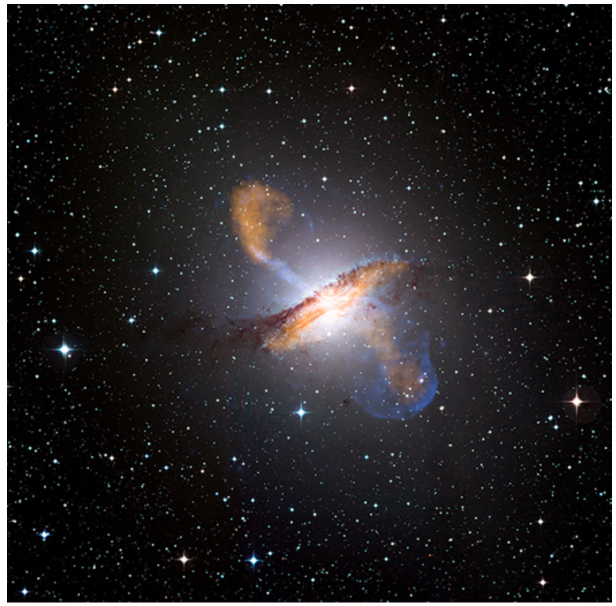
HIGH ENERGY ASTROPHYSICS - Lecture 10



PD Frank Rieger
ITA & MPIK Heidelberg
Wednesday

Pair Plasmas in Astrophysics

1 Overview

- Pair Production $\gamma + \gamma \rightarrow e^{+} + e^{-}$ (threshold)
- Compactness parameter
- Example I: ”internal” $\gamma\gamma$-absorption in AGN (Mkn 421)
- Example II: EBL & limited transparency of the Universe to TeV photons
- Pair Annihilation $e^{+} + e^{-} \rightarrow \gamma + \gamma$ (no threshold)
- Example: Annihilation-in-flight (energetics)

2 Two-Photon Pair Production

Generation of e^+e^--pairs in environments with very high radiation energy density:

$$\gamma + \gamma \to e^+ + e^-$$

- Reaction threshold for pair production (see lecture 9):

$$\boxed{e_1 e_2 - p_1 p_2 \cos\theta = m_1 m_2 c^2 + \delta m\ c^2\ (m_1 + m_2 + 0.5\ \delta m)}$$

  With $m_i = 0$, $e_i := \epsilon_i / c = p_i$ (photons) and $\delta m = 2m_e$:

$$\boxed{\epsilon_1 \epsilon_2 (1 - \cos\theta) = 0.5\,(\delta m)^2 c^4 = 2 m_e^2 c^4}$$

$$\Rightarrow \epsilon_1 \epsilon_2 = m_e^2 c^4$$

  for producing e^+e^- at rest in head-on collision (lab frame angle $\cos\theta = -1$).

- *Example:* for TeV photon $\epsilon_1 = 10^{12}$ eV, interaction possible with soft photons $\epsilon_2 \geq (0.511)^2$ eV $= 0.26$ eV (infrared photons).

- Cross-section for pair-production:

$$\boxed{\sigma_{\gamma\gamma}(\beta^*) = \frac{\pi r_0^2}{2}(1-\beta^{*2})\left[2\beta^*(\beta^{*2}-2) + (3-\beta^{*4})\ \ln\left(\frac{1+\beta^*}{1-\beta^*}\right)\right]}$$

with $\beta^* = v^*/c$ velocity of electron (positron) in centre of momentum frame, and $\pi r_0^2 = 3\sigma_T/8$.

In terms of photon energies and collision angle θ (cf. above):

$$\epsilon_1\epsilon_2(1-\cos\theta) = 2E_e^{*2} \quad \text{and} \quad E_e^* = \gamma^* m_e c^2 = m_e c^2 (1-\beta^{*2})^{-1/2}$$

where E_e^*=total energy of electron (positron) in CoM frame, so

$$\epsilon_1\epsilon_2(1-\cos\theta) = 2\gamma^{*2} m_e^2 c^4 = 2m_e^2 c^4 \frac{1}{(1-\beta^{*2})}$$

$$\Rightarrow \boxed{\beta^* = \sqrt{1 - \frac{2m_e^2 c^4}{\epsilon_1\epsilon_2(1-\cos\theta)}}}$$

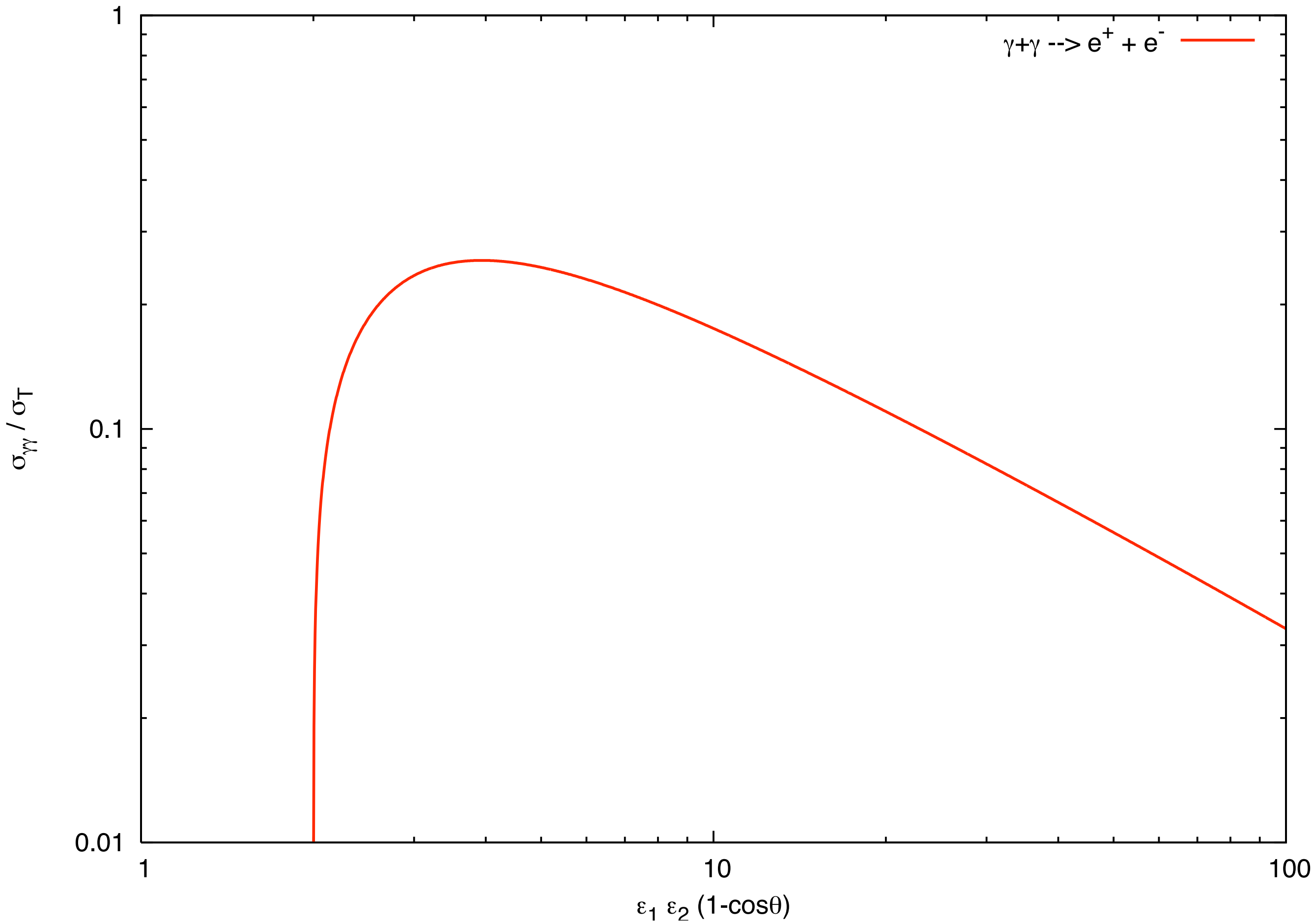


Figure 1: Cross-section for $\gamma\gamma$-pair production in units of the Thomson cross-section σ_T as a function of interacting photon energies $(\epsilon_1/m_ec^2)\,(\epsilon_2/m_ec^2)\,(1-\cos\theta)$. The cross-section rises sharply above the threshold $\epsilon_1\epsilon_2(1-\cos\theta) = 2m_e^2c^4$ and has a peak of $\simeq \sigma_T/4$ at roughly twice this value, i.e. at $\epsilon_1\epsilon_2(1-\cos\theta) = 4m_e^2c^4$.

- At low energies, large annihilation probability of created pairs (cf. later).
- Maximum of cross-section (isotropic radiation field, average of $\cos\theta \to 0$)

$$\sigma_{\gamma\gamma} \simeq 0.25\sigma_T \quad \text{at} \quad \epsilon_1\epsilon_2 \simeq 4m_e^2c^4$$

$\Rightarrow$ *TeV photons interact most efficiently with infrared photons*

$$\boxed{\epsilon_t := \epsilon_2 \simeq 1\ \left(\frac{1\ \text{TeV}}{\epsilon_1}\right)\ \text{eV}}$$

$\Rightarrow$ produced e^+e^- will be highly relativistic and tend to move in direction of initial VHE γ-ray.

- High-energy limit ($\beta^* \to 1$):

$$\begin{aligned}
\sigma_{\gamma\gamma}(\beta^*) &= \frac{\pi r_0^2}{2}(1-\beta^{*2})\left[2\beta^*(\beta^{*2}-2)+(3-\beta^{*4})\ \ln\left(\frac{1+\beta^*}{1-\beta^*}\right)\right] \\
\Rightarrow \sigma_{\gamma\gamma}(\beta^*) &\to \frac{3\sigma_T}{16}(1-\beta^{*2})\left[2(1-2)+(3-1)\ \ln\left(\frac{(1+\beta^*)(1+\beta^*)}{(1-\beta^*)(1+\beta^*)}\right)\right] \\
\sigma_{\gamma\gamma}(\beta^*) &\simeq \frac{3}{8}\frac{\sigma_T}{\gamma^{*2}}\left[\ln(4\gamma^{*2})-1\right] \propto \frac{1}{\epsilon_1\epsilon_t}
\end{aligned}$$

$\Rightarrow$ γ-rays with ϵ_1 can interact with all photons above threshold, but cross-section decreases.

- Delta-function approximation for cross-section in power-law soft photon field (cf. Zdziarski & Lightman 1985):

(e.g., γ-ray with ϵ_1 interacting with power-law differential energy density $n(\epsilon_2) \propto \epsilon_2^{-\alpha_1}$; approximating interaction by cross-section peak at $\epsilon_1\epsilon_2 \simeq 2m_e^2c^4$ [head-on] and accounting for proper normalization)

$$\sigma_{\gamma\gamma}^{\delta}(\epsilon_1, \epsilon_2) \simeq \frac{\sigma_T}{3}\,\frac{\epsilon_2}{m_e c^2}\,\delta\left(\frac{\epsilon_2}{m_e c^2} - \frac{2m_e c^2}{\epsilon_1}\right)$$

- Compare (noting $\delta(ax) = (1/|a|)\delta(x)$):

$$\sigma_{\gamma\gamma}^{\max} \sim \frac{1}{4}\sigma_T\,\delta\left(\frac{\epsilon_1\epsilon_2}{2m_e^2c^4} - 1\right) \;=\; \frac{\sigma_T}{4}\frac{2m_e c^2}{\epsilon_1}\,\delta\left(\frac{\epsilon_2}{m_e c^2} - \frac{2m_e c^2}{\epsilon_1}\right)$$
$$= \frac{\sigma_T}{4}\frac{\epsilon_2}{m_e c^2}\,\delta\left(\frac{\epsilon_2}{m_e c^2} - \frac{2m_e c^2}{\epsilon_1}\right)$$

with

$$\epsilon_1\epsilon_2 \simeq 2m_e^2c^4 \quad \text{(from peak location for head} - \text{on)}$$

- In general optical depth for $\gamma\gamma$-absorption of a γ-ray photon in differential soft photon field $n_{ph}(\epsilon_2, \Omega, x)$ including interaction probability $\propto (1 - \cos\theta)$:

$$\boxed{\tau_{\gamma\gamma}(\epsilon_1) = \int_0^R dx \int_{4\pi} d\Omega \; (1 - \cos\theta) \int_{\frac{2m_e^2c^4}{\epsilon_1(1-\cos\theta)}}^{\infty} d\epsilon_2 \; n_{ph}(\epsilon_2, \Omega, x) \; \sigma_{\gamma\gamma}(\epsilon_1, \epsilon_2, \cos\theta)}$$

Example: For isotropic power-law soft photon field ($F_\nu \propto \nu^{-\alpha}$) with energy spectral index $\alpha = \alpha_1 - 1$, i.e. $n_{ph}(\epsilon_2) \propto \epsilon_2^{-\alpha_1}$ using δ-approximation:

$$\begin{aligned} \tau_{\gamma\gamma}(\epsilon_1) &\simeq \frac{\sigma_T}{3} R \int_{m_e^2c^4/\epsilon_1}^{\infty} d\epsilon_2 \; n_{ph}(\epsilon_2) \; \epsilon_2 \; \delta\left(\epsilon_2 - \frac{2m_e^2c^4}{\epsilon_1}\right) \\ &\simeq \frac{\sigma_T}{3} R \, \frac{2m_e^2c^4}{\epsilon_1} \; n_{ph}\left(\frac{2m_e^2c^4}{\epsilon_1}\right) \propto \frac{1}{\epsilon_1} \, \epsilon_1^{\alpha_1} \; \propto \epsilon_1^{\alpha} \end{aligned}$$

$\Rightarrow \tau_{\gamma\gamma}$ increase with increasing ϵ_1 (there are more targets if $\alpha > 0$).

- Estimating optical depth for homogeneous **non-relativistically moving** source:

$$\tau_{\gamma\gamma}(\epsilon_1) \simeq n_{\epsilon_t}\sigma_{\gamma\gamma}R \simeq \frac{L(\epsilon > \epsilon_t)}{4\pi R c \epsilon_t}\sigma_{\gamma\gamma} \simeq \frac{L_{\epsilon_t}\sigma_{\gamma\gamma}}{4\pi R m_e c^3}$$

with $\sigma_{\gamma\gamma} \simeq \sigma_T/4$ and n_{ϵ_t} number density of target photons, and where last equality holds for typical photon energies $< \epsilon_t > \sim m_e c^2$.

$\Rightarrow$ *source can be opaque for high target luminosities* L_{ϵ_t} *and small sizes* R

- If absorbing radiation field is **external** to source (e.g., BLR in AGN, EBL interactions), $\gamma\gamma$-absorption leads to exponential suppression (lect. 3):

$$\boxed{F_\nu^{obs}(\epsilon) = F_\nu^{int}(\epsilon)\; e^{-\tau_{\gamma\gamma}(\epsilon)}}$$

- If absorption is **internal**, i.e. happens within source, from formal solution of radiation transfer equation: (cf. lecture 3, no background source $I_\nu(0) \simeq 0$, and constant source function $S_\nu := j_\nu/\alpha_\nu$ with $\tau_\nu = \alpha_\nu R$):

$$I_\nu(\tau_\nu) \simeq S_\nu \left(1 - e^{-\tau_\nu}\right) = \frac{j_\nu R}{\alpha_\nu R} \left(1 - e^{-\tau_\nu}\right) = \frac{j_\nu R}{\tau_\nu} \left(1 - e^{-\tau_\nu}\right)$$

$$\Rightarrow F_\nu^{obs}(\epsilon) \simeq F_\nu^{int}(\epsilon) \frac{1 - e^{-\tau_{\gamma\gamma}(\epsilon)}}{\tau_{\gamma\gamma}(\epsilon)} \simeq \frac{F_\nu^{int}(\epsilon)}{\tau_{\gamma\gamma}(\epsilon)} \quad \text{for} \quad \tau_{\gamma\gamma} \gg 1$$

- Compactness parameter l_c for measuring relevance of internal $\gamma\gamma$-absorption:

$$l_c := \frac{L_\gamma}{4\pi R} \frac{\sigma_T}{m_e c^3}$$

(Note: Sometimes, l_c is defined without 4π in denominator)

At MeV energies, $\tau_{\gamma\gamma} \gtrsim 1 \Leftrightarrow l_c \gtrsim 4$.

$\Rightarrow$ if $l_c \gg 1$, then high-energy photons are likely to be absorbed.

3 Example I: "Internal" $\gamma\gamma$-absorption in AGN: Mkn 421 ($d \sim 140$ Mpc)

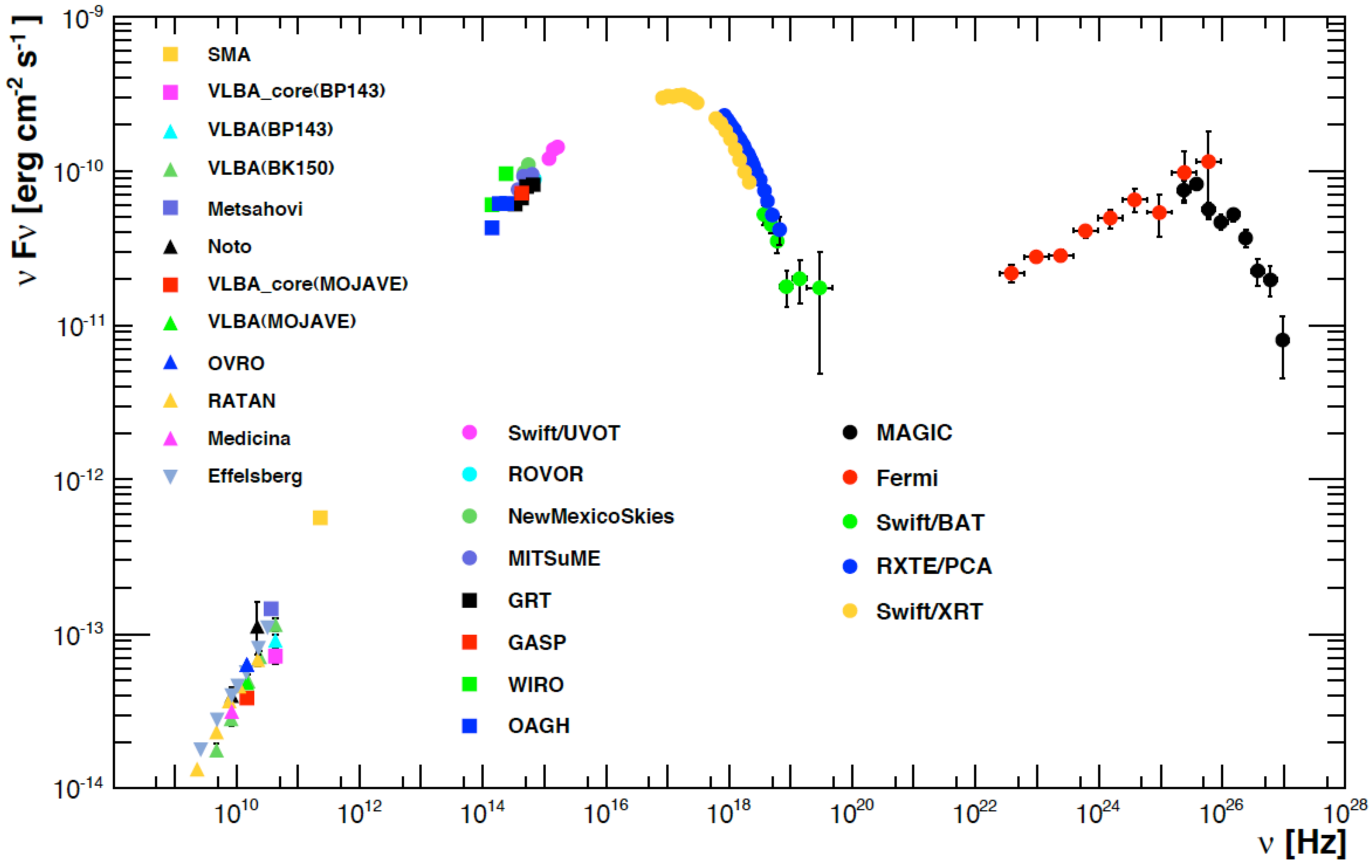


Figure 2: Spectral Energy Distribution (SED) for the BL Lac object Markarian 421 as seen by different instruments [Credits: Abdo, A. et al., ApJ 736 (2011)].

- **Application** to Markarian 421 – assume "characteristic" numbers: Schwarzschild $r_S \sim 10^{14}$ cm, $L_{IR} \sim 10^{44}$ erg/s (cf. SED), $\epsilon_{IR} \sim 1$ eV $\sim 10^{-12}$ erg

$$\Rightarrow \tau(\epsilon_{VHE}) \simeq \frac{L_{IR}\sigma_{\gamma\gamma}}{4\pi R c \epsilon_{IR}} \sim 3 \times 10^5 \left(\frac{r_s}{R}\right) \gg 1$$

  $\Rightarrow$ *IR flux from extended region $R \gg r_s$ and/or jetted-AGN emission* (variability may help to distinguish)

- **Note:** Modification for **relativistically moving source** ("blob"):
  (1) Apparent vs intrinsic luminosity (flux enhancement by beaming, lect. 3):

$$L_t = D^4 L'_t$$

  (2) Short-term variability vs size of emitting region (Doppler formula):

$$\Delta t_{\rm obs} = \Delta t'/D \quad \Rightarrow \quad R' = cD\Delta t_{\rm obs}$$

  (3) Photon energy boosting with $D = 1/[\gamma_b(1 - \beta_b \cos i)]$:

$$\epsilon_t = D\epsilon'_t \quad \Leftrightarrow \quad \epsilon'_t = \epsilon_t/D$$

  $\Rightarrow$ Optical depth:

$$\tau' \simeq \frac{L'_t \sigma_{\gamma\gamma}}{4\pi R' c \epsilon'_t} \propto \frac{L'_t}{R'\epsilon'_t} \propto \frac{L_t}{D^4}$$

4 Example II: "External" Absorption of TeV photons in the Extragalactic Background Light

- Extragalactic background light (EBL) = accumulated light from all galaxies (optical/UV) reprocessed by dust (infrared)

  $\Rightarrow$ TeV photons from a distant AGN traversing EBL will get absorbed.

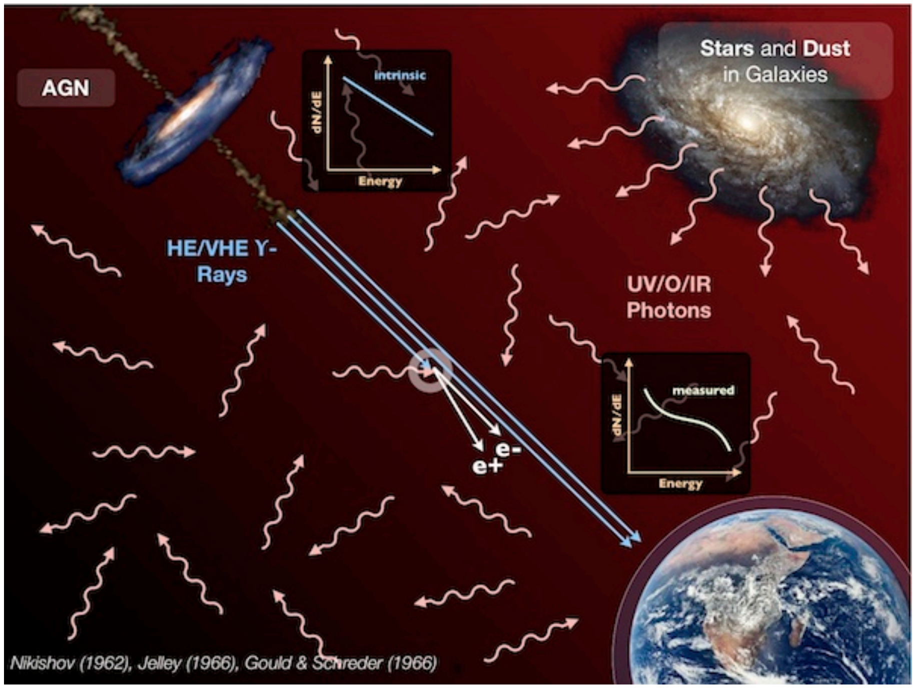


Figure 3: Sketch illustrating absorption of VHE gamma-rays from distant AGN by interaction with the EBL [Credits: M. Raue].

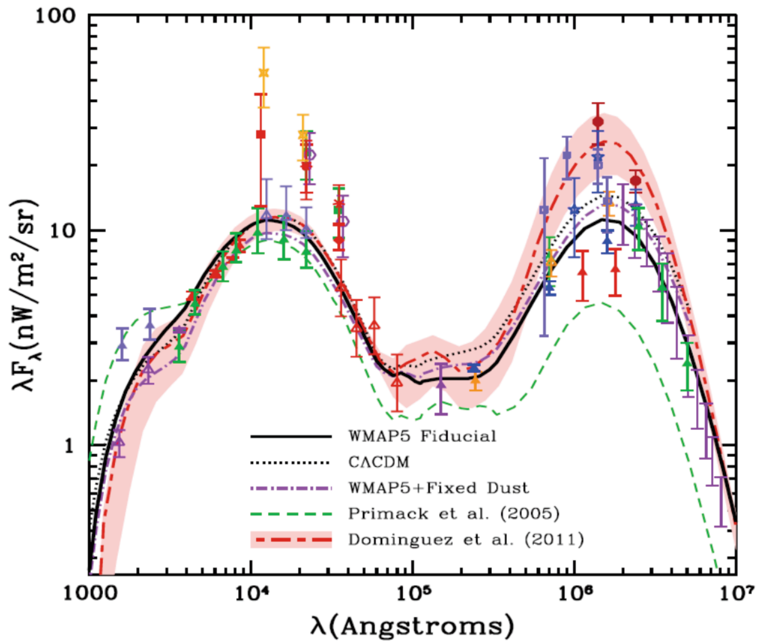


Figure 4: EBL energy density in the local Universe ($z = 0$) with model curves to observations (data points) [Credits: J. Primack]. The EBL is usually difficult to measure due to foreground emission from within the solar system and the Milky Way. Note: λ =1 Angstroem= 10^{-8} cm corresponds to $\nu = c/\lambda = 3 \times 10^{18}$ Hz.

- TeV photons of energy ϵ_1 primarily interact with infrared photons of energy $\epsilon_2 \simeq 1 \ \left(\frac{1 \text{ TeV}}{\epsilon_1}\right)$ eV, corresponding to

$$\lambda_2 = \frac{h\, c}{\epsilon_2} = 1.2 \times 10^4 \ (\epsilon_1/1 \text{ TeV}) \text{ Angstrom} = 1.2 \ (\epsilon_1/1 \text{ TeV}) \ \mu\, m$$

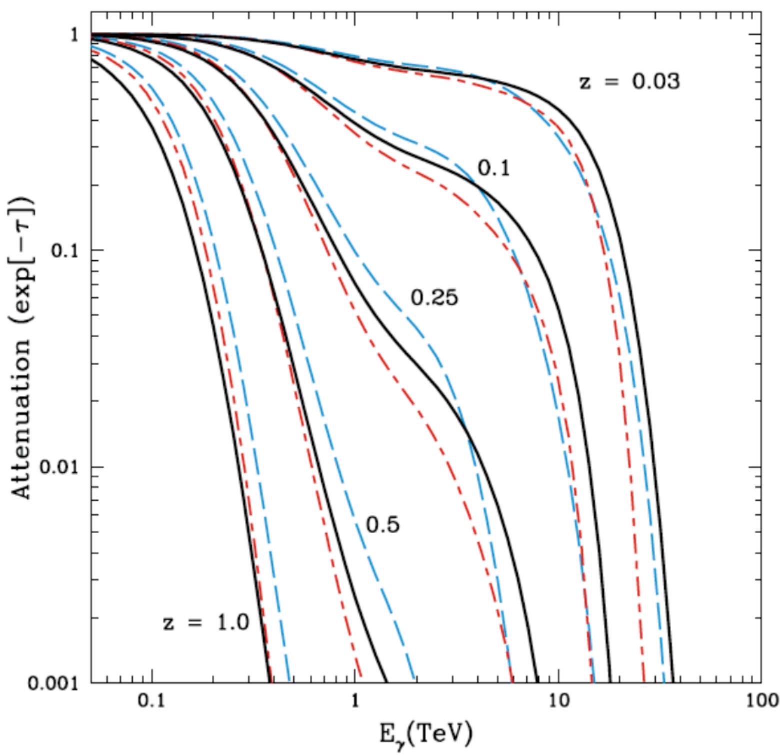


Figure 5: Gamma-ray flux attenuation for γ-ray photons of energy E_γ from sources at different redshifts z. The plateau between 1 and 10 TeV at low redshifts is a consequence of the mid-IR valley in the EBL spectrum [Credits: J. Primack].

- EBL spectrum and intensity depends on cosmological time (star formation history) and hence on redshift z. "Knowing" intrinsic source spectra, $\gamma\gamma$ absorption imprint can be used to diagnose EBL evolution.

5 Pair Annihilation

Decay of e^+e^--pairs into two photons (no threshold):

$$e^+ + e^- \rightarrow \gamma + \gamma$$

- Annihilation cross-section in center of momentum (CoM) frame:

$$\boxed{\sigma_{e^+e^-}(\gamma^*) = \frac{\pi r_0^2}{\gamma^* + 1}\left[\frac{\gamma^{*2} + 4\gamma^* + 1}{\gamma^{*2} - 1}\ \ln\left(\gamma^* + \sqrt{\gamma^{*2} - 1}\right) - \frac{\gamma^* + 3}{\sqrt{\gamma^{*2} - 1}}\right]}$$

  with $\gamma^* = E^*/m_e c^2$ lepton energy in CoM frame, $r_0 = e^2/m_e c^2$ (cgs) classical electron radius. Note: Thomson cross-section $\sigma_T := 8\pi r_0^2/3$.

- Low-energy ($\beta^* << 1$, thermal electrons) limit:

$$\sigma_{e^+e^-} \simeq \frac{1}{\beta^*}\pi r_0^2 = \frac{3\sigma_T}{8\beta^*}$$

  $\Rightarrow$ Annihilation probability is high for leptons nearly at rest.
  $\Rightarrow$ Decay produces two γ's very close to rest energy of electron: $h\nu \sim m_e c^2$
  $\Rightarrow$ 0.511 MeV "annihilation line"

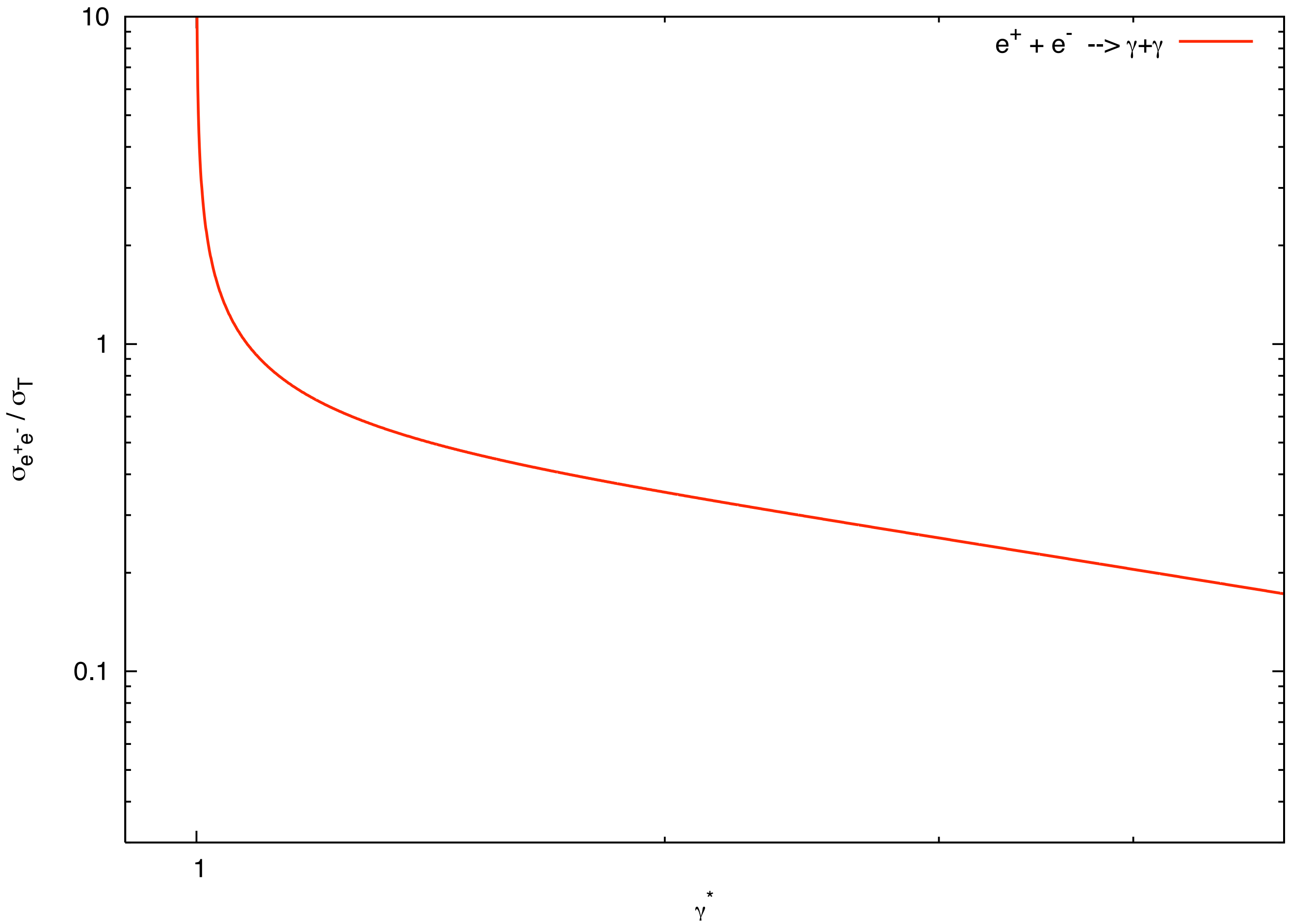


Figure 6: Cross-section $\sigma_{e^+e^-}$ for pair annihilation in units of the Thomson cross-section σ_T as a function of CoM electron/positron Lorentz factor γ^*.

- High energy ($\gamma^* \gg 1$) limit:

$$\sigma_{e^+e^-} \simeq \frac{\pi r_0^2}{\gamma^*} (\ln 2\gamma^* - 1)$$

Decay produces two photons, but photons have broader energy spread $\Rightarrow$ "annihilation spectrum".

- If ambient density of electrons is n_e, characteristic positron annihilation time scale (assuming $\sigma_{e^+e^-} \sim 3\sigma_T/8$) is:

$$t_{ann} \simeq \frac{1}{c n_e \sigma_{e^+e^-}} \sim 4 \times 10^6 \left(\frac{n_e}{\mathrm{cm}^{-3}}\right) \text{ year}$$

- **Note:** e^+ are created in π^+-decay, i.e. $\pi^+ \rightarrow \mu^+ + \nu_\mu$ with $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$, the charged pion's being created in pp-collisions (lect. 9), $p+p \rightarrow p+n+\pi^+ \ldots$. Since pp-collisions also produce π^0 which decay into γ-rays, flux of interstellar positrons created by this process can be estimated from γ-ray luminosity of interstellar gas.

6 Example: Positron-electron-annihilation in flight

Maximum/minimum photon energy in case of relativistic e^+ and stationary e^-:

4-vectors $P_{e^+} = \gamma m_e(c, \vec{v})$, $P_{e^-} = m_e c(1, 0)$, $P_{\gamma_1} = e_1(1, -\vec{n})$, $P_{\gamma_2} = e_2(1, \vec{n})$, with $\vec{n} = \vec{v}/|\vec{v}|$:

$$P_{e^+} + P_{e^-} = P_{\gamma_1} + P_{\gamma_2}$$

$$\Rightarrow P_{\gamma_1}^2 = (P_{e^+} + P_{e^-} - P_{\gamma_2})^2$$

$$\Rightarrow 0 = P_{e^+}^2 + P_{e^-}^2 + P_{\gamma_2}^2 + 2P_{e^+}P_{e^-} - 2P_{e^+}P_{\gamma_2} - 2P_{e^-}P_{\gamma_2}$$

$$\Rightarrow 0 = 2m_e^2c^2 + 2\gamma m_e^2 c^2 - 2\gamma m_e c e_2 \left(1 - \frac{v}{c}\right) - 2m_e c e_2$$

$$\Rightarrow e_i := \frac{\epsilon_i}{c} = m_e c \frac{1+\gamma}{\gamma(1 \mp \frac{v}{c}) + 1} = m_e c^2 \frac{1+\gamma}{\gamma \mp \sqrt{\gamma^2 - 1} + 1}$$

(minus sign for e_2; plus sign for e_1 once solved for P_{γ_2})

Have

$$\Rightarrow e_i := \frac{\epsilon_i}{c} = m_e c^2 \frac{1+\gamma}{\gamma \mp \sqrt{\gamma^2-1}+1}$$

In terms of kinetic e^+-energy: $T = E - m_e c^2 = (\gamma - 1) m_e c^2 \Leftrightarrow \gamma = \frac{T}{m_e c^2} + 1$:

$$\epsilon_i = \frac{m_e c^2 (T + 2m_e c^2)}{T + 2m_e c^2 \mp \sqrt{2m_e c^2 T + T^2}} = \frac{m_e c^2}{1 \mp \frac{T\sqrt{1+2m_e c^2/T}}{T(1+2m_e c^2/T)}} = \frac{m_e c^2}{1 \mp (1 + 2m_e c^2/T)^{-1/2}}$$

For $T \gg m_e c^2$, expansion $(1+x)^{-1/2} \simeq 1 - \frac{x}{2}$,

$$\epsilon_i \simeq \frac{m_e c^2}{1 \mp (1 - m_e c^2/T)}$$

Hence

$$\epsilon_1 \simeq \frac{m_e c^2}{2} (\text{plus sign})$$

$$\epsilon_2 \simeq T(\text{ for minus sign})$$

$\Rightarrow$ forward-going photon (maximum) takes almost all the kinetic energy of the positron, while energy of backward-moving photon (minimum) is only half the rest mass energy.

$\Rightarrow$ in lab. frame, photon spectrum by annihilation will spread over interval $[\epsilon_1, \epsilon_2]$.