Rotating and vibrating tetrahedrons in heavy nuclei

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I want to report to you how I came to the conclusion that in each heavy nucleus there are some states described by a rotating and vibrating tetrahedron.

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Since 50 years I am studying from time to time the doubly magic nucleus $^{208}\text{Pb}$.

It began when I met Peter von Brentano who just found out that the proton decay of isobaric analog resonances (IAR) allows to obtain several hundred amplitudes of particle-hole configurations in heavy nuclei.

In 1968 we performed our own experiments on $^{208}\text{Pb}(p, p')$.

Fifteen years later a complementary experiment on $^{209}\text{Bi}(d, {^3\text{He}})$ was performed.

By the combination of all experimental data, the complete orthogonal transformation matrix from 20 particle-hole configurations to 20 negative parity states was derived and their spins determined.

Then I had to wait for another twenty years until the Q3D magnetic spectrograph at Garching was finished and obtained the final resolution of 3 keV. It is limited by the knockout of atomic electrons from the $M$-shell.

Ten years later I identified 90 negative parity states in $^{208}\text{Pb}$, their spin and the major particle-hole amplitudes.

Last year I identified the complete set of the 150 lowest states in $^{208}\text{Pb}$ with a reliability of 95%.
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From the very beginning I wondered about some low-lying states. They are badly described by the shell model.
Some of them are strongly excited by electromagnetic probes, \((p, p')\), \((d, d')\), \((\alpha, \alpha')\), \((e, e')\).
(top) In these \(^{208}\text{Pb}(e, e')\) spectra the \(3^-, 2^+, 4^+\) yrast states show up among 30 known states.
(bottom) Even with five times better resolution only a dozen shell-model states (of natural parity) weakly show up among 60 states in addition to the \(3^-, 2^+, 4^+\) (and \(6^+\)) yrast states.

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The \(3^-, 2^+, 4^+\) (and \(6^+\)) states are assumed to be collective excitations.
Especially the lowest \(3^-\) state was described as an octupole vibration.
This started a long search for double octupole vibrations.
Indeed in 1996 a \(0^+\) state was identified at exactly twice the energy of the lowest \(3^-\) state.
The double octupole vibration multiplet should contain states with spin \(0^+, 2^+, 4^+, 6^+\).
However from my analysis done in the last year, I am sure that the \(4^+, 6^+\) members don't show up as expected.

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By chance I heard from the study of metal clusters with 50-500 atoms.
Magic numbers of the electronic density are explained by geometrical shapes, here clusters with 216 and 92 Na atoms.
I started to calculate shapes for \(^{208}\text{Pb}\) with 208 nucleons and 82 protons.
The tetrahedron is the simplest shape.
The 208 nucleons of \(^{208}\text{Pb}\) can be perfectly arranged in a tetrahedron.
Also 41 \(\alpha\)-particles with the neutron excess acting as a mortar do so.

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Yet after some time I concluded that only the symmetry is relevant.
It does not matter whether there are 4 or 41 \(\alpha\)-particles, 16 or 208 nucleons or a continuous shape.
The application of group theory to a system of fermions was started by Tisza in 1933.

Exactly 80 years ago Wheeler predicted states $^{16}\text{O}$ in the shape of a rotating and vibrating tetrahedron.

The Pauli exclusion principle does not allow certain spins; for a rotating tetrahedron spins 1, 2, 5 are forbidden.

The rotation spectrum starts with the spins $0^+$, $3^-$, $4^+$ and a $6^\pm$ doublet.

There should be three vibration modes.

- The dilation mode where four parts are moving isotropically back and forth; here the spin sequence is the same as for the rotation band.
- The flattening mode where two halves are moving against each other; here the spin sequence starts with a $2^\pm$ doublet and a $4^\pm$ doublet.
- The torsion mode where two halves are twisting around each other; here the spin sequence starts with $1^-$, $2^+$, and a $4^\pm$ doublet.

Just in January this year a paper by Bijker and Iachello appeared where the excitation energies of $^{16}\text{O}$ are calculated.

The rotation is described by the usual dependence on the angular momentum with $J(J+1)$.

For the $3^-$ and $4^+$ states the ratio should be $5:3$. 

Indeed, the ratio for the $3^-$ and $4^+$ states in $^{208}\text{Pb}$ is exactly 5:3.
The $1^-$ and $2^+$ states which are badly described by the shell model clearly correspond to the heads of the torsion and flattening mode.
Because the $3^-$ state is now described as a pure rotator there is no longer any double octupole state.
Hence the $0^+$ state is identified as the head of the dilation mode.
The fact that the energy of the $0^+$ state is exactly twice the energy of the $3^-$ state turns out to be pure chance.

A comparison of the band heads in $^{208}\text{Pb}$ and $^{16}\text{O}$ confirm the description by a rotating and vibrating tetrahedron.
In both nuclei the $3^-$ and $4^+$ states have large electromagnetic moments.
The ratio of the B(E3) values agrees with the ratio of the parameter $\kappa_r$ within the uncertainty; it corresponds to the momenta of inertia.
The relative values of the vibration parameters $\omega_1, \omega_2, \omega_3$ are similar; especially the ratio $\omega_2/\omega_3$ for the flattening and torsion modes is identical.

With the knowledge of the four basic parameters more states can be predicted. Five more states are found.
I will explain the identification for three states (marked by a cross).

Below 6.2 MeV nine $2^+$ states are known.
The $2^+$ state which was formerly counted as double octupole state is clearly the second member of the torsion mode.

The resonant neutron capture on the stable isotope $^{207}\text{Pb}$ shows a strong resonance at 472 keV; the spin $3^-$ is firmly assigned.
It corresponds to the second member of the dilation mode.
The deviation from the predicted energy is less than 1%.
The band head of the flattening mode should be a doublet with spin $2^\pm$. The $2^-$ state at 4320 keV contains more than 90% of the $g_{9/2}f_{5/2}$ particle-hole configuration.

It cannot be the $2^-$ state searched for.

The new spectrum for $^{208}$Pb($d, d'$) taken with the Q3D magnetic spectrograph clearly show the existence of the 4142 state.

I am re-analyzing the spectra for $^{208}$Pb($p, p'$) taken in 1968 with modern tools.

Here the 4142 state shows up at backward scattering angles.

The comparison of the excitation energies for the rotating and vibrating tetrahedrons in $^{208}$Pb and $^{16}$O are shown here.

In $^{208}$Pb the head of the dilation mode lies much higher; it was up to now declared as the “double octupole” vibration.

The biggest difference is the splitting of the $2^\pm$ band head of the flattening mode.

In $^{208}$Pb the relative distance between the $2^+$ and $2^-$ state is 1.3%, in $^{206}$Pb the doublet is even closer.

In $^{16}$O the deviation amounts to 30%.

The deviation of the experimental energies for three rotating and vibrating members from the prediction is 100 times smaller than in $^{16}$O.
I wondered why nobody has found similar rotating and vibrating tetrahedrons in other heavy nuclei.

The reason is clear: In most nuclei the assignment of spin and parity is rather often uncertain.

In the Nuclear Data Sheets I found 60 nuclei with firm assignments corresponding to the $4^+$ yrast state in $^{208}\text{Pb}$.

I included 90 pairs of states with tentative and apparently incorrect assignments.

The ratio of the excitation energies for the $3^-$ and $4^+$ states agrees with the prediction of $5 : 3$ within about 1%.

The mean value is 1% smaller than predicted. The deviation is the same as for $^{208}\text{Pb}$.

A rotating system of fermions in the shape of a tetrahedron predicts a rotation band starting with spins $0^+, 3^-, 4^+$, and $6^\pm$.

The ratio of the excitation energies for the pair of the $3^-$ and $4^+$ states should be $5 : 3$.

In 60 heavy nuclei pairs of $3^-$ and $4^+$ states are identified with firm assignments and another 90 ones with less reliability.

The ratio of their excitation energies deviates from the prediction by less than 1%.

In $^{208}\text{Pb}$, eight members of rotating and vibrating tetrahedrons are identified.

The excitation energies deviate from the prediction by less than 1% down to a few parts in $10^{-4}$.