Renormalization Group Evolution of Neutrino Mass Parameters

T30d Theoretische Elementarteilchenphysik

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`hep-ph/0108005`, `hep-ph/0110366`, `hep-ph/0203027`, `hep-ph/0203233`

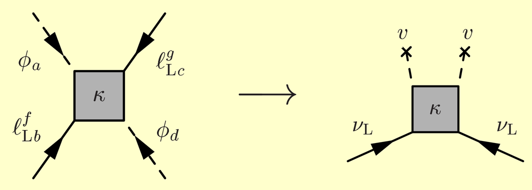
In order to compare predictions for neutrino masses, mixings and CP phases from high-energy theories with experimental results, it is necessary to take into account the renormalization group evolution of these quantities. We present techniques for obtaining the relevant differential equations in various models and show numerical examples.

Effective Neutrino Mass Operators

Lowest dimensional neutrino mass operator in the SM:

$$\mathscr{L}_\kappa = \tfrac{1}{4}\kappa_{gf}\,\overline{\ell^{C}_{\mathrm{L}c}}^{g}\varepsilon^{cd}\phi_d\;\ell^{f}_{\mathrm{L}b}\varepsilon^{ba}\phi_a + \text{h.c.}$$

➥ Majorana masses for the left-handed neutrinos after electroweak symmetry breaking:



Similar operators can appear in the MSSM and in Two-Higgs-Doublet Models (2HDM's).

Renormalization Group Equations

Loop corrections induce dependence of physical quantities on the renormalization scale μ, e.g.

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\kappa = \beta_\kappa$$

Calculation (↷ `hep-ph/0108005`):

Divergences of loop diagrams

⬇

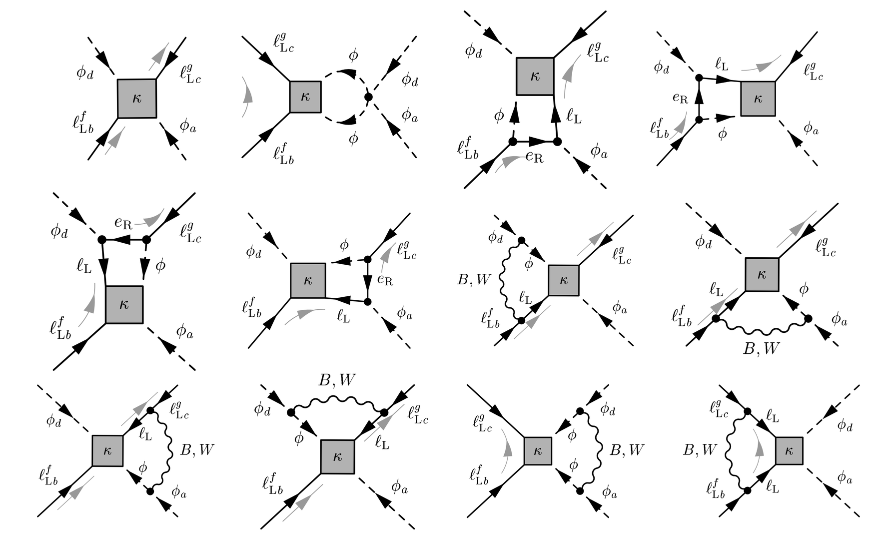
Renormalization constants

⬇

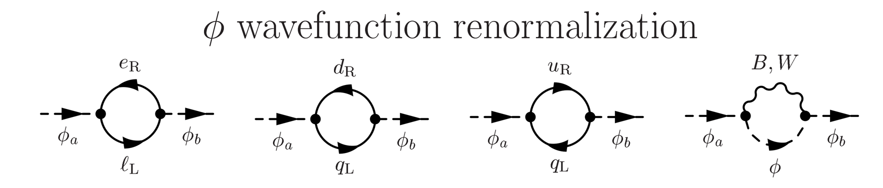
Renormalization group equations

β_κ in the Standard Model

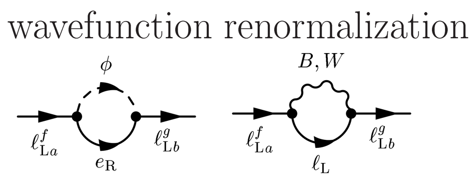
Vertex renormalization



ϕ wavefunction renormalization



ℓ_{L} wavefunction renormalization



⬇ ↷ `hep-ph/0108005` ⬇

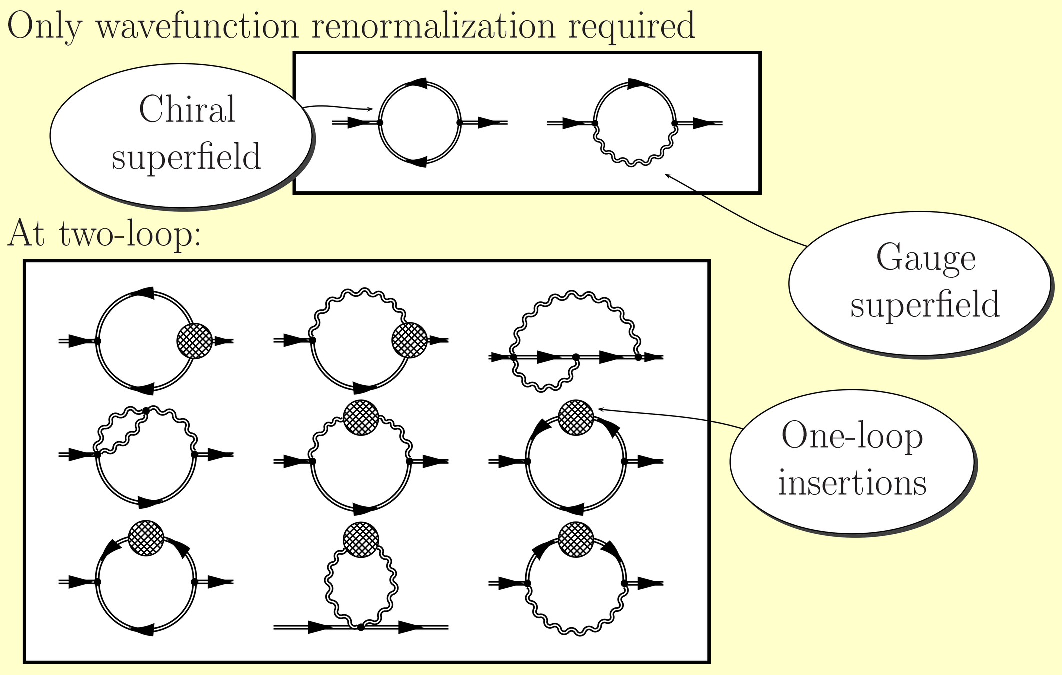
$$16\pi^2\,\beta_\kappa = -\frac{3}{2}\kappa\,(Y_e^\dagger Y_e) - \frac{3}{2}(Y_e^\dagger Y_e)^T\kappa + \lambda\kappa - 3g_2^2\kappa + 2\,\mathrm{Tr}(Y_e^\dagger Y_e)\,\kappa + 6\,\mathrm{Tr}(Y_u^\dagger Y_u)\,\kappa + 6\,\mathrm{Tr}(Y_d^\dagger Y_d)\,\kappa$$

β-functions for the two neutrino mass operators in a class of 2HDM's ↷ `hep-ph/0110366`

MSSM — Supergraph Techniques

Only wavefunction renormalization required

At two-loop:



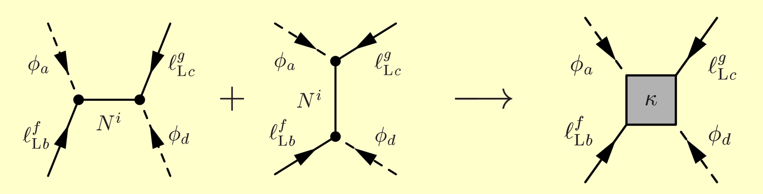
➥ Two-loop β-function for the effective neutrino mass operator in the MSSM ↷ `hep-ph/0203027`

See-Saw Scenario

Standard Model or MSSM enlarged by additional neutrinos N^i:

- Singlets under the SM gauge groups
- Large Majorana masses M_i
- Yukawa couplings Y_ν to Higgs and lepton doublets

➥ Effective neutrino mass operator arises at low energies by integrating out the heavy neutrinos:



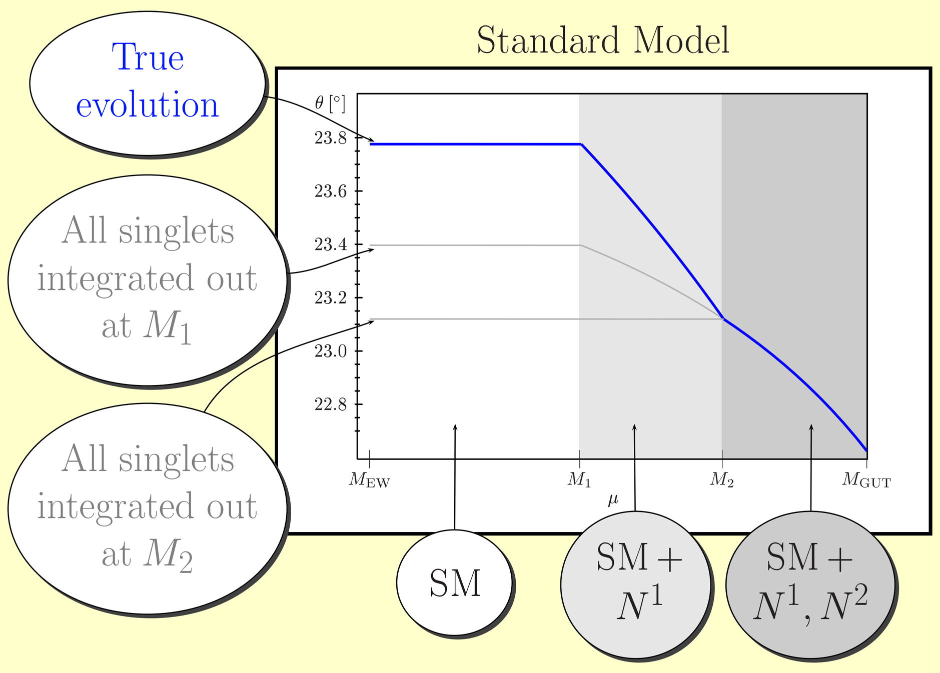
➥ See-saw suppressed neutrino masses: $m_\nu = \frac{v^2}{2}Y_\nu^T M^{-1} Y_\nu$

Two-Neutrino Case

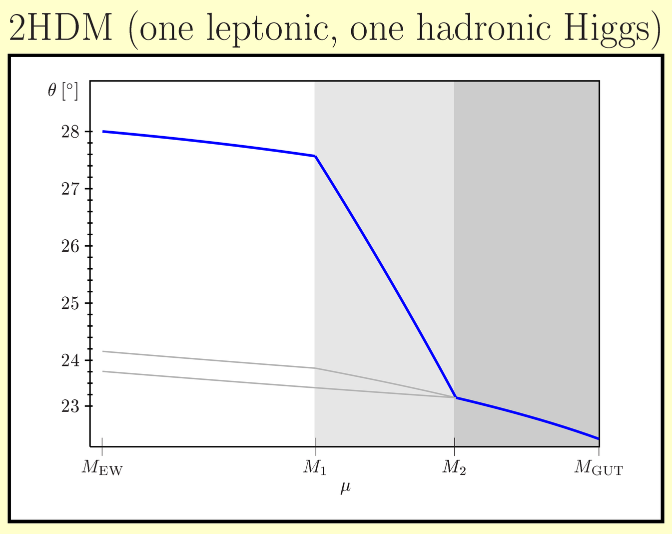
Numerical example (↷ `hep-ph/0203233`):

- 2 lepton doublets
- 2 singlet neutrinos
- Initial conditions at the GUT scale, $M_{\mathrm{GUT}} = 10^{16}$ GeV:
  - Singlet masses: $M_1 = 10^8$ GeV and $M_2 = 10^{12}$ GeV
  - Neutrino Yukawa couplings: real matrices with untuned entries between 0.025 and 1
- Non-degenerate spectrum of heavy singlet masses
  - ➥ Effective theories with singlets partly integrated out

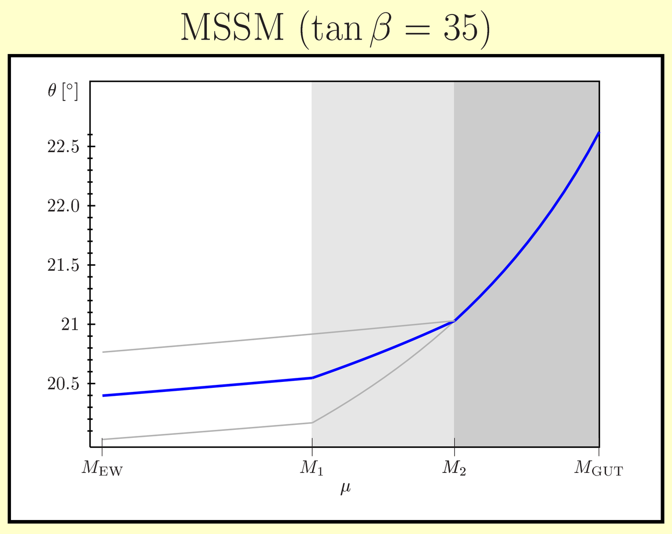
Standard Model



2HDM (one leptonic, one hadronic Higgs)



MSSM ($\tan\beta = 35$)



Correct running cannot be reproduced by integrating out all heavy neutrinos at a common mass scale in general.

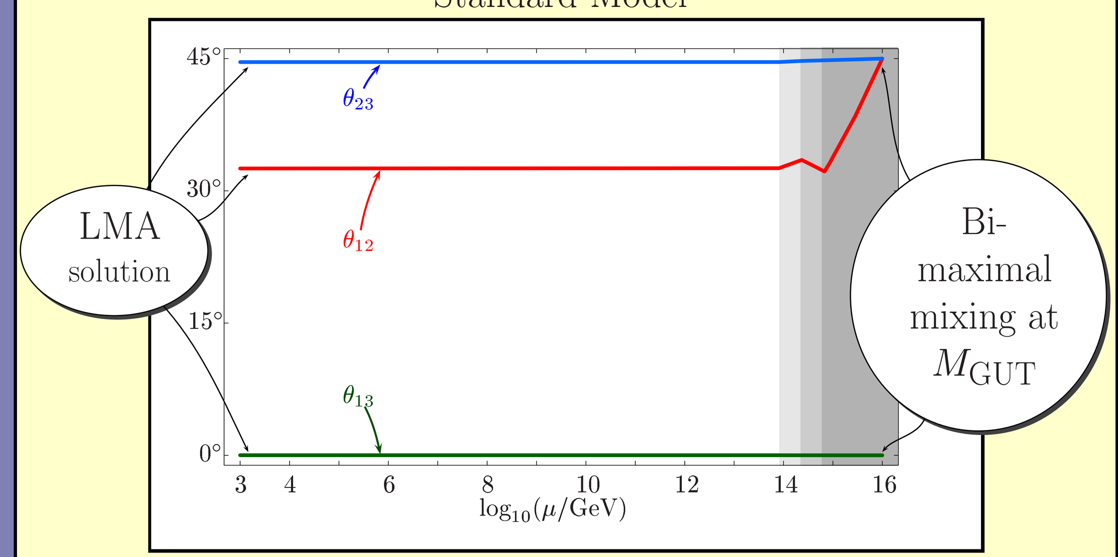
LMA Solution from Bi-Maximal Mixing at the GUT Scale

- 3 lepton doublets
- 3 singlet neutrinos
- All parameters real (➡ no $\not{C}\not{P}$)
- Initial conditions at $M_{\mathrm{GUT}} = 10^{16}$ GeV:
  - Bi-maximal mixing for the light neutrinos
  - See-saw formula $m_\nu = \frac{v^2}{2}Y_\nu^T M^{-1} Y_\nu$

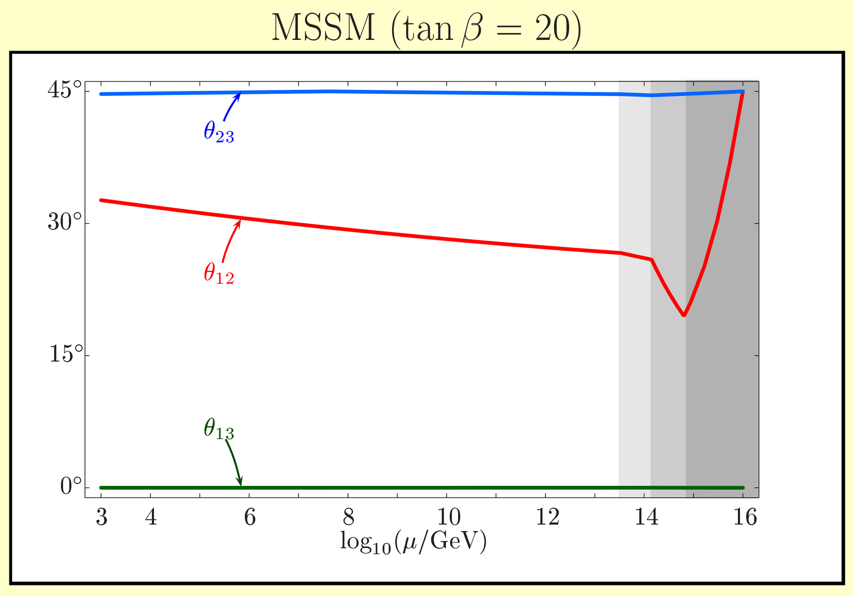
  } ➡ Many choices for the neutrino Yukawa matrix Y_ν possible
- For example, Y_ν diagonal at M_{GUT} (with diagonal Y_e)
  - $Y_\nu = \mathrm{diag}(\varepsilon^2, \varepsilon, 1)$ ➡ wrong sign of $\Delta m^2_{\mathrm{sol}}$
  - $Y_\nu = \mathrm{diag}(1, \varepsilon, \varepsilon^2)$ ➡ evolution to LMA solution possible

Evolution of the Mixing Angles

Standard Model



MSSM ($\tan\beta = 20$)



Generic effect at M_{GUT}:

$$\Delta\theta_{12} \gg \Delta\theta_{23},\,\Delta\theta_{13} \quad \text{for} \quad \min\{m_i\} \gg \sqrt{\Delta m^2_{\mathrm{sol}}}$$

θ_{12} runs away from $\frac{\pi}{4}$, while θ_{13} and θ_{23} stay roughly constant.

➥ Renormalization group evolution may account for the experimentally favored deviation from bi-maximal mixing.

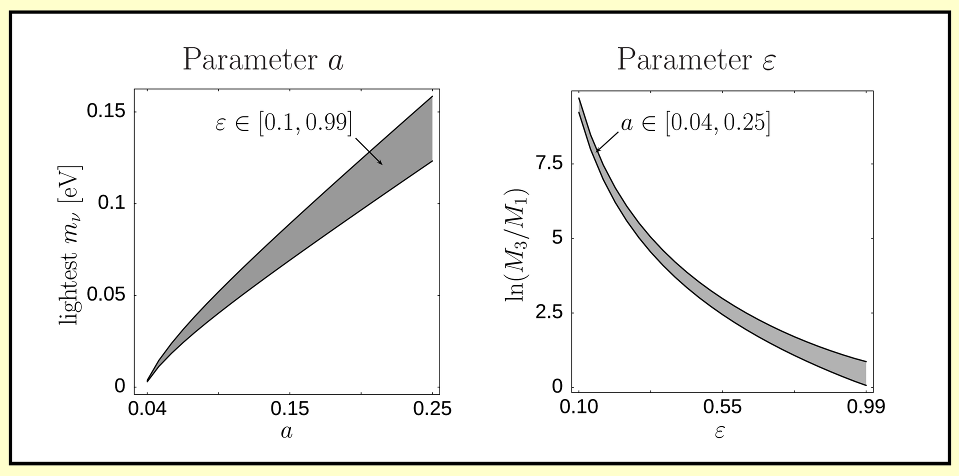
Parameter Space Regions Compatible with LMA Solution of Solar Neutrino Problem

Parameterization of the bi-maximal mass matrix at the GUT scale:

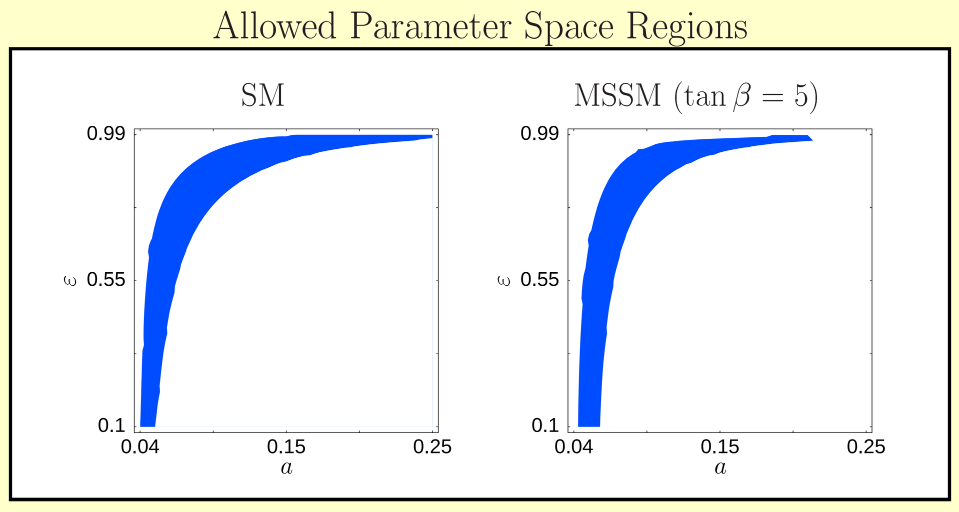
$$m_\nu(M_{\mathrm{GUT}}) = \begin{pmatrix} a-b & c & -c \\ c & a & b \\ -c & b & a \end{pmatrix}$$

- Parameters b and c adjusted to produce the experimentally favored mass squared differences
- a controls the mass of the lightest neutrino (hierarchical or degenerate mass spectra)

In this example: $Y_\nu(M_{\mathrm{GUT}}) = \mathrm{diag}\left(1, \varepsilon, \varepsilon^2\right)$. Parameter ε controls the degeneracy of the see-saw scales.



Allowed Parameter Space Regions



Bi-maximal mixing at the GUT scale can be compatible with the LMA solution for hierarchical and degenerate neutrino masses as well as for degenerate and non-degenerate see-saw scales.

Summary

- The renormalization group evolution of neutrino mass parameters can have sizeable effects.
- In a see-saw scenario with non-degenerate heavy neutrinos, the evolution between mass thresholds has to be taken into account.
- If there is bi-maximal mixing at a high energy scale, θ_{12} typically evolves towards smaller values, while the other angles remain nearly constant.