 4 degrees of freedom $(B \propto \text{density})$

In spherical polars: $\frac{\dot{p}}{p} = \frac{(1-\mu^2)}{2}\frac{\dot{B}}{B} \qquad \frac{\dot{\mu}}{\mu} = -\frac{(1-\mu^2)}{2}\frac{\dot{B}}{B}$ So that Liouville's eq. is $\frac{\partial f}{\partial t} + \frac{\dot{B}}{2B}(1-\mu^2)\left(p\frac{\partial f}{\partial p} - \mu\frac{\partial f}{\partial \mu}\right) = 0$ Introducing strong isotropization: $\Rightarrow PB^{-4/3} = \text{constant}$ Adiabat for 6 degrees of freedom



Fokker-Planck approach
Stochastic interactions with
$$\Delta p/p \ll 1$$
.
Evolution of an isotropic distribution on timescale long
compared to that between individual interactions:

$$\frac{df}{dt} = \frac{1}{p^2} \frac{\partial}{\partial p} \left[-Fp^2 f + \frac{\partial}{\partial p} \left(Dp^2 f \right) \right]$$
dynamical friction: $F = \frac{\langle \Delta p \rangle}{\Delta t}$ diffusion: $D = \frac{\langle (\Delta p)^2 \rangle}{2\Delta t}$
Computation of *F* and *D*:
turbulent waves two-body collisions

Two-body collisions:

$$\vec{p} + \vec{P} = \vec{p'} + \vec{P'}$$

$$\sqrt{p^2 + m^2} + \sqrt{P^2 + M^2} = \sqrt{p'^2 + m^2} + \sqrt{P'^2 + M^2}$$

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For non-relativistic targets, and $|p' - p| \ll p$

$$\frac{\vec{p}' - \vec{p}}{p} \approx \frac{\sqrt{\vec{p}^2 + m^2}}{p^2 M} \left[\vec{P} \cdot \Delta \vec{p} + \text{second-order terms} \right]$$
$$\sim \frac{V}{v} \ll 1$$

Head-on collisions $(\vec{P} \cdot \Delta \vec{p} > 0) \Rightarrow$ energy gain Tail-on collisions $(\vec{P} \cdot \Delta \vec{p} < 0) \Rightarrow$ energy loss

Fokker-Planck coefficients:

$$\begin{pmatrix} \frac{(p'-p)}{p\Delta t} \\ \frac{((p'-p)^2)}{p^2\Delta t} \end{pmatrix} = \int d^3 P f_{\rm T}(\vec{P}) \int d\Omega' \frac{d\sigma}{d\Omega'} v_{\rm rel} \begin{pmatrix} \frac{V}{V} \vec{P} \cdot \Delta \vec{p} \\ (\frac{V}{V})^2 (\vec{P} \cdot \Delta \vec{p})^2 \end{pmatrix}$$

$$(\vec{P} = \vec{P}/P, \Delta \vec{p} = \Delta \vec{p}/P.)$$
If head-on and tail-on collisions equally probable (to lowest order in $|p' - p|/p$) then $F \sim (p' - p)^2/p^2 \sim D$
 \Rightarrow second order Fermi process
Under special conditions (e.g., anisotropic $f_{\rm T}$) $F \sim |p' - p|/p$
 \Rightarrow first order Fermi process

Introductor non-thermal particles and electromagnetic fields Trajectories Non-stochastic (angle-shot) acceleration Stochastic acceleration in energy Maxwellian targets, density $n_{\rm T}$, temperature $T = MV_{\rm th}^2$ $(k_{\rm B} = c = 1)$: $\frac{\langle (\Delta \rho)^2 \rangle}{\rho^2 \Delta t} = 2n_{\rm T} \frac{\sigma V_{\rm th}^2}{\nu} \left(1 + \frac{1}{2} \langle \cos \theta \rangle \right)_{\theta = \rm scattering angle}$ Requiring the equilibrium solution $f \propto e^{-\sqrt{\rho^2 + m^2}/T}$ to be stationary gives (specialising to relativistic particles, $\nu = 1$): $F = -n_{\rm T} \overline{\sigma} p^2 / M + n_{\rm T} V_{\rm th}^2 \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left(\overline{\sigma} p^4 \right)_{\theta = \sigma(1 + \langle \cos \theta \rangle / 2)}$ second derivative of D cancels! Kinematical effect, true for all distributions, not just equilibrium Introduction: nonthermal particles and electromagnetic fields Trajectories Non-stochastic (elegie-shot) acceleration Stochastic acc Diffusion in energy $\frac{df}{dt} = \frac{1}{p^2} \frac{\partial}{\partial p} \left[n_{\rm T} \bar{\sigma} p^4 \left(\frac{1}{M} f + V_{\rm th}^2 \frac{\partial f}{\partial p} \right) \right]$ For heavy scatterers ($T, M \to \infty$, $V_{\rm th}$ finite) $\frac{df}{dt} = \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D \frac{\partial f}{\partial p} \right)$ = Diffusion equation $\dot{f} = -\nabla_p \cdot \vec{F}$, with flux $\vec{F} = -D\nabla_p f$ = Applies also to wave turbulence ($\hbar k \gg \hbar \omega / v, v \gg v_{\rm phase}$) = Almost always acceleration (e.g., for $D \propto p^q$, with q > -2)

Summary, first lecture

• Particle acceleration is associated with entropy production, either in the background plasma (e.g., during a single encounter with a shock front or a dissipative current sheet) or in the energetic particle distribution itself by a scattering process

ion: nonthermal particles and electromagnetic fields Trajectories Non-stochastic (single-shot) acceleration Stochastic

- Single-shot mechanisms are important for relatively low energy accelerated particles, e.g., in the heliosphere, but not for acceleration over many order of magnitude in energy (cosmic rays)
- $\bullet~$ Second order Fermi mechanisms tend to be slow: $\sim~$ scattering frequency $\times~$ (wave speed/particle speed)^2 ~



Diffusive shock acceleration: Test particles DSA. Nonlinear effects DSA. Time dependence DSA Anomalous transport Scale-free spectra General case: acceleration events vs escape If $\Delta E > 0$ in every event $\Rightarrow E(t)$ Let P(p) be probability that a particle escapes with momentum > p ($p = \sqrt{E^2 - m^2 c^2/c}$) Acceleration $\dot{p} = ap$ Escape rate b P(p + dp) = P(p)(1 - b dt) $\dot{P} = -bP \Rightarrow \frac{dP}{dp} = -\frac{bP}{ap}$ $P \propto p^{-b/a}$ Power-law (scale-free) provided b/a independent of p



Diffusive shock acceleration: Test particles 26A Nonlinear effects 2004 Time dependence 26A Anomalous transport
Average over isotropic distribution (prob. of crossing proportional to relative speed
$$|\mu v|$$
):

$$\frac{\langle \Delta p \rangle}{p} = \int_{0}^{+1} d\mu \int_{-1}^{0} d\mu' |\mu \mu'| \Delta p / p / \int_{0}^{+1} d\mu \int_{-1}^{0} d\mu' |\mu \mu'|$$

$$\Rightarrow \langle \Delta p \rangle / p = 4 \Delta V / 3 v$$
Density $n = 2\pi p^2 \int_{-1}^{+1} d\mu f$
N^{o.} entering/sec $= 2\pi p^2 \int_{0}^{+1} d\mu (\mu v + V_2) f = nv/4$
N^{o.} leaving/sec $= 2\pi p^2 \int_{-1}^{+1} d\mu (\mu v + V_2) f = nV_2$

$$\Rightarrow Escape Prob. = 4V_2/v$$

Diffusive shock acceleration: Test particles DSA: Nonlinear effects DSA: Time dependence DSA: Anomalous transport In terms of the phase-space density $f(p) \propto p^{-3-P_{esc}/\langle \Delta p/p \rangle} \equiv p^{-s}$, $s = \frac{3u}{\Delta u} = \frac{3r}{r-1}$ where $r = V_1/V_2$ is the *compression ratio* of the shock. A strong shock in a gas with $C_p/C_V = 5/3$, has r = 4, and s = 4.

Diffusion approximation
$$f(p, \mu) = f^{(0)}(p) + f^{(1)}(p, \mu)$$

 $\frac{\partial}{\partial x} \left(\kappa \frac{\partial f^{(0)}}{\partial x} \right) - V \frac{\partial f^{(0)}}{\partial x} = 0$
(constant $V = V_{1,2}$, unmodified shock). General solution:
 $f_{1,2}^{(0)} = A_{1,2}(p) + C_{1,2}(p) \exp\left(\int_0^x dx' V_{1,2}/\kappa_{1,2}\right)$
Exponential decay upstream and growth downstream
 $\Rightarrow C_2 = 0$
No diffusive flux downstream $f_2^{(1)} = 0$

Diffusive shock acceleration: Test particles DSA: Nonlinear effects DSA: Time dependence DSA: Anomalous trans

Diffusive shock acceleration: Test particles DEA. Nonlinear effects DEA. Time dependence (DEA. Anomalous triangle of pi in shock frame)

$$p_2 \approx p_1 (1 + \Delta V \mu_1 / v_1)$$
At shock front, Liouville's theorem gives
$$f_1(p_1, \mu_1) = f_2(p_2, \mu_2) \\ = f_2^{(0)}(p_2) \\ \approx f_2^{(0)}(p_1) + \mu_1 \frac{\Delta V}{v_1} p_1 \frac{\partial f_2^{(0)}}{\partial p} \Big|_{p_1}$$
Integrate over $\mu_1: \Rightarrow A_1 + C_1 = A_2$
No density jump across shock

$$-\kappa_1 \frac{\partial f_1^{(0)}}{\partial x} = \frac{v_1}{2} \int_{-1}^{+1} d\mu_1 \mu_1 f_1$$

Diffusive shock acceleration: Test particles DSA: Nonlinear effects DSA: Time dependence DSA: Anomalous tran

so that

$$V_1 C_1 \approx \left. \frac{\Delta V}{3} \rho_1 \left. \frac{\partial f_2^{(0)}}{\partial \rho} \right|_{\rho_1, x=0}$$

and the matching condition is

_

$$\frac{\Delta V}{3V_1} p \frac{\mathrm{d} f_2^{(0)}}{\mathrm{d} p} + f_2^{(0)} = A_1$$

Shock modification

Two feedback effects important for the spectrum in nonlinear case:

sive shock acceleration: Test particles DSA: Nonlinear effects DSA: Time dependence DSA: Anomalous tra

- Relativistic particles have $\hat{\gamma}=4/3,$ and so soften the equation of state
 - overall compression ratio increases
 - high energy particles (with long mean free path) get a harder spectrum
- Pressure gradient decelerates and heats incoming plasma
 - Mach number (strength) of sub-shock reduced
 - low energy particles (with short mean free path) get a softer spectrum

 \Rightarrow concave spectrum

Ways around excessive feedback:

- Allow for a steady state with escape of high energy particles
- Follow the time dependence explicitly
- Problems:
- A delicate balance with the *injection* mechanism needed

sive shock acceleration: Test particles DSA: Nonlinear effects DSA: Time dependence DSA: Anomalous transp

- Possible feedback effect of magnetic amplification not fully understood
- Transport properties in driven turbulence uncertain

Review article: Malkov & Drury, Rep. Prog. Phys. 64, 429 (2001) Modelling of SNR: Berezhko & Völk, Astroparticle Phys. 14, 201 (2000)

Cycle time

Total number of particles upstream

$$\int dx \, n_1 = n_1(0) \int_{-\infty}^0 dx \exp(xV_1/\kappa_1) \\ = n_1(0)\kappa_1/V_1$$

usive shock acceleration: Test particles DSA: Nonlinear effects DSA: Time dependence DSA: Anomalous

Rate at which particles cross into upstream = $vN_1(0)/4$. Average residence time upstream

$$t_1 = \frac{4\kappa_1}{vV_1}$$

Downstream?



Diffusive shock acceleration. Test particles DSA: Nonlinear effects DSA: Time dependence DSA: Anomalous transport $\begin{array}{l} & (\text{cycle time}) = \frac{4}{v} \left(\frac{\kappa_1}{V_1} + \frac{\kappa_2}{V_2} \right) \\ & (\text{acceleration timescale}) = \frac{(\text{cycle time})}{\frac{(\Delta \rho)}{\rho}} \\ & = \frac{3}{\Delta V} \left(\frac{\kappa_1}{V_1} + \frac{\kappa_2}{V_2} \right) \\ & \sim \kappa/V_1^2 > r_{gyro} c/V_1^2 \end{array}$ Too slow for SNR to make cosmic rays? $\Rightarrow \text{ magnetic field amplification} \end{array}$ Preview: Bell Astroparticle Physics 43, 56 (2013)

Dinusive shock acceleration, rest particles	BOX. Noniniear enects	DSA. Time dependence	DSA: Anomaious transport
Non-diffusive trans	port		
 Small anisotrop ⇒ Fick's law, the Parallel shock: a scattering centre Small for fas ⇒ Random Oblique shock: by compression Anisotropy in Density inho fluctuations Diffusion app 	y $f = f^{(0)}(p) +$ a diffusion app anisotropy drive as $\epsilon \sim \Delta V/v$ t particles (at now walk in $x: \langle \Delta x^2 \rangle$ additional anise of <i>B</i> -field. In the gyro-phase mogeneities from proximation fails	$\epsilon f^{(1)}(\vec{p})$ roximation en by relative mo nrelativistic shock $\propto t$. otropy/inhomoge distribution for we n compressed ma	tion of). neity driven eak scattering gnetic







 Otherwise shock acceleration
 — Summary

 Bitterwe shock acceleration
 — Summary





Bulk relativistic motion	First-order Ferm	ni mechanism at re	lativistic shocks	Pulsar winds
Relativistic bulk r	notion			
Object	Evidence	Lorentz factor	Radiation mechanism	
Radio Galaxies Micro-Quasars γ -ray Bursts γ -ray Blazars Pulsar Winds	direct direct indirect indirect theory	10 3 250 50 10 ⁵	synchrotron synchrotron synchrotron/IC synchrotron/IC synchrotron	
In	all cases γ (\Rightarrow <i>Particle</i>	particle) ≫ e Accelera	> Γ(bulk) ation	



• Well-defined kinematical problem: given u_+ and u_- , find s

Solve using methods developed for neutron/radiation transfer:

First-order Fermi mechanism at relativistic shocks

- Attempt to find an analytical solution No consistent analytic treatment available
- Expansion in suitably chosen eigenfunctions Converges rapidly for well-behaved D_{μμ}
- Convert to integral equation and solve numerically Might be better for finite-angle scattering
- Monte-Carlo Flexible: allows e.g., prescribed turbulence, regular magnetic field, escape boundaries, etc.
- 3D relativistic, electron proton, particle-in-cell simulation Still out of reach
- Best bet: use methods as a check on each other

Bulk relativistic motion First-order Fer	mi mechanism at relativistic shocks Pulsar win
Nonrelativistic (DSA) vs.	
pitch-angle diffusion \Rightarrow near-isotropy \Rightarrow spatial diffusion	pitch-angle diffusion, particles in narrow, forward directed cone
solution of PDE in <i>x</i> , <i>p</i> required	solution of PDE in μ , x , p required
small escape probability small $\left< \Delta p \right> / p$ per cycle	, escape probability \sim 0.5, $\left< \Delta p \right> / p \sim \Gamma^2$ for first cycle, then \sim 2
power-law of index s = 3r/(r-1), independer of scattering law	asymptotically, $s = 4.23$, weakly dependent on scat- tering law









First-order Fermi mechanism at relativistic shocks



















Non-MHD fields		
Raw fields	1.0 0.5 ¹⁰ 0.0 ¹⁰ 0.0 ¹⁰ 0.0	AAAAA
Non-MHD fields $\vec{E}_y = E_y - v_x$ $\vec{E}_z = E_z + v_x$	$B_{z} = \begin{bmatrix} 0.4 \\ 0.2 \\ 0.2 \\ 0.4 \\ 0.2 \\ 0.4 \\ 0.2 \\ 0.4 \\ 0.2 \\ 0.4 \\ 0.2 \\ 0.4 \\ 0.2 \\ 0.4 \\ 0.2 \\ 0.4 \\ 0.2 \\ 0.4 \\ 0.2 \\ 0.4 \\ 0.2 \\ 0.4 \\ 0.2 \\ 0.4 \\ 0.2 \\ 0.4 \\ 0.2 \\ 0.4 \\ 0.2 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ $	2.28 ×10 ³





