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# Structure of neutrino mass matrix and CP violation


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Based on

M.F. and Alexei Yu. Smirnov, hep-ph/0202247 and in preparation

# Introduction

- The structure of neutrino mass matrix gives informations on the underlying theory of masses and possible flavor symmetries.

- In flavor basis, the matrix elements are directly measurable, because of the Majorana nature of neutrinos.

  [K.Zuber, hep-ph/0008080]

- There are two Majorana CP violating phases: the matrix structure can strongly depend on them.

- Different mass spectra are possible: normal hierarchy (ordering), degeneracy, inverted hierarchy (ordering).

- The allowed ranges for oscillation parameters ($\sin\theta_{13}$ , $\theta_{sol}$ and $\Delta m^2_{sol}$ , $\theta_{atm}$ and $\Delta m^2_{atm}$) put serious constraints on the matrix structure.

- Constraints come also from $2\beta_{0\nu}$-decay.

# Mass matrix in flavor basis

- Majorana mass term:

$$\frac{1}{2}\,\overline{\nu^c} M \nu + \text{h.c.} = \frac{1}{2}\left(\overline{\nu_e^c} \quad \overline{\nu_\mu^c} \quad \overline{\nu_\tau^c}\right) \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ \dots & M_{\mu\mu} & M_{\mu\tau} \\ \dots & \dots & M_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} + \text{h.c.}$$

- Mixing:

$$\nu_{\alpha L} = U_{\alpha i} \nu_{iL} \;, \quad \alpha = e, \mu, \tau \;, \quad i = 1, 2, 3$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij} \;, \qquad s_{ij} \equiv \sin\theta_{ij}$$

(the CP violating Dirac phase $\delta$ is always associated with $s_{13}$)

- Mass matrix in flavor basis:

$$M = U^* \begin{pmatrix} m_1 e^{-2i\rho} & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 e^{-2i\sigma} \end{pmatrix} U^\dagger$$

($\rho$ and $\sigma$ are the two CP violating Majorana phases)

$$m_2^2 - m_1^2 \equiv \Delta m^2_{sol} \qquad\qquad |m_3^2 - m_2^2| \simeq |m_3^2 - m_1^2| \equiv \Delta m^2_{atm}$$

- Matrix elements moduli: $m_{\alpha\beta} \equiv |M_{\alpha\beta}|$ , $\quad \alpha, \beta = e, \mu, \tau$

- Adimensional quantities:

$$\tilde{m}_{\alpha\beta} \equiv \frac{m_{\alpha\beta}}{m_2} \;, \qquad r \equiv \frac{m_3}{m_2} \;, \qquad k \equiv \frac{m_1}{m_2} \;.$$

# Experimental Input

- Solar Neutrinos. LMA MSW solution. Allowed intervals (90% C.L.):

$$\tan^2\theta_{12} = 0.35^{+0.2}_{-0.1}\,, \qquad \Delta m^2_{sol} = (5^{+10}_{-2})\cdot 10^{-5}\mathrm{eV}^2$$

[SuperKamiokande, hep-ex/0103032,0103033
SNO, nucl-ex/0106015,0204008,0204009]

- Atmospheric neutrinos. Allowed intervals (90% C.L.):

$$\tan\theta_{23} = 1^{+0.4}_{-0.3}\,, \qquad \Delta m^2_{atm} = (2.5^{+1.5}_{-0.9})\cdot 10^{-3}\mathrm{eV}^2$$

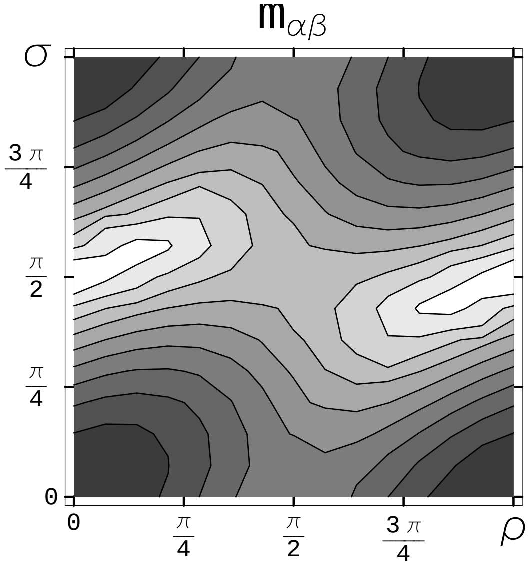
[SuperKamiokande, hep-ex/0105023]

- Bound on $\theta_{13}$ (90% C.L.): $\sin\theta_{13} \lesssim 0.2$

[CHOOZ, hep-ex/9907037]

In the figures we will take best fit values for solar and atmospheric neutrinos and $s_{13} = 0.1$, $\delta = \pi/2$.

# $\rho - \sigma$ plot of a matrix element



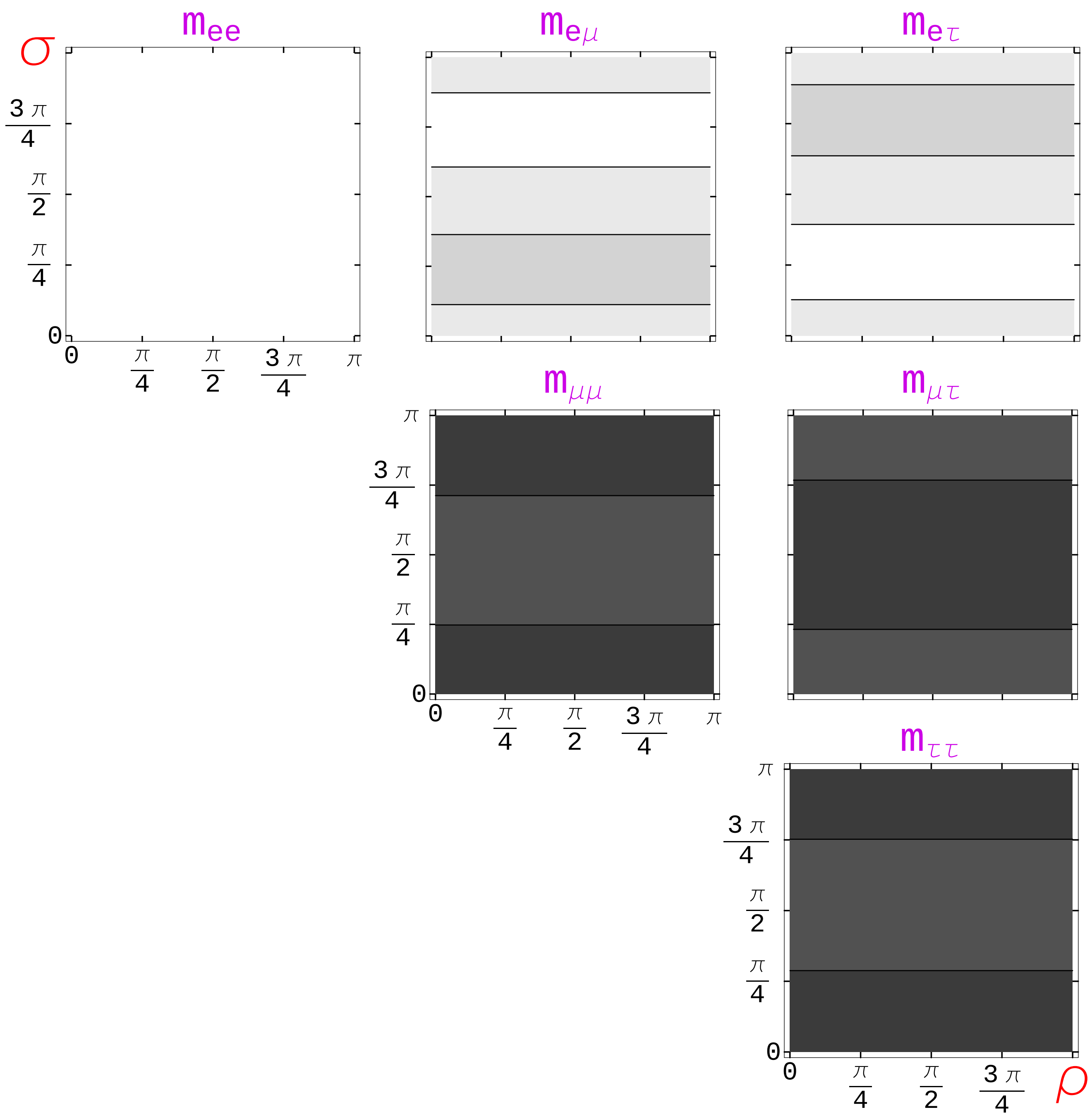

**CP parities**

| $\rho$ | $\sigma$ | $m_1$ | $m_2$ | $m_3$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| $\pi/2$ | 0 | $-1$ | 1 | 1 |
| 0 | $\pi/2$ | 1 | 1 | $-1$ |
| $\pi/2$ | $\pi/2$ | -1 | 1 | -1 |

Darkest regions for $0.9\ m_{max} < m_{\alpha\beta} < m_{max}$
White regions for $0 < m_{\alpha\beta} < 0.1\ m_{max}$

# Normal hierarchy:

$r = 7$, $m_{max} = 0.027$ eV




Dominant $\mu\tau$-block: $m \approx m_3 \begin{pmatrix} 0 & 0 & 0 \\ \dots & * & * \\ \dots & \dots & * \end{pmatrix}$

(with "$*$" we denote order 1 elements)

# Normal hierarchy: dominant $\mu\tau$-block

$$m_1 \ll m_2 \ll m_3 \quad \Rightarrow \quad m_1 \approx 0\ , \quad m_2^2 \approx \Delta m^2_{sol}\ , \quad m_3^2 \approx \Delta m^2_{atm}$$

$$r \approx \sqrt{\frac{\Delta m^2_{atm}}{\Delta m^2_{sol}}} = 7^{+5}_{-4}\ , \qquad k \approx 0\ .$$

• In the limit $s_{13} = k = 0$:

$$m = m_3 \begin{pmatrix} s_{12}^2/r & c_{23}s_{12}c_{12}/r & s_{23}s_{12}c_{12}/r \\ \dots & |c_{23}^2 c_{12}^2/r + s_{23}^2 e^{-2i\sigma}| & s_{23}c_{23}|-c_{12}^2/r + e^{-2i\sigma}| \\ \dots & \dots & |s_{23}^2 c_{12}^2/r + c_{23}^2 e^{-2i\sigma}| \end{pmatrix}$$

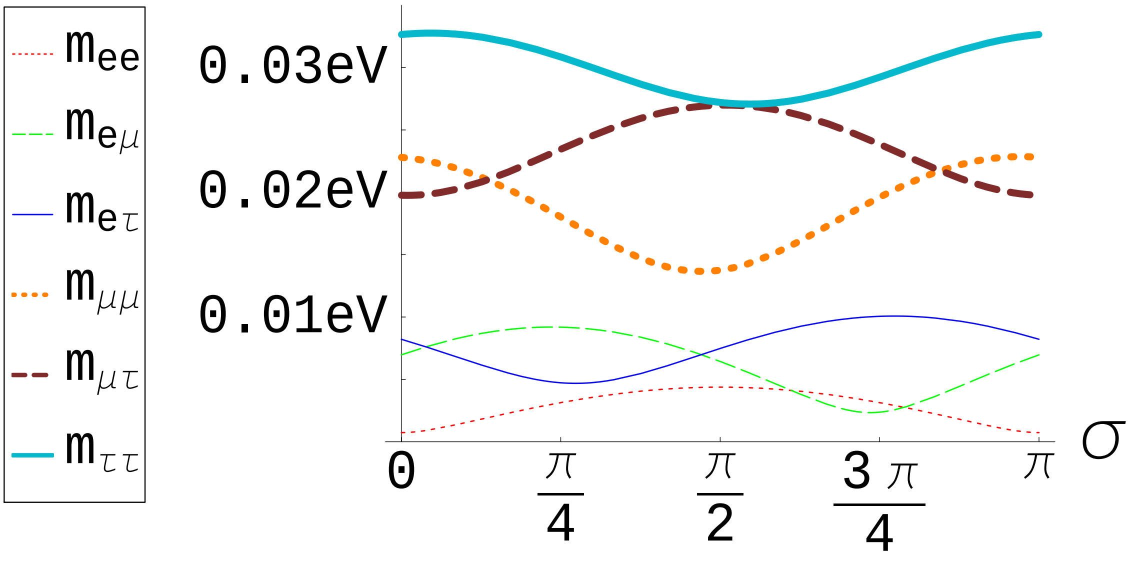
The matrix has a dominant $\mu\tau$-block.

[F.Vissani, hep-ph/9810435,0102236,0111373
J.Sato and T.Yanagida, hep-ph/0009205]

• The uncertainties in r, $\theta_{23}$ and $s_{13}$ strongly influence the matrix structure. Consider the gap between $\mu\tau$-block elements and $e$-row elements:

$$\frac{m_{\mu\tau-\text{block}} - m_{e-\text{row}}}{m_{\mu\tau-\text{block}}} \approx 1 - |\cos 2\theta_{23}| - \frac{1}{r}(c_{12}^2 + \sqrt{2}c_{12}s_{12}) - \sqrt{2}s_{13}$$

For $\Delta m^2_{sol} = 10^{-4}$ eV, $\tan\theta_{23} = 0.8$ and $s_{13} = 0.2$, the gap is strongly reduced:



# The dominant structure of the mass matrix

- The dominant structure of the matrix can be found in the limit $s_{13} \to 0$, because order $s_{13}$ terms are small corrections to the large matrix elements.
  Therefore, the Dirac phase $\delta$ does not influence the dominant structure.

- We have discussed separately the case of normal hierarchy, where $\Delta m^2_{sol}$ can be important. For the other mass spectra, that is for $r \lesssim 2 \div 3$, $m_1$ and $m_2$ are equal up to $\sim 5\%$ corrections. Therefore we can consider the limit $k \to 1$.

- For $s_{13} = 0$ and $k = 1$, the mass matrix becomes:

$$\boxed{\tilde{m} = \begin{pmatrix} x & c_{23}\sqrt{1-x^2} & s_{23}\sqrt{1-x^2} \\ \dots & |c_{23}^2 x + r s_{23}^2 e^{-2i\sigma_x}| & s_{23}c_{23}|-x + re^{-2i\sigma_x}| \\ \dots & \dots & |s_{23}^2 x + r c_{23}^2 e^{-2i\sigma_x}| \end{pmatrix}}$$

  where:

$$x \equiv |s_{12}^2 e^{-2i\rho} + c_{12}^2| = \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \rho}\ ,$$
$$\sigma_x = \sigma + \tfrac{1}{2}\arg(s_{12}^2 e^{-2i\rho} + c_{12}^2)\ .$$

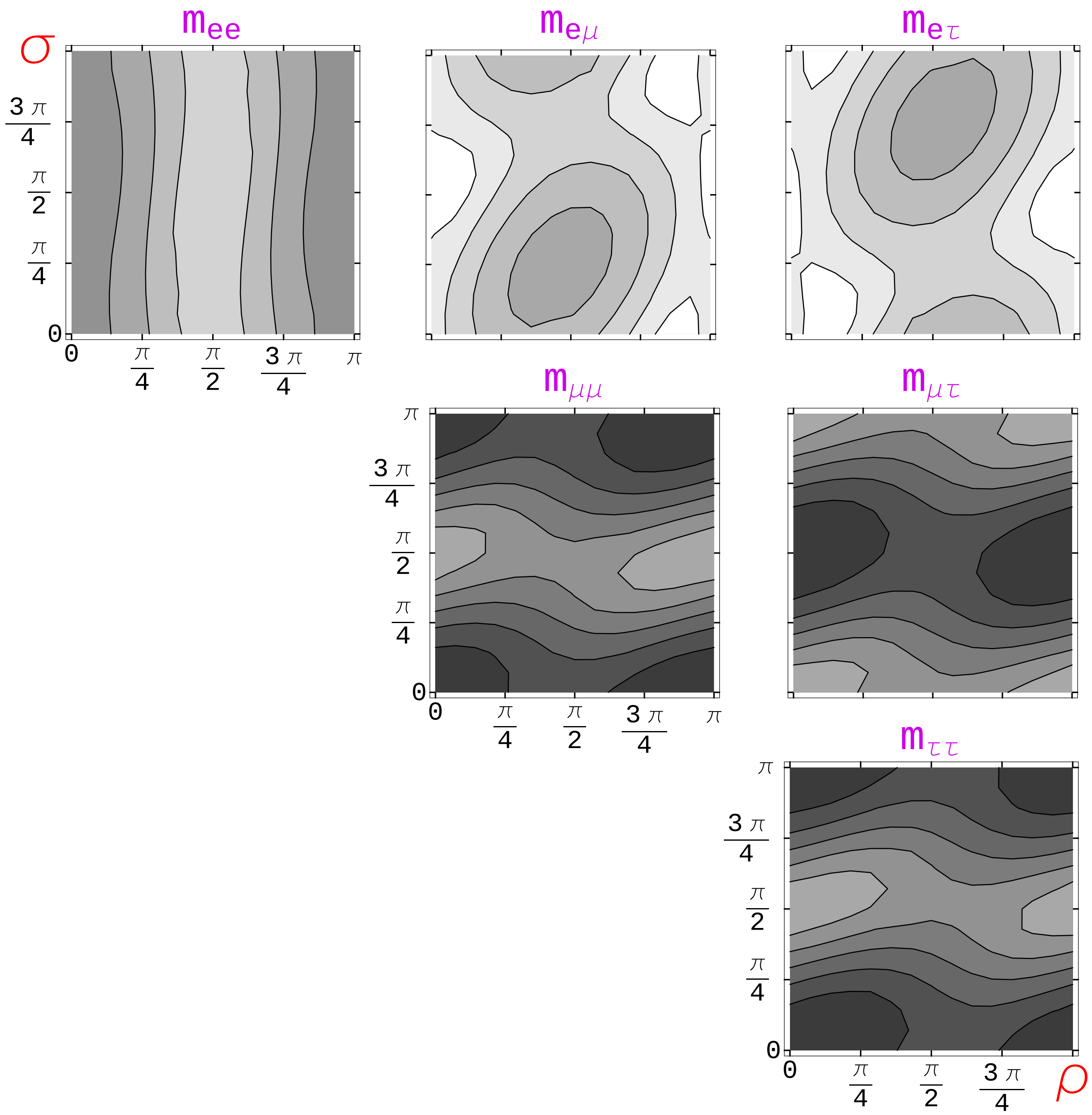
- $\tilde{m}$ depends only on four independent parameters, restricted in the following ranges (90% C.L. for the mixing angles):

$$\begin{aligned} r &\in [0,3)\ , \\ c_{23}^2 &\in [0.35, 0.65]\ , \\ x(\rho,\theta_{12}) &\in [\cos 2\theta_{12}, 1] = [0.2, 1]\ , \\ \sigma_x(\sigma,\rho,\theta_{12}) &\in [0,\pi)\ . \end{aligned}$$

- All the possible hierarchical structures of the mass matrix can be found studying when some elements are zero in $\tilde{m}$.

# Normal ordering:

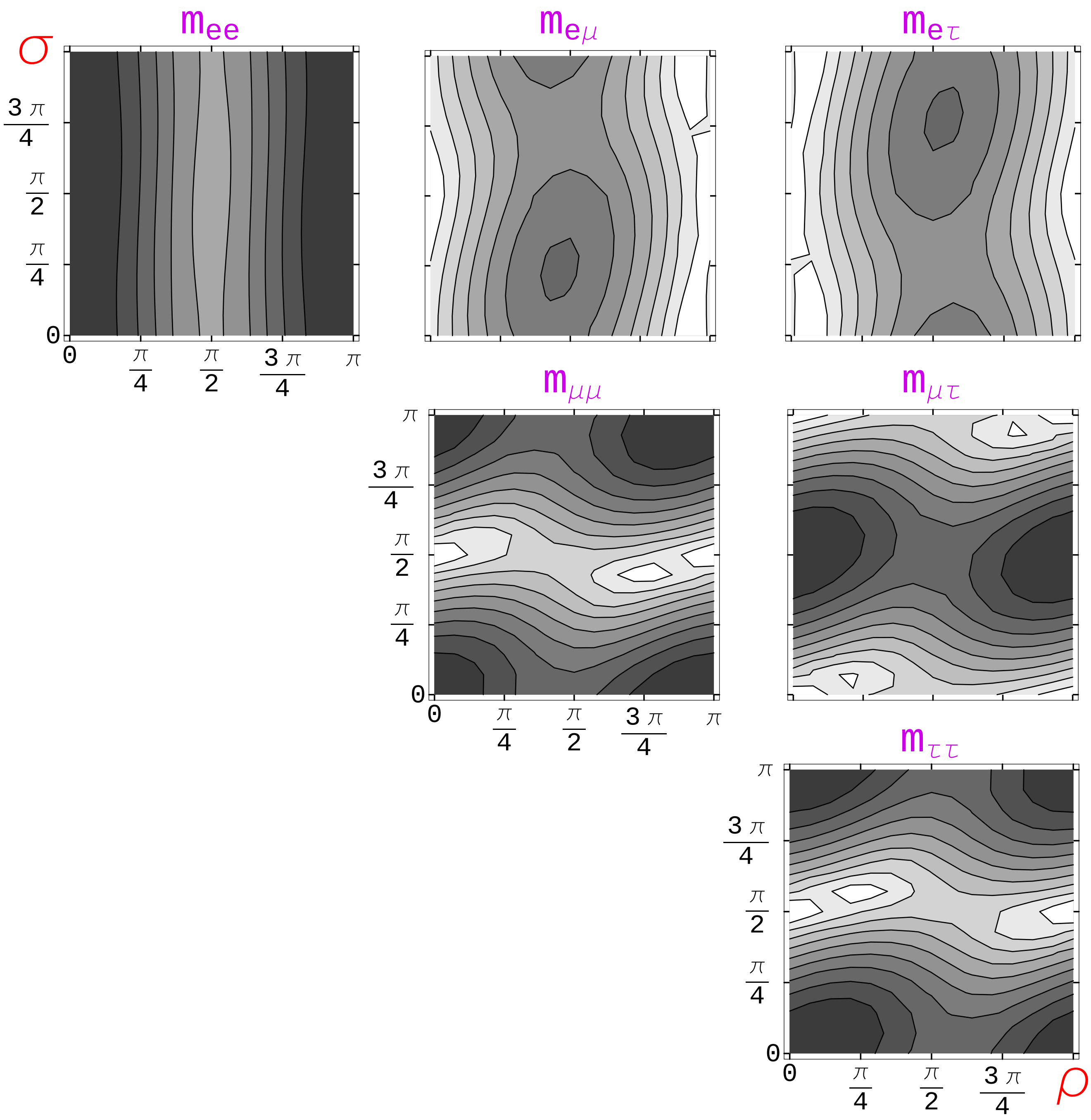
$r = 2.5$, $m_{max} = 0.037$ eV




In this case $m_1 \approx m_2 < m_3$. The dominant $\mu\tau$-block is still present. $m_{ee}$ is larger. The dependence of matrix elements on $\rho$ and $\sigma$ becomes stronger.

# Degeneracy:

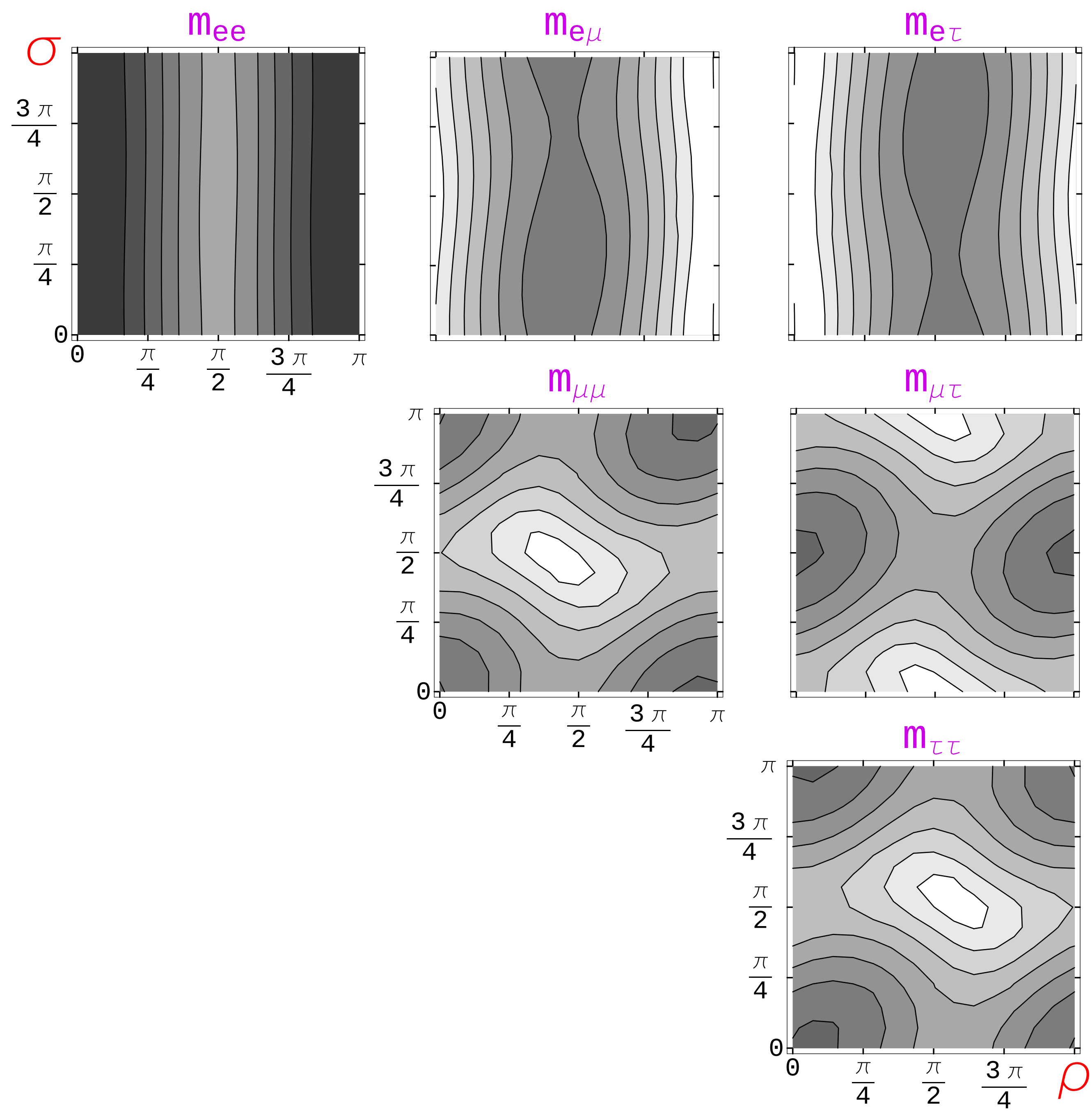
$r = 0.995,\ m_{max} = 0.5$ eV




In this case $m_1 \approx m_2 \approx m_3$. Regions with very small $\mu\tau$-block elements appear. The dependence on $\rho$ and $\sigma$ is very strong.

# Inverted ordering:

$r = 0.4$, $m_{max} = 0.054$ eV

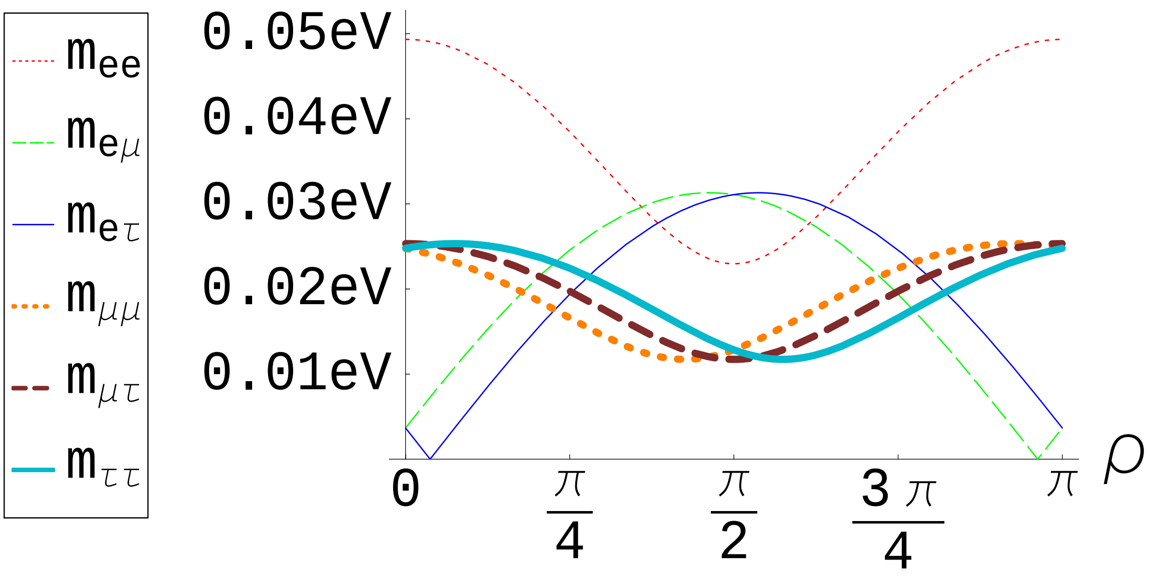



In this case $m_1 \approx m_2 > m_3$. The element $m_{ee}$ is the heaviest. The dependence on $\sigma$ becomes weaker.

# Inverted Hierarchy

$$m_3 \ll m_1 \approx m_2 \quad \Rightarrow \quad m_3 \approx 0\,, \quad m_1^2 \approx m_2^2 \approx \Delta m^2_{atm}$$

$$r \approx 0\,, \qquad k \approx 1\,.$$




• Hierarchical structure for $\rho \approx 0, \pi$ $(x \approx 1)$; up to order $s_{13}$ corrections, we get:

$$\tilde{m} \approx \begin{pmatrix} 1 & 0 & 0 \\ \dots & 1/2 & 1/2 \\ \dots & \dots & 1/2 \end{pmatrix}.$$

[G.Altarelli and F.Feruglio, Phys.Rept.320(1999)295;
H.J.He, D.A.Dicus and J.N.Ng, hep-ph/0203237]

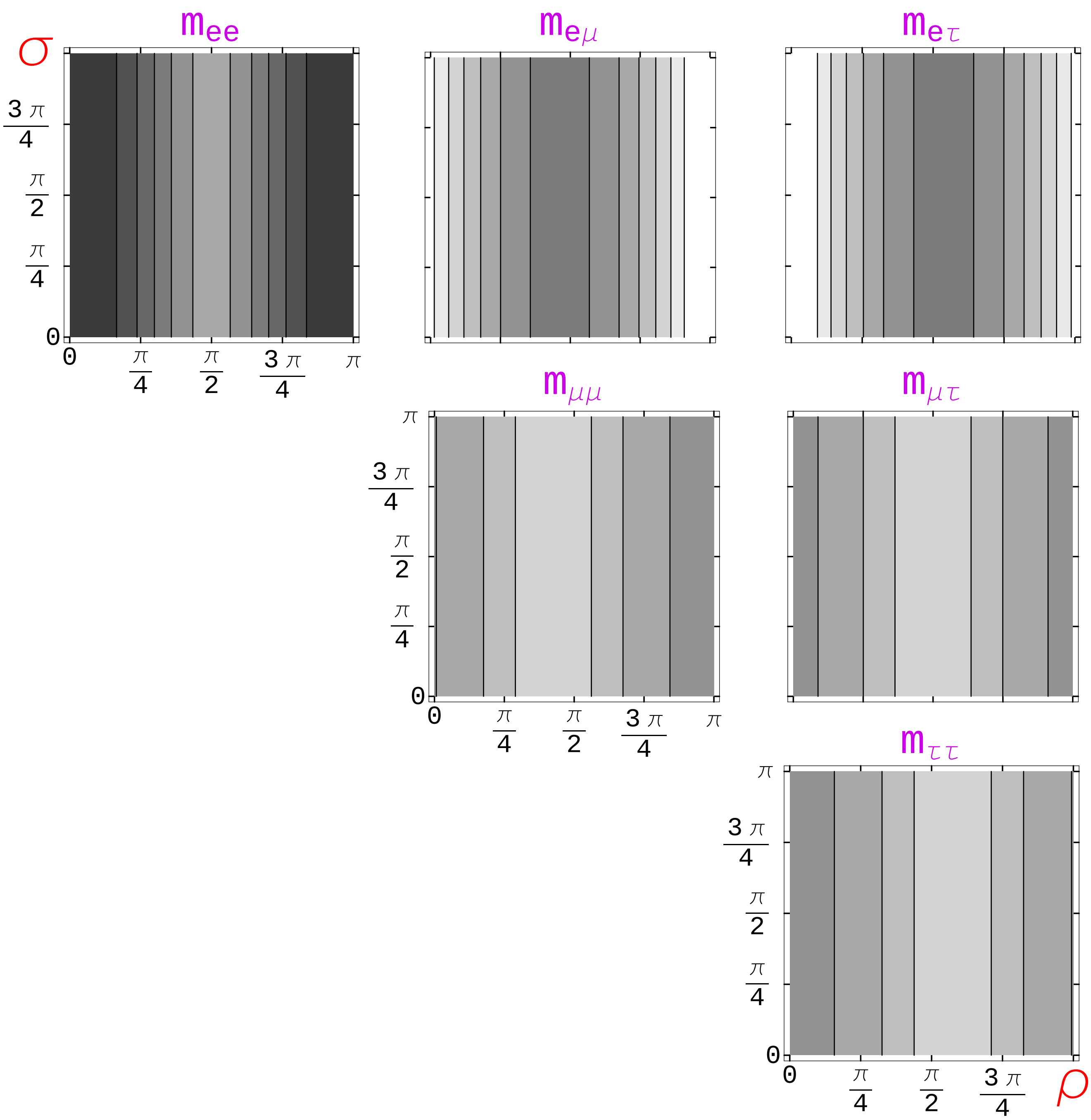
• Ordering structures in the other cases. Some examples, up to order $s_{13}$ corrections:

| $\sin\rho \approx \frac{2}{3}$ | $\sin\rho \approx \sqrt{\frac{2}{3}}$ | $\sin\rho \approx \frac{2\sqrt{2}}{3}$ |
|---|---|---|
| $\sqrt{\frac{2}{3}}\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \dots & \frac{1}{2} & \frac{1}{2} \\ \dots & \dots & \frac{1}{2} \end{pmatrix}$ | $\sqrt{\frac{1}{2}}\begin{pmatrix} 1 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ \dots & \frac{1}{2} & \frac{1}{2} \\ \dots & \dots & \frac{1}{2} \end{pmatrix}$ | $\sqrt{\frac{1}{3}}\begin{pmatrix} 1 & 1 & 1 \\ \dots & \frac{1}{2} & \frac{1}{2} \\ \dots & \dots & \frac{1}{2} \end{pmatrix}$ |

In all these three cases CP is violated.

# Inverted hierarchy:

$$r = 0,\ m_{max} = 0.050 \text{ eV}$$




The dependence on $\sigma$ disappears completely, therefore the determination of $m_{ee}$ in $2\beta_{0\nu}$-decay would also determine all other matrix elements.

# Hierarchical matrix structures

| $\mathcal{O}(s_{13})$ entries | Range for $r$ | $\tilde{m}$ |
|---|---|---|
| $m_{e\mu} \approx m_{e\tau} \approx 0$ | $0 \leq r \lesssim 3$ | $\begin{pmatrix} * & 0 & 0 \\ \dots & * & * \\ \dots & \dots & * \end{pmatrix}$ |
| $m_{\mu\mu} \approx 0$ | $0.4 \lesssim r \lesssim 1.4$ | $\begin{pmatrix} * & * & * \\ \dots & 0 & * \\ \dots & \dots & * \end{pmatrix}$ |
| $m_{\tau\tau} \approx 0$ | $0.4 \lesssim r \lesssim 1.4$ | $\begin{pmatrix} * & * & * \\ \dots & * & * \\ \dots & \dots & 0 \end{pmatrix}$ |
| $m_{\mu\mu} \approx m_{\tau\tau} \approx 0$ | $0.5 \lesssim r \lesssim 0.8$ | $\begin{pmatrix} * & * & * \\ \dots & 0 & * \\ \dots & \dots & 0 \end{pmatrix}$ |
| $m_{\mu\tau} \approx 0$ | $0.5 \lesssim r \lesssim 0.8$ | $\begin{pmatrix} * & * & * \\ \dots & * & 0 \\ \dots & \dots & * \end{pmatrix}$ |
| $m_{e\mu} \approx m_{e\tau} \approx m_{\mu\mu} \approx 0$ | $0.7 \lesssim r \lesssim 0.8$<br>$1.2 \lesssim r \lesssim 2$ | $\begin{pmatrix} * & 0 & 0 \\ \dots & 0 & * \\ \dots & \dots & * \end{pmatrix}$ |
| $m_{e\mu} \approx m_{e\tau} \approx m_{\tau\tau} \approx 0$ | $0.7 \lesssim r \lesssim 0.8$<br>$1.2 \lesssim r \lesssim 2$ | $\begin{pmatrix} * & 0 & 0 \\ \dots & * & * \\ \dots & \dots & 0 \end{pmatrix}$ |
| $m_{e\mu} \approx m_{e\tau} \approx$<br>$m_{\mu\mu} \approx m_{\tau\tau} \approx 0$ | $r \approx 1$ | $\begin{pmatrix} * & 0 & 0 \\ \dots & 0 & * \\ \dots & \dots & 0 \end{pmatrix}$ |
| $m_{e\mu} \approx m_{e\tau} \approx m_{\mu\tau} \approx 0$ | $r \approx 1$ | $\begin{pmatrix} * & 0 & 0 \\ \dots & * & 0 \\ \dots & \dots & * \end{pmatrix}$ |

(with "$*$" we denote order 1 elements)

[Using different assumptions, hierarchical structures are studied in G.Altarelli and F.Feruglio, Phys.Rept.320(1999)295; P.H.Frampton, S.L.Glashow and D.Marfatia,hep-ph/0201008]

# Ordering matrix structures

In large part of the $\rho - \sigma$ plane, all the matrix elements are of the same order (intermediate gray level). Equalities among the elements can be reached. One possibility is that masses decrease with transition from the $\tau$-flavor to the $e$-flavor (flavor ordering), but parameters can be found such that any element can be the smallest or the largest one.

Some interesting examples, up to order $s_{13}$ corrections:

- The matrix has a democratic structure for

$$\theta_{23} = \frac{\pi}{4}, \quad x = \sqrt{\frac{1}{3}}, \quad r = 1\,, \quad \cos 2\sigma_x = 0\ :$$

$$\tilde{m} = \sqrt{\frac{1}{3}} \begin{pmatrix} 1 & 1 & 1 \\ \ldots & 1 & 1 \\ \ldots & \ldots & 1 \end{pmatrix}\,.$$

[H.Fritzsch and Z.Z.Xing, hep-ph/9509389;
E.K.Akhmedov et al., hep-ph/0008010;
K.Fukuura et al., hep-ph/9909415]

- The matrix has equal $e$-row elements and equal $\mu\tau$-block elements for

$$\theta_{23} = \frac{\pi}{4}, \quad x = \sqrt{\frac{1}{3}}, \quad \cos 2\sigma_x = 0\ :$$

$$\tilde{m} = \sqrt{\frac{1}{3}} \begin{pmatrix} 1 & 1 & 1 \\ \ldots & \frac{1}{2}\sqrt{1+3r^2} & \frac{1}{2}\sqrt{1+3r^2} \\ \ldots & \ldots & \frac{1}{2}\sqrt{1+3r^2} \end{pmatrix}\,.$$

- The matrix has equal diagonal elements and equal off-diagonal elements for

$$\theta_{23} = \frac{\pi}{4}, \quad r = 1\,, \quad x = \frac{2}{3}\cos 2\sigma_x\ :$$

$$\tilde{m} = \begin{pmatrix} x & \sqrt{\frac{1-x^2}{2}} & \sqrt{\frac{1-x^2}{2}} \\ \ldots & x & \sqrt{\frac{1-x^2}{2}} \\ \ldots & \ldots & x \end{pmatrix}$$

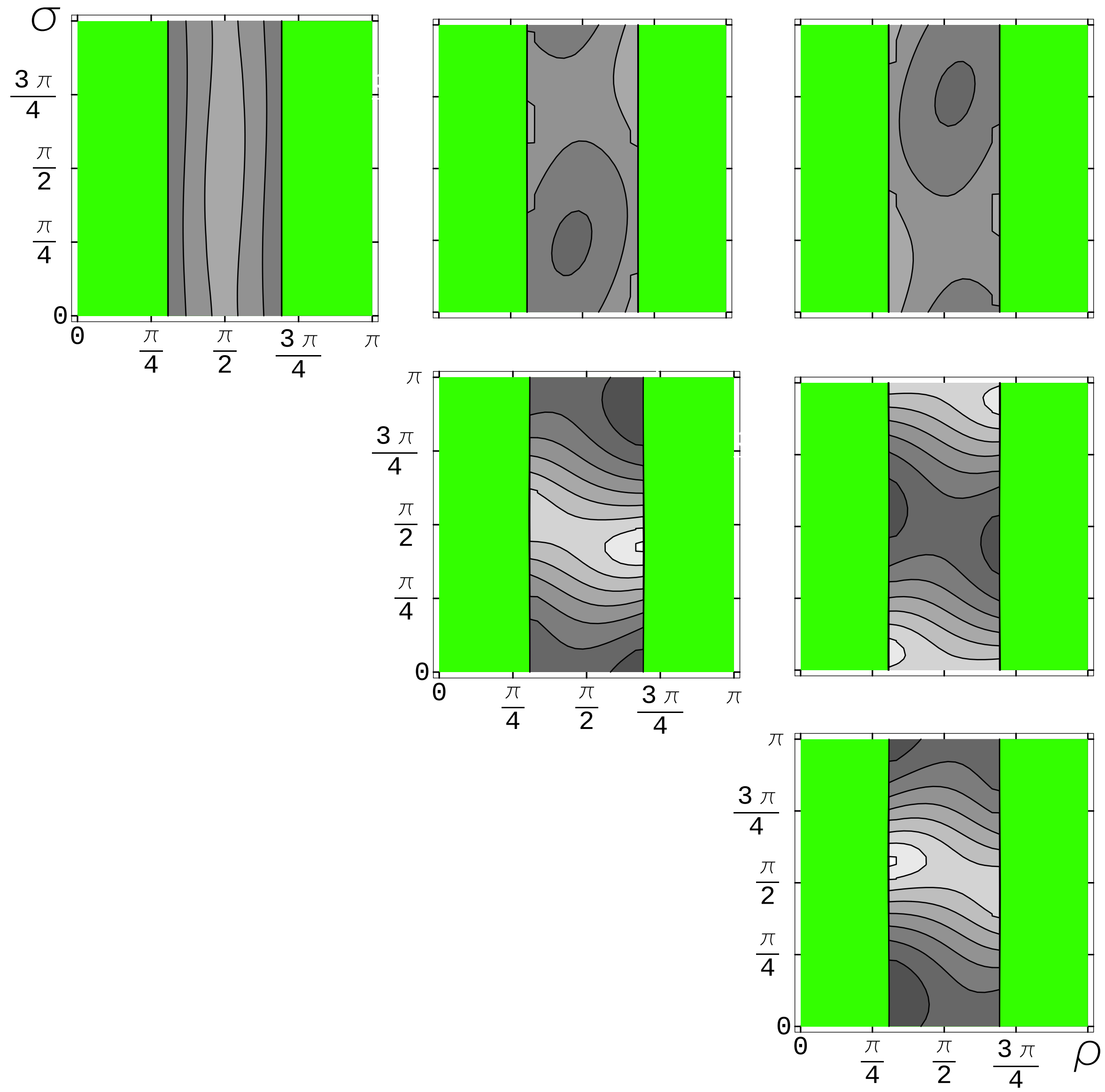
# Constraints from $2\beta_{0\nu}$-decay

Experimental upper bound: $m_{ee} < 0.35$ eV.

[Heidelberg-Moscow Collaboration, hep-ph/0103062,0103074]

The regions, in $\rho - \sigma$ plots, where $m_{ee}$ is larger than the upper bound, are excluded for all the matrix elements.

E.g., in the case $m_{max} = 0.5$ eV, the situation is the following:




[Thanks to D.Perini for the graphic arts]

- Hierarchical structures are almost excluded.
- $e$-row elements are approximately determined.
- $\mu\tau$-block elements are not determined, because of the dependence on $\sigma$.

# Conclusions

- Normal hierarchy: the existence of a dominant $\mu\tau$-block will be established improving the upper bounds on

$$\sin\theta_{13}\ , \qquad \cos 2\theta_{atm}\ , \qquad \Delta m^2_{sun}/\Delta m^2_{atm}\ .$$

  In this case the dependence of the matrix on CP phases would be small.

- For all the other mass spectra, the dependence of the matrix structure on the Majorana phases is very strong.
  - Only for particular values of $\rho$ and $\sigma$ the matrix has a hierarchical structure (nine combinations of very small elements are possible).
  - A very frequent situation is a matrix with all elements of the same order (intermediate gray-level in $\rho - \sigma$ plots): ordering structure.

- How to select a unique structure for the neutrino mass matrix?
  - Different possible mass spectra: improvements of kinematic bound on $m_{\nu_e}$ , of cosmological bound on $\sum_i m_i$ and/or positive signal from $2\beta_{0\nu}$ experiments could select a unique spectrum;
  - Majorana phase $\rho$: it can be constrained (determined) by $2\beta_{0\nu}$ experiments;
  - Majorana phase $\sigma$: it is unknown (in the case of degeneracy the matrix structure cannot be determined completely, while, in the inverted hierarchy case, $\sigma$ is negligible).