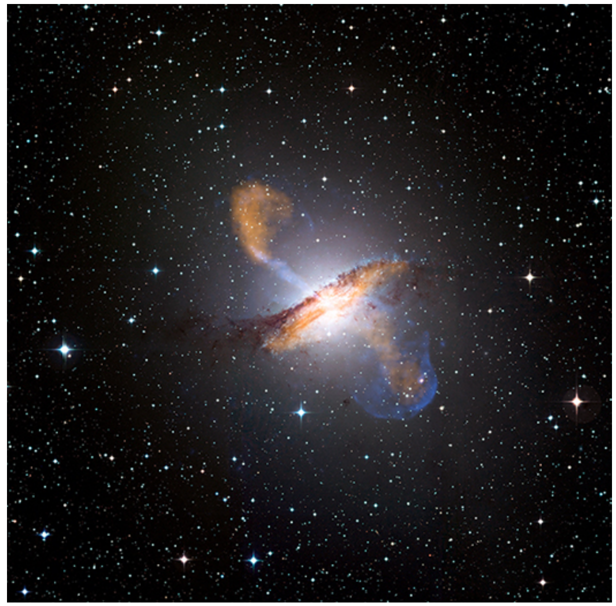
HIGH ENERGY ASTROPHYSICS - Lecture 6



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Wednesday

Synchrotron Emission II + Curvature Emission

1 Overview

- Radiation of charged particles in magnetic fields.
- Synchrotron Self-Absorption, cut-off in the spectrum at low frequencies.
- Synchrotron cooling effects for injected electron distribution (kinetic equation), resultant emission spectrum.
- Minimum energy considerations and "equipartition"
- Curvature Emission

2 Synchrotron Radiation (recap)

Total power per unit frequency [erg/s/Hz] for single electron with Lorentz factor γ (relativistic case $\beta \simeq 1$)

$$\boxed{P_\nu(\gamma) = \sqrt{3}\,\frac{e^3 B \sin\theta}{mc^2}\,F\left(\frac{\nu}{\nu_c}\right)}$$

with $F(x) := x \int_x^\infty K_{5/3}(x')dx' \simeq 1.8\ x^{0.3}e^{-x}$, $K_{5/3}$ modified Bessel function of order 5/3, and γ entering via

$$\boxed{\nu_c = \frac{3}{4\pi}\,\gamma^2\,\frac{eB}{mc}\sin\theta = \frac{3}{4\pi}\,\gamma^2\,\Omega_0 \sin\theta}$$

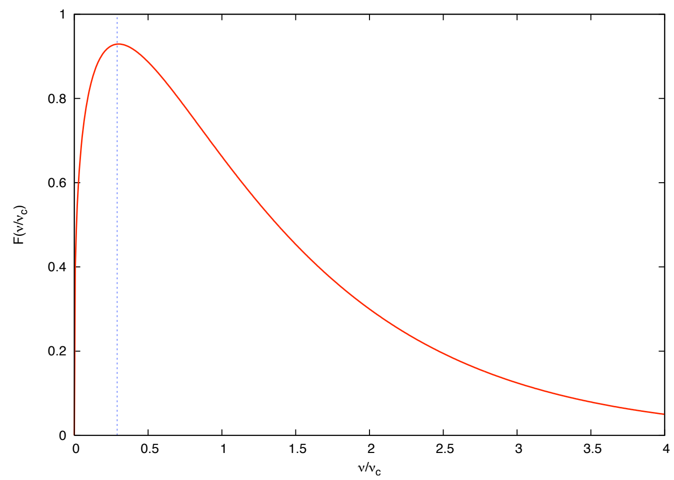


Figure 1: $F(\nu/\nu_c)$ as function of frequency ν/ν_c. The spectrum has a maximum at $\nu_{max} = 0.29\nu_c$.

3 Synchrotron Self-Absorption

Inverse Process = free electron in a magnetic field can absorb a photon. Happens when source is compact. Heuristic Derivation:

- Compare with thermal radiation, where source function $S_\nu = B_\nu(T)$, i.e.

$$S_\nu = \left(\frac{2\nu^2}{c^2}\right)\left(\frac{h\nu}{e^{h\nu/kT}-1}\right) \propto \nu^2 \bar{E}$$

  first term corresponds to phase-space factor, second to mean energy (e.g., $\bar{E} \propto kT$ in Rayleigh Jeans limit $kT \gg h\nu$).

- For non-thermal synchrotron, kT must be replaced by mean energy of electron emitting synchrotron at ν, $\bar{E} = \gamma m_e c^2$ with $\nu \simeq \gamma^2 \nu_L$ (where $\nu_L := \Omega_0/2\pi$), i.e. $\gamma \simeq (\nu/\nu_L)^{1/2}$, thus

$$\boxed{S_\nu \simeq \left(\frac{2\nu^2}{c^2}\right)\left(\frac{\nu}{\nu_L}\right)^{1/2} m_e c^2 \propto B^{-1/2}\nu^{5/2}}$$

  $\Rightarrow$ Source function is power law with index $5/2$, *independent of the value of electron power law index p.*

- **Note:** Spectral index is different from thermal Rayleigh-Jeans $S_\nu \propto \nu^2$!

4 Total Synchrotron Spectrum

Recall solution of transfer equation for constant source function (lecture 3)

$$I_\nu = S_\nu \left(1 - e^{-\tau_\nu}\right)$$

where $S_\nu := j_\nu/\alpha_\nu$, and $\tau_\nu = \int \alpha_\nu ds$ integrated through source.

From above, $\alpha_\nu = j_\nu/S_\nu \propto B^{1/2}\nu^{-5/2} j_\nu \propto B^{(p+2)/2}\nu^{-(p+4)/2}$ [cm^{-1}] with synchrotron $j_\nu \propto B^{(p+1)/2}\nu^{-(p-1)/2}$ for electron power law.
$\Rightarrow$ α_ν decreases towards higher ν

Limiting cases:

$$I_\nu \to S_\nu \qquad for \quad \tau_\nu \gg 1$$
$$I_\nu \to S_\nu \alpha_\nu s = j_\nu s \quad for \quad \tau_\nu \ll 1\,.$$

Thus for source with $\tau_{\nu_B} = 1$ at some frequency ν_B, we have low-frequency (optically thick) range:

$$I_\nu \propto \nu^{5/2}$$

and high frequency (optically-thin) range:

$$I_\nu \propto \nu^{-(p-1)/2}$$

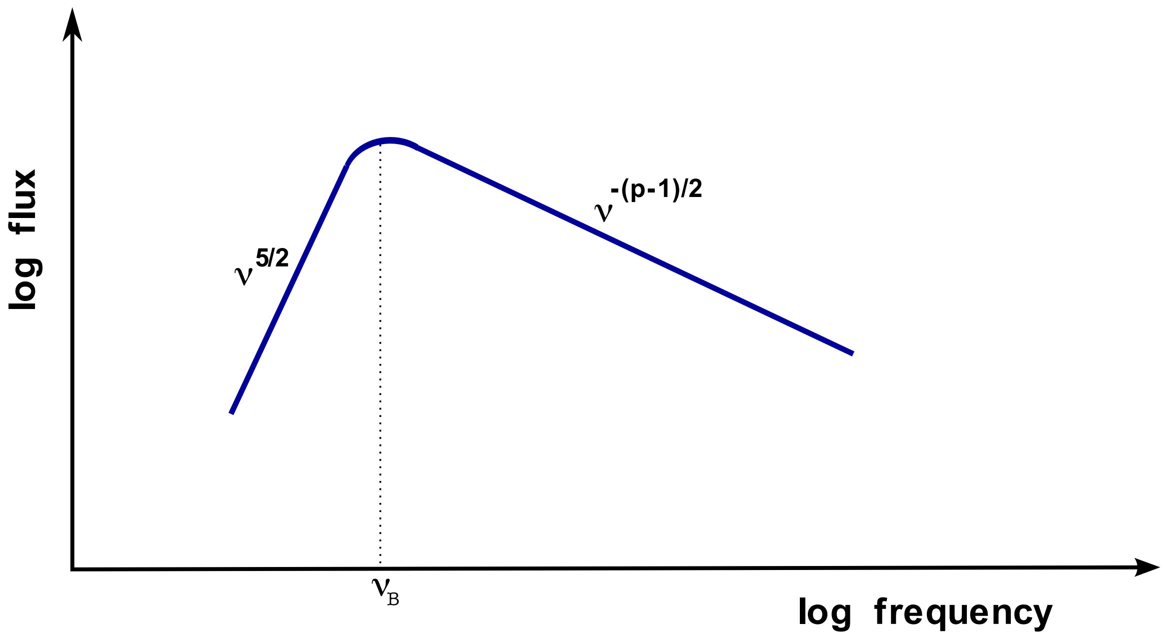


Figure 2: Total synchrotron spectrum - Shape for electron power law $n(\gamma) \propto \gamma^{-p}$, with $j_\nu \propto \nu^{5/2}$ in the low-frequency, optically-thick regime ($\nu < \nu_B$), and $j_\nu \propto \nu^{-(p-1)/2}$ in the optically thin ($\nu > \nu_B$) regime. Note that for an electron distribution with low- and high-energy cut-off γ_{min}, γ_{max}, the optically thin spectrum could be more complex with low-frequency part $j_\nu \propto \nu^{1/3}$, mid-frequency part $j_\nu \propto \nu^{-(p-1)/2}$ and high-energy frequency part $j_\nu \propto e^{-\nu/\nu_{max}}$, $\nu_{max} \simeq \gamma_{max}^2 \nu_L$.

5 Kinetic Equation for Electron Distribution

Consider electrons experiencing synchrotron losses before escaping from source.

- particles injected at rate $Q(E,t)dV$ undergoing energy changes within dV:
$$-\frac{dE}{dt} = \eta(E)$$
For synchrotron, $dE/dt = -c_1 E^2$ (lecture 5), $c_1 = \frac{4}{3} c \sigma_T \frac{1}{(m_e c^2)^2} \frac{B^2}{8\pi}$.

- At time t, number density of particles in energy range E to $E+\Delta E$: $n(E)\Delta E$

- At later $t + \Delta t$, these particles are replaced by those having had energy E' to $E' + \Delta E'$ at t, with:
$$E' = E + \eta(E)\Delta t \qquad \text{and :}$$
$$\begin{aligned} E' + \Delta E' &= (E + \Delta E) + \eta(E + \Delta E)\Delta t \\ &\simeq (E + \Delta E) + \eta(E)\Delta t + \frac{\partial \eta(E)}{\partial E}\Delta E \Delta t \end{aligned}$$
using Taylor expansion for small ΔE.

- So

$$\begin{aligned}
\Delta E' &= [E' + \Delta E'] - E' \\
&= \left[(E + \Delta E) + \eta(E)\Delta t + \frac{\partial \eta(E)}{\partial E}\Delta E \Delta t\right] - (E + \eta(E)\Delta t) \\
&= \Delta E + \frac{\partial \eta(E)}{\partial E}\Delta E \Delta t
\end{aligned}$$

- Change in $n(E)\Delta E$ in time interval Δt:

$$\Delta n(E)\Delta E = n(E', t)\Delta E' - n(E, t)\Delta E = n\left(E + \eta(E)\Delta t, t\right)\Delta E' - n(E, t)\Delta E$$

applying Taylor expansion for small $\eta(E)\Delta t$:

$$\Delta n(E)\Delta E = -n(E, t)\Delta E + n(E, t)\Delta E' + \frac{\partial n(E)}{\partial E}\eta(E)\Delta t \Delta E'$$

Substituting $\Delta E' = \Delta E + \frac{\partial \eta(E)}{\partial E}\Delta E \Delta t$ from above:

$$\begin{aligned}
\Delta n(E)\Delta E &= \\
-n(E, t)\Delta E &+ n(E, t)\left(\Delta E + \frac{\partial \eta(E)}{\partial E}\Delta E \Delta t\right) + \left(\frac{\partial n(E)}{\partial E}\eta(E)\Delta t\right)\left(\Delta E + \frac{\partial \eta(E)}{\partial E}\Delta E \Delta t\right)
\end{aligned}$$

For small ΔE and $\eta(E)\Delta t$ end up with (two 2nd order):

$$\Delta n(E)\Delta E = n(E,t)\frac{\partial \eta(E)}{\partial E}\Delta E \Delta t + \frac{\partial n(E)}{\partial E}\eta(E)\Delta t \Delta E$$

Thus

$$\boxed{\frac{\partial n(E,t)}{\partial t} := \frac{\Delta n(E)}{\Delta t} = n(E,t)\frac{\partial \eta(E)}{\partial E} + \eta(E)\frac{\partial n(E,t)}{\partial E} = \frac{\partial}{\partial E}\left[\eta(E)n(E,t)\right]}$$

Including injection term $Q(E,t)dV$:

$$\boxed{\frac{\partial n(E,t)}{\partial t} = \frac{\partial}{\partial E}\left[\eta(E)n(E,t)\right] + Q(E,t)}$$

$\Rightarrow$ Kinetic equation *describing evolution of particle distribution in the presence of a cooling function* $\eta(E)$.

6 Application: Cooling - Distortion of HE Injection Spectrum

For synchrotron: $\eta(E) = c_1 E^2$, thus

$$\frac{\partial n(E,t)}{\partial t} = \frac{\partial}{\partial E}\left[c_1 E^2 n(E,t)\right] + Q(E,t)$$

In (quasi) steady-state, $\partial n/\partial t = 0$, with continuous injection:

$$\frac{\partial}{\partial E}\left[c_1 E^2 n(E)\right] = -Q(E)$$

$$\Rightarrow \quad n(E) = \frac{1}{c_1 E^2}\left(\mathrm{const.} - \int_{\mathrm{E}} \mathrm{Q(E')dE'}\right)$$

Examples:

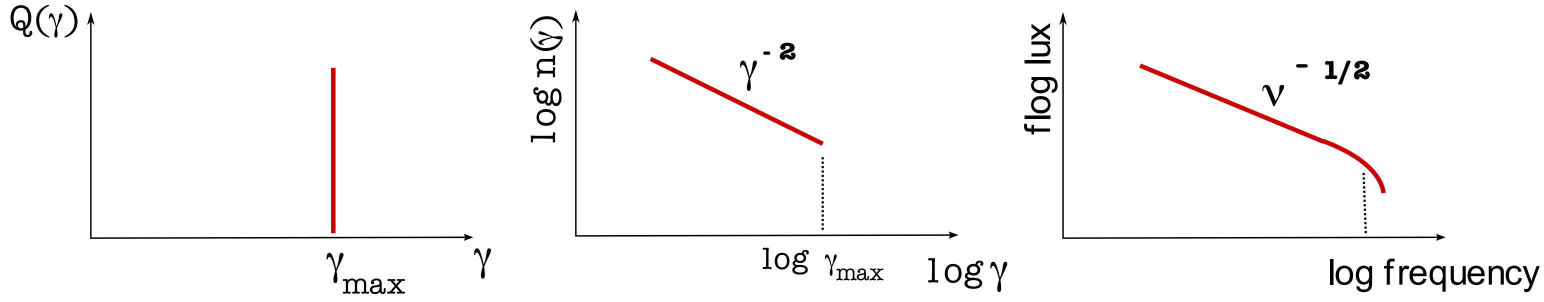
- Mono-energetic injection: $Q(E) = Q_0 \delta(E - E_{max})$ with $n(E > E_{max}) = 0$:

$$\Rightarrow \quad n(E) = \frac{Q_0}{c_1 E^2}\, H(E_{max} - E) \propto \frac{1}{E^2}$$

- Power law injection: $Q(E) = Q_0 E^{-p}$, $p > 1$ with $n(E > E_{max}) = 0$,

$$\Rightarrow \quad n(E) \propto \frac{E^{-p+1}}{E^2} \propto E^{-(p+1)} \quad \text{below} \quad E_{max}$$

Monoenergetic injection

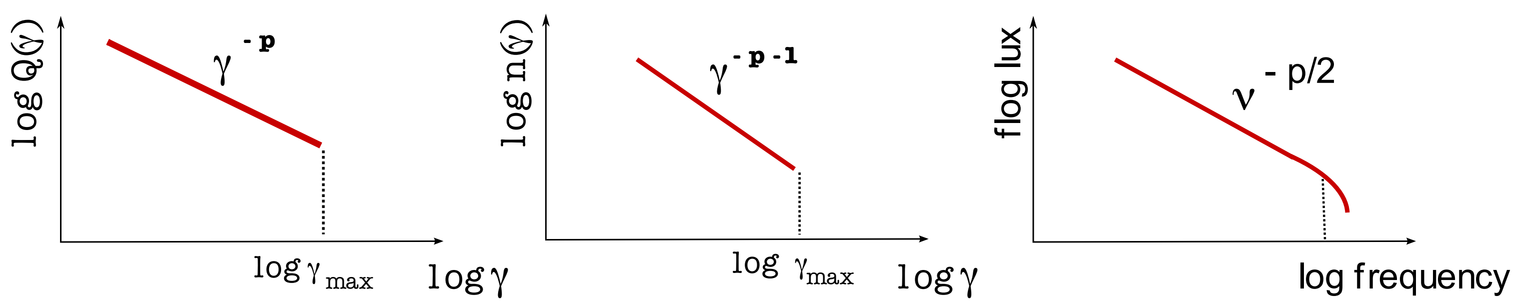


Particle injection

Cooled particle distribution

Resultant synchrotron spectrum

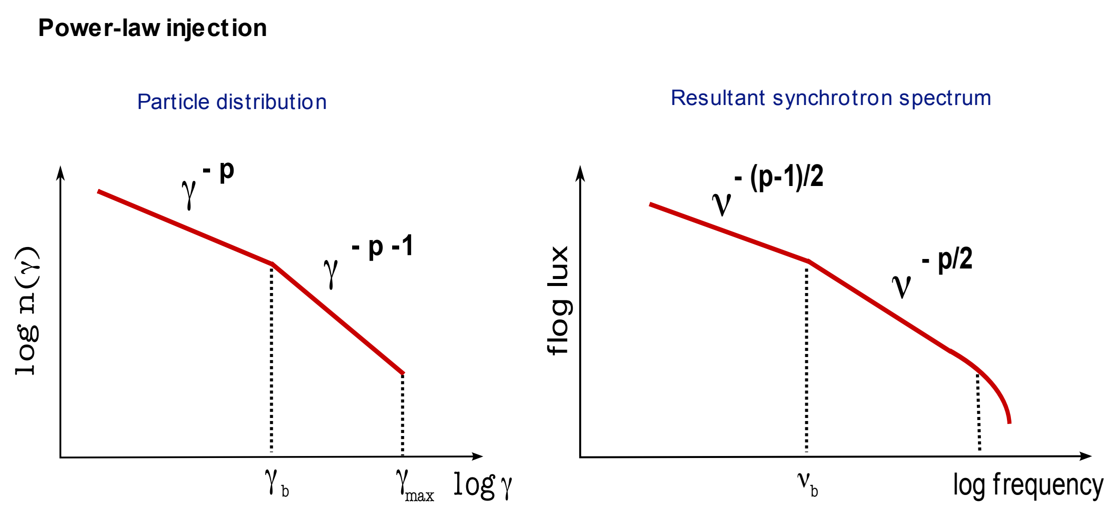
Power-law injection



Consequence of Cooling for Spectral Evolution:

- *Continuous Mono-Energetic Injection:*
  Electron distribution develops power-law tail $n(E) \propto E^{-2}$ below injection, extending with time down to $E(t) = E(t_0)/(1 + c_1 \, [t - t_0] \, E(t_0)) \simeq 1/(c_1 t)$ (lecture 5).
  $\Rightarrow$ Synchrotron emission shows $j_\nu \propto \nu^{-1/2}$ behavior.

- *Power-Law Injection:*
  Electrons distribution steepens, with index $p \rightarrow p + 1$. Break roughly at energy $E_b(t) \simeq 1/(c_1 t)$. Electrons with $E > E_b$ had sufficient time to cool.
  $\Rightarrow$ Synchrotron spectrum above $\nu_b \simeq \frac{1}{2\pi} \gamma_b^2 \Omega_0$ steepens by $1/2$ to $j_\nu \propto \nu^{-p/2}$.



7 Equipartition and Minimum Energy Considerations

How much energy in particle and fields is needed to produce observed synchrotron radiation?

- Energy density in magnetic field: $u_B = \frac{B^2}{8\pi}$

- Energy density in relativistic particles:

$$u_e = n_0 \int_{\gamma_{min}}^{\gamma_{max}} (\gamma m_e c^2) \gamma^{-p} d\gamma \simeq \frac{n_0 m_e c^2}{2-p} \gamma_{min}^{-(p-2)}$$

  for $p > 2$ and $\gamma_{max} \gg \gamma_{\rm min}$.

  ($\gamma_{\rm min}$ can be constrained assuming lowest observed frequency, typically $\sim 10^7$ Hz, to correspond to $\gamma_{min}^2 \nu_L$.)

- For a given source luminosity L_ν with

$$L_\nu = \int j_\nu dV \propto \nu^{-(p-1)/2}$$

  minimising $(u_B + u_e)V$ for a homogeneous source implies minimising $(u_B + u_e)$ for a given j_ν.

- Emissivity for a power law distribution (lecture 5):

$$j_\nu \propto n_0 \, \frac{u_B}{\nu_L} \left(\frac{\nu}{\nu_L} \right)^{-(p-1)/2} ,$$

with $\nu_L \propto B \Rightarrow$ fixed j_ν for given ν implies $n_0 \propto \nu_L \, \nu_L^{-(p-1)/2} \, \frac{1}{u_B} = CB^{-(p+1)/2}$.

- Thus, u_e changes as $u_e \propto n_0 \propto B^{-(p+1)/2}$, while $u_B \propto B^2$. Minimising means

$$\frac{d}{dB} \left(u_e + u_B \right) = -\frac{(p+1)u_e}{2B} + \frac{2u_B}{B} \stackrel{!}{=} 0$$

$\Rightarrow$

$$\boxed{u_e = \frac{4}{p+1} u_B \simeq u_B} \quad (\text{for } 2 < p < 3)$$

$\Rightarrow$ **Equipartition requirement**:
Minimum energy constraint for an optically thin synchrotron source places similar amount of energy in particles as in magnetic fields.

8 Example: Radio Galaxy Cygnus A

Radio synchrotron emission of lobes with $B \sim 10^{-4}$ G. Approximating two lobes by spheres of radius $R \sim 10$ kpc, required minimum energy

$$E_{min} \simeq (u_B + u_e)V \simeq (2\ u_B)\ 2\ \frac{4\pi}{3} R^3 \sim 10^{60}\ \mathrm{erg}$$

$\Rightarrow$ enormous energy ($\sim 10^9$ SN explosions)!

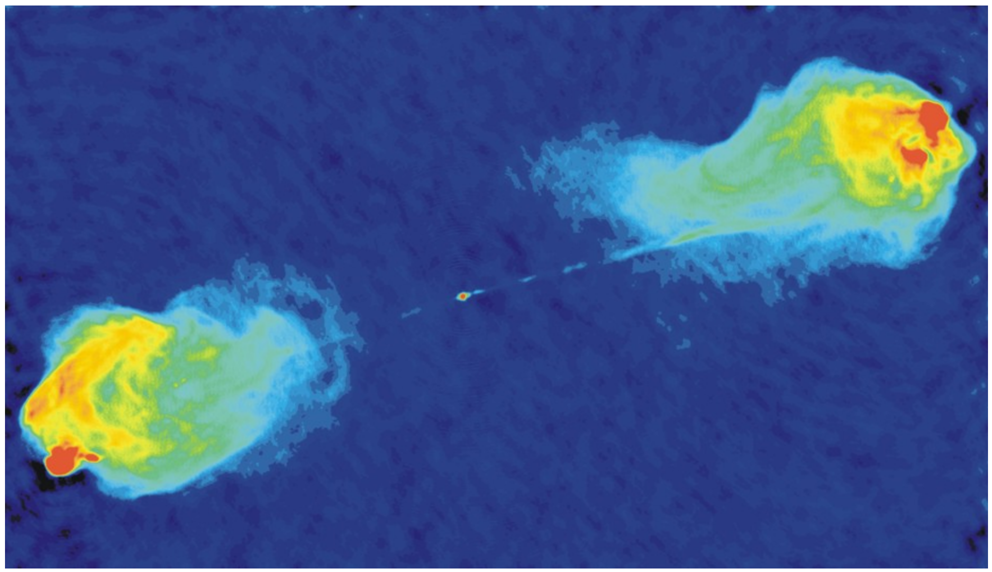


Figure 3: The powerful FR II radio galaxy Cygnus A ($z = 0.057$, $d \simeq 230$ Mpc) with estimated black hole mass of $\sim 10^9 M_\odot$: Radio (5 GHz) false color image of its jet and lobes (stretching 50 kpc from centre). Red shows regions with brightest radio emission, while blue shows regions of fainter emission. The radio structure has an angular extend of ~ 100 arcsec corresponding to ~ 100 kpc [Credits: NRAO/AUI]

9 Curvature Radiation

Curvature Radiation = Synchrotron variant for charged particle moving along curved magnetic field line.

- Radiation primarily due to field line curvature, not gyro-acceleration.

- Recall total synchrotron power for single particle

$$P_{syn} \propto B^2 \gamma^2 \sin^2 \theta \,,$$

  θ pitch angle (angle between magnetic field and particle motion).

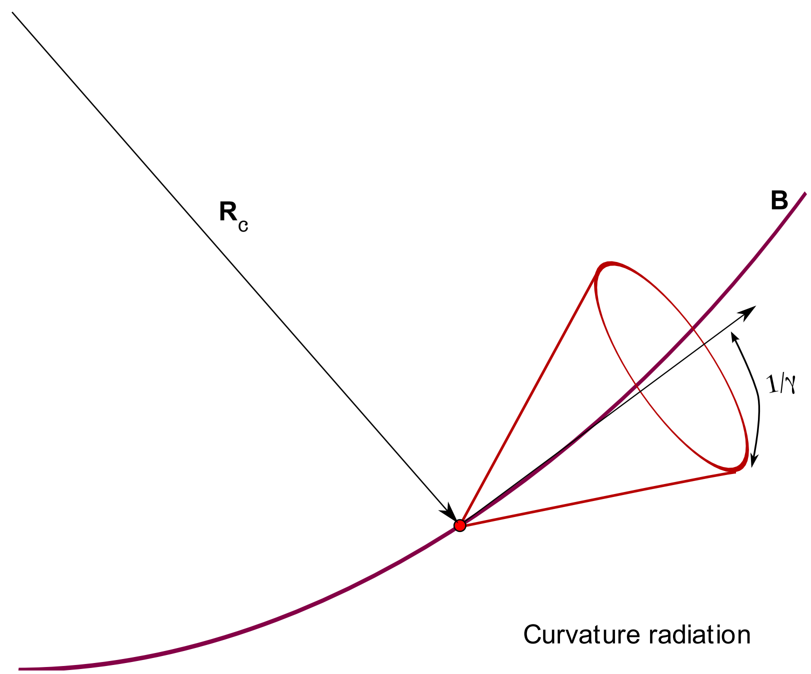
  $\Rightarrow$ in strong B, perpendicular momentum is quickly radiated away.

  $\Rightarrow$ Particle "slides" along field line.

- Curvature of field line can lead to curvature radiation.
  $\Rightarrow$ Expect similar characteristics as for synchrotron.

  $\Rightarrow$ Comparison of acceleration $a = \frac{v_\perp^2}{r_L} = \frac{v^2}{R_c} \rightarrow R_c = \frac{v^2}{v_\perp^2} r_L$

  $\Rightarrow$ replace $\frac{r_L}{\sin^2\theta} = \frac{\gamma m c^2}{eB \sin\theta}$ by curvature radius of field line R_c.



Curvature radiation

10 Total Power Emitted and Characteristic Frequency

Have for synchrotron emission from single particle (lecture 5, for $\beta \sim 1$)

$$P_{syn} = \frac{2}{3}\frac{e^2\gamma^4}{c^3}\frac{v_\perp^2 e^2 B^2}{\gamma^2 m^2 c^2} = \frac{2}{3}\frac{e^4 B^2}{m^2 c^3}\gamma^2 \sin^2\theta$$

$$\nu_c = \frac{3}{4\pi}\gamma^2 \frac{eB}{mc}\sin\theta = \frac{3}{4\pi}\gamma^2\,\Omega_0 \sin\theta$$

So for **curvature**, using $\frac{\gamma m c^2}{eB\sin\theta} \leftrightarrow R_c$, or $B \leftrightarrow \frac{mc^2}{e}\frac{\gamma}{R_c \sin\theta}$, total emitted power:

$$\boxed{P_{curv} = \frac{2}{3}\frac{e^2 c}{R_c^2}\gamma^4 \propto \gamma^4}$$

at characteristic frequency:

$$\boxed{\nu_{curv} = \frac{3c}{4\pi R_c}\gamma^3 \propto \gamma^3}$$

Corresponding cooling timescale:

$$\boxed{t_{cool} = \frac{E}{|dE/dt|} = \frac{\gamma m c^2}{P_{curv}} = 180\ R_c^2 \left(\frac{m}{m_e}\right)\frac{1}{\gamma^3} \quad [\mathrm{sec}]}$$

11 Example: VHE Emission from AGN Black Hole Magnetosphere

- Suppose unscreened electric field $E \sim B$ is available for particle acceleration.
- Estimate B from equipartition $\frac{B^2}{8\pi} \sim \frac{L_{Edd}}{4\pi r_s^2 c}$, $L_{Edd} \simeq 10^{46} M_{BH,8}$ erg/s, using $r_s = 2GM_{BH}/c^2 \simeq 3 \times 10^{13} M_{BH,8}$ cm $\Rightarrow B \sim 10^4$ G.
- Particle acceleration timescale $t_{acc} = \frac{\epsilon}{|d\epsilon/dt|}$ with $\epsilon = \gamma mc^2$ and $d\epsilon/dt = eEr/(r/c) = eEc$ (potential $\Phi = Er$), so

$$t_{acc} \simeq \frac{\gamma mc}{eB}$$

- Maximum achievable particle energy in the presence of curvature losses:

$$t_{acc} = t_{cool} = 180\ R_c^2 \left(\frac{m}{m_e}\right) \frac{1}{\gamma^3}$$

$$\Rightarrow \gamma_{max} \simeq 10^{10} \left(\frac{R_c}{r_s}\right)^{1/2} \left(\frac{B}{10^4\ \mathrm{G}}\right)^{1/4}$$

$$\Rightarrow \nu_{curv}^{max} = \frac{3c}{4\pi R_c} \gamma^3 \sim 2 \times 10^{26}\ \mathrm{Hz} = 1\ \mathrm{TeV}$$

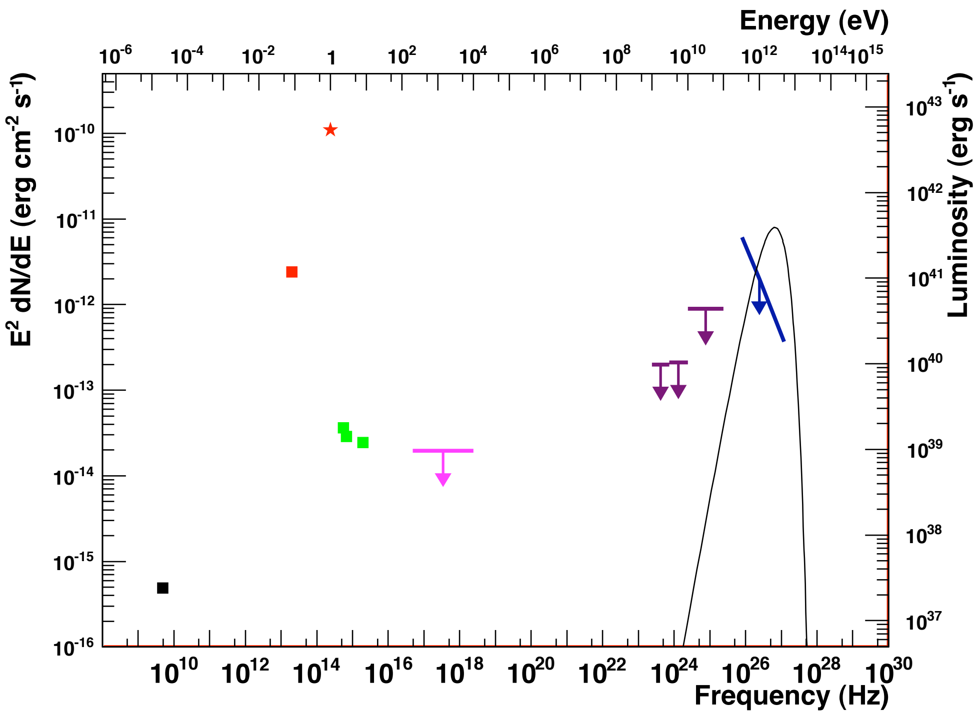


Figure 4: Constraints on Curvature Emission in the nearby ($d \sim 20$ Mpc) giant elliptical Fornax cluster galaxy NGC 1399. Observations and flux upper limits derived for the gamma-ray regime are below the anticipated curvature output (thick black line) suggesting that the available potential is much reduced [from Pedaletti et al. 2011, ApJ 738, 142].