Bilarge Mixing of Leptons using Abelian Flavor Symmetries

T. Ohlsson and G. Seidl

Technische Universität München

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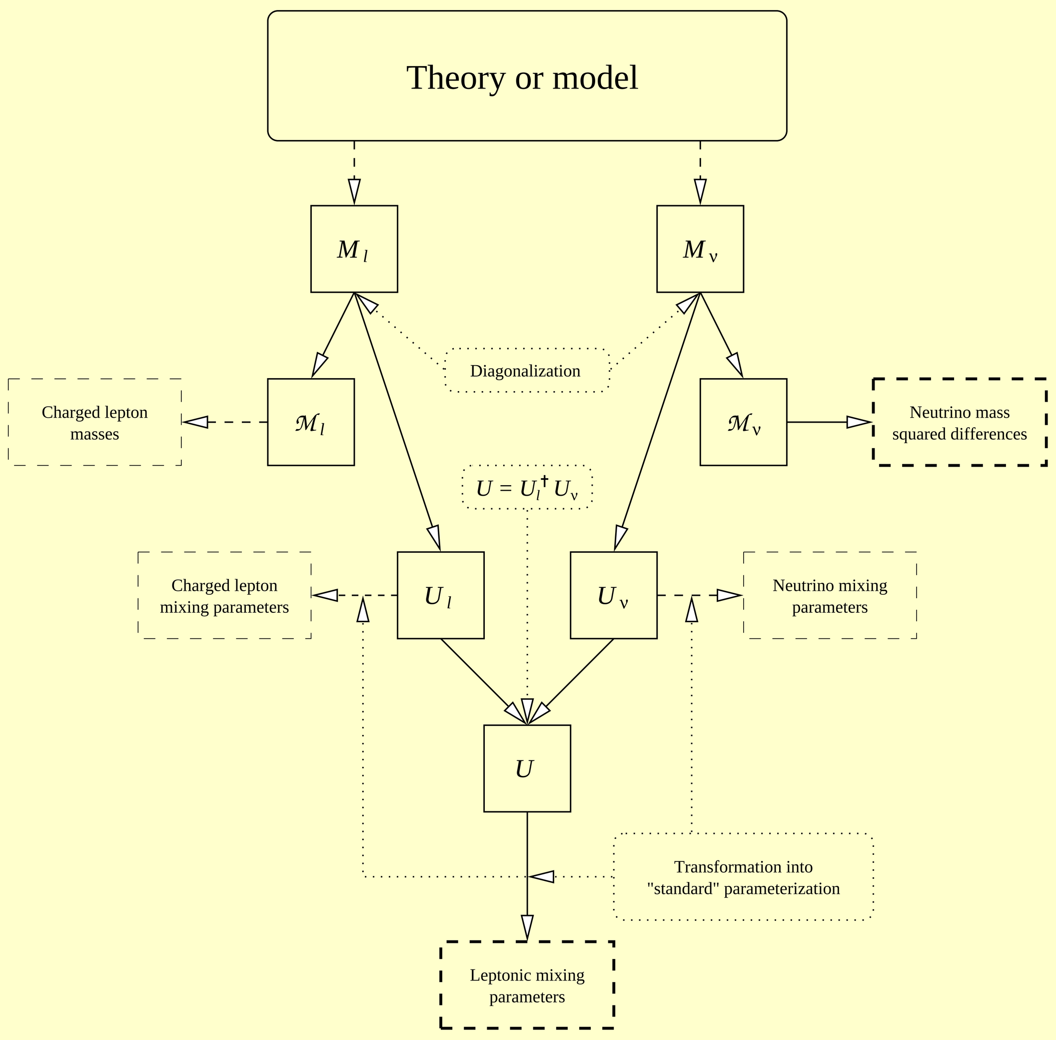
We present a model for the mixing of leptons based on non-renormalizable terms generated by the Froggatt–Nielsen mechanism and Abelian flavor symmetries. The model yields bilarge leptonic mixing, the hierarchical pattern of charged lepton masses and an inverted neutrino mass hierarchy. The leptonic mixing parameters and neutrino mass squared differences which are obtained are all consistent with the atmospheric neutrino data and the Mikheyev–Smirnov–Wolfenstein large mixing angle solution for the solar neutrino problem.

Leptonic mixing

The leptonic mixing parameters result from the mixings of the charged leptons as well as the neutrinos:



Particle representations

We extend the two-Higgs-doublet standard model (SM) by additional superheavy right-handed Dirac neutrinos $F_1, F_2, N_e, N_\mu, N_\tau$, and a number of superheavy fundamental charged fermions as well as by additional SM singlet scalar fields. The neutrinos N_e, N_μ, N_τ have masses of a common order M_2, whereas the common order of mass scale of the remaining additional fermions is denoted by M_1. The fields are assigned anomaly-free gauged U(1) charges Q_1, Q_2, and Q_3 as follows:

Fermions	(Q_1, Q_2, Q_3)
L_e, E_e	$(1,0,0)$
$L_\mu, L_\tau, E_\mu, E_\tau$	$(0,1,0)$
N_e	$(1,0,0)$
N_μ, N_τ	$(0,1,0)$
F_1	$(1,0,0)$
F_2	$(-1,0,1)$

Scalars	(Q_1, Q_2, Q_3)
H_1, H_2	$(0,0,0)$
ϕ_1, ϕ_2	$(1,-1,2)$
ϕ_3, ϕ_4	$(0,0,0)$
$\phi_1', \phi_2', \phi_3', \phi_4'$	$(0,0,0)$
ϕ_5, ϕ_6	$(0,0,1)$
ϕ_7, ϕ_8	$(-1,-1,0)$
ϕ_9	$(-2,0,1)$
ϕ_{10}	$(0,0,0)$
θ	$(0,0,-1)$

The approximately conserved U(1) charges are mainly responsible for the generation of a hierarchical lepton mass matrix pattern. To account for the maximal atmospheric mixing we assume the presence of permutation symmetries:

$$\mathcal{P}_1 : \begin{cases} L_\mu \to -L_\mu, \; E_\mu \to -E_\mu, \; N_\mu \to -N_\mu, \\ \phi_1' \leftrightarrow \phi_2', \quad \phi_1 \leftrightarrow \phi_2, \quad \phi_7 \to -\phi_7, \end{cases}$$

$$\mathcal{P}_2 : \begin{cases} L_\mu \to -L_\mu, \; N_\mu \to -N_\mu, \; \phi_3' \leftrightarrow \phi_4', \\ \phi_3 \leftrightarrow \phi_4, \quad \phi_5 \leftrightarrow \phi_6, \quad \phi_7 \to -\phi_7, \end{cases}$$

$$\mathcal{P}_3 : \begin{cases} L_\mu \leftrightarrow L_\tau, \; E_\mu \leftrightarrow E_\tau, \; N_\mu \leftrightarrow N_\tau, \\ \phi_2 \to -\phi_2, \; \phi_4 \to -\phi_4, \; \phi_6 \to -\phi_6, \\ \phi_7 \leftrightarrow \phi_8. \end{cases}$$

These permutation symmetries establish specific exact relations among the Yukawa couplings as well as among the parameters in the corresponding renormalizable many-scalar potential.

Cyclic symmetries

For $P \equiv e^{2\pi i/n}$ $(n \geq 5)$ we can forbid specific Yukawa couplings and terms in the many-scalar potential by imposing the $\mathbb{Z}_n$ symmetries:

$$\mathcal{C}_1 : \begin{cases} E_e \to P^{-4}E_e, \; E_\mu \to P^{-1}E_\mu, \; E_\tau \to P^{-1}E_\tau, \\ \phi_1 \to P\phi_1, \quad \phi_2 \to P\phi_2, \quad \phi_3 \to P\phi_3, \\ \phi_4 \to P\phi_4, \quad \phi_5 \to P\phi_5, \quad \phi_6 \to P\phi_6, \end{cases}$$

$$\mathcal{C}_2 : \begin{cases} \phi_1' \to -\phi_1', \; \phi_3' \to -\phi_3', \; \phi_1 \to -\phi_1, \\ \phi_3 \to -\phi_3, \; \phi_5 \to -\phi_5, \end{cases}$$

$$\mathcal{C}_3 : \begin{cases} E_e \to P^{-(4k+1)}E_e, \; N_e \to PN_e, \; \phi_1' \to P^{-k}\phi_1', \\ \phi_2' \to P^{-k}\phi_2', \quad \phi_3' \to P^{-l}\phi_3', \; \phi_4' \to P^{-l}\phi_4', \\ \phi_1 \to P^k\phi_1, \quad \phi_2 \to P^k\phi_2, \quad \phi_3 \to P^l\phi_3, \\ \phi_4 \to P^l\phi_4, \quad \phi_5 \to P^l\phi_5, \quad \phi_6 \to P^l\phi_6, \\ \phi_9 \to P^{-1}\phi_9, \quad \phi_{10} \to P\phi_{10}. \end{cases}$$

Here k and l denote appropriately chosen integers and the right-handed neutrinos are $\mathcal{C}_1$ singlets, whereas the $\mathcal{D}_6$ charges of the superheavy charged fermions are integral multiples of $2\pi k/n$ or $2\pi l/n$.

Charged lepton mass operators

The most general charged lepton mass terms, which are invariant under transformations of the symmetries of our model, are given by

$$\mathcal{L}^\ell = \overline{L_\alpha} H_2 \left[(Y^1_{\rm eff})_{\alpha\beta} + (Y^2_{\rm eff})_{\alpha\beta} \right] E_\beta + \text{h.c.},$$

where the relevant effective Yukawa interaction matrices are

$$Y^1_{\rm eff} = \begin{pmatrix} A_1 & B_1 - B_2 & B_1 + B_2 \\ 0 & C_1 - C_2 & 0 \\ 0 & 0 & C_1 + C_2 \end{pmatrix},$$

$$Y^2_{\rm eff} = \text{diag}\left(0, \; D_1 - D_2, \; D_2 + D_2\right).$$

Here the dimensionful coefficients in $Y^1_{\rm eff}$ and $Y^2_{\rm eff}$ are generated via the Froggatt–Nielsen mechanism by higher-dimensional operators of the form

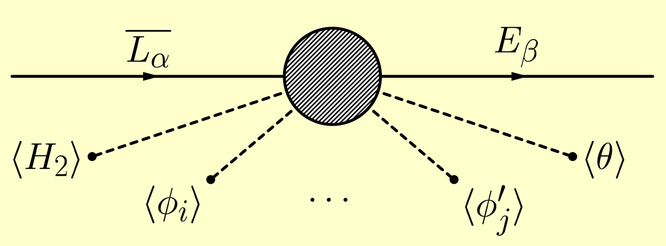


Fig. 1: Non-renormalizable terms generating the effective Yukawa couplings in $Y^1_{\rm eff}$ and $Y^2_{\rm eff}$.

and can – after formally integrating out the heavy fermionic degrees of freedom – be written as

$$A_1 = Y_a^\ell \frac{\phi_{10}}{(M_1)^5}[(\phi_3)^4 + (\phi_4)^4],$$

$$B_1 = Y_b^\ell \frac{\theta^2}{(M_1)^4}\phi_1\phi_1', \quad C_1 = Y_c^\ell \frac{\phi_3'\phi_3}{(M_1)^2}, \quad D_1 = Y_d^\ell \frac{\phi_3'\phi_5}{(M_1)^3},$$

$$B_2 = Y_b^\ell \frac{\theta^2}{(M_1)^4}\phi_2\phi_2', \quad C_2 = Y_c^\ell \frac{\phi_4'\phi_4}{(M_1)^2}, \quad D_2 = Y_d^\ell \frac{\phi_4'\phi_6}{(M_1)^3},$$

where the quantities $Y_a^\ell, Y_b^\ell, Y_c^\ell$, and Y_d^ℓ are order unity coefficients. Note that the pairs $\{B_1, B_2\}$, $\{C_1, C_2\}$, and $\{D_1, D_2\}$ are characterized by the same Yukawa couplings, respectively.

Neutrino Yukawa interaction matrix

The most general Yukawa interactions of the neutrinos, which are invariant under transformations of the symmetries of our model, are to leading order given by

$$\mathcal{L}^\nu = \overline{L^c_\alpha}\frac{(H_1)^2}{M_2}(Y^3_{\rm eff})_{\alpha\beta}L_\beta + \text{h.c.}, \quad Y^3_{\rm eff} = \begin{pmatrix} A_2 & B_3 & B_4 \\ B_3 & 0 & 0 \\ B_4 & 0 & 0 \end{pmatrix},$$

where the effective Yukawa interaction matrix is on the approximate bimaximal mixing form and the dimensionful entries A_2, B_3, and B_4 again arise from non-renormalizable interactions.

Neutrino mass operators

The entries A_2, B_3, and B_4 in the effective neutrino Yukawa interaction matrix $Y^3_{\rm eff}$ are generated via the Froggatt–Nielsen mechanism

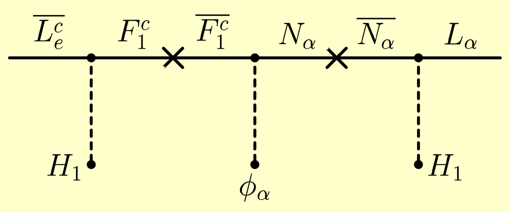


Fig. 2: The dimension six operator for $\alpha = \mu, \tau$ and $\phi_\mu = \phi_7$, $\phi_\tau \equiv \phi_8$, generating the e-μ- and e-τ-elements in $Y^3_{\rm eff}$.

which yields for the dimensionful coefficients in $Y^3_{\rm eff}$ the expressions

$$A_2 = Y_a^\nu \frac{\phi_9\phi_{10}\theta}{(M_1)^3}, \quad B_3 = Y_b^\nu \frac{\phi_7}{M_1}, \quad B_4 = Y_b^\nu \frac{\phi_8}{M_1},$$

where the quantities Y_a^ν and Y_b^ν are order unity coefficients. Note that B_3 and B_4 involve the same Yukawa coupling Y_b^ν.

Alignment mechanism

Suppose that the SM singlet scalar fields aquire their VEVs at some high mass scale and thereby give rise to a small symmetry breaking parameter

$$\epsilon \simeq \frac{\langle\theta\rangle}{M_1} \simeq \frac{\langle\phi_i\rangle}{M_1} \simeq \frac{\langle\phi_j'\rangle}{M_1} \simeq 10^{-1},$$

where $i = 1, 2, \ldots, 10$ and $j = 1, 2, 3, 4$. Then, for a range of parameters in the many scalar potential an alignment mechanism is operative which exactly relates the VEVs of the singlet scalar fields in such a way that after SSB the lepton mass matrices take on the forms

$$M_\ell \simeq m_\tau \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon^4 \\ \epsilon^3 & \epsilon & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \quad \text{and} \quad \text{M}_\nu \sim \begin{pmatrix} \epsilon^2 & 1 & 1 \\ 1 & \epsilon^4 & \epsilon^4 \\ 1 & \epsilon^4 & \epsilon^4 \end{pmatrix}.$$

As a result, we obtain for the charged lepton masses m_e, m_μ, and m_τ the order of magnitude relations

$$m_e/m_\mu \simeq \epsilon^2 \simeq 10^{-2} \quad \text{and} \quad m_\mu/m_\tau \simeq \epsilon \simeq 10^{-1}.$$

For the neutrino masses m_1, m_2, and m_3 we obtain an inverted hierarchy $m_1 \simeq m_2$ and $m_3 \simeq 0$ consistent with the MSW LMA neutrino mass squared differences

$$\Delta m^2_\odot \sim 10^{-5}\text{eV}^2 \quad \text{and} \quad \Delta m^2_{\rm atm} \sim 10^{-3}\text{eV}^2.$$

In the "standard" parameterization, the model predicts a nearly maximal atmospheric mixing angle θ_{23}, a significant deviation from a maximal solar mixing angle θ_{12}, and a relation between θ_{12} and θ_{13}. The model is in perfect agreement with the MSW LMA solution, since, *e.g.*, a choice $Y_b^\ell/Y_d^\ell \simeq 2$ of the order unity Yukawa couplings satisfies

$$\theta_{12} \simeq 37^\circ, \quad \theta_{13} \simeq 8^\circ, \quad \theta_{23} \simeq 44^\circ,$$

which lies in the 90% confidence level region of the MSW LMA solution and which is consistent with the CHOOZ upper bound ($|\theta_{13}| \lesssim 9.2^\circ$).

Summary & Conclusions

Model predictions:

- ✔ Realistic charged lepton mass spectrum
- ✔ Inverse hierarchical neutrino mass spectrum
- ✔ Large (but not necessarily close to maximal) solar mixing angle θ_{12}
- ✔ Approximately maximal atmospheric mixing angle θ_{23}
- ✔ Relation between θ_{12} and θ_{13}

⇒ The MSW LMA solution can be naturally obtained!

References

☞ T. Ohlsson and G. Seidl, Phys. Lett. B **537**, 95 (2002), `hep-ph/0203117`.

☞ T. Ohlsson and G. Seidl, *in preparation*.