T-Violating Effects in Neutrino Oscillations with Three Flavors in Matter

T30d
Theoretische Elementarteilchenphysik

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We investigate the interplay of fundamental and matter-induced T-violating effects in three flavor neutrino oscillations in matter. In addition, we present an approximative analytical formula for the T-violating probability asymmetry valid for an arbitrary density profile. We also discuss some implications of the obtained results. Since there are no T-violating effects in the two flavor case, the T-violating probability asymmetry can, in principle, provide a way to measure θ_{13} and Δm^2_{21}. Finally, we apply our approximative analytical formula to different scenarios for neutrino factories using two layer matter density profiles. We also show for terrestrial experiments that matter-induced T violation can safely be ignored and cannot hinder the determination of fundamental T violation.

T-violating effects

Interplay of fundamental and matter-induced T violation:

T transformation = time reversal transformation

Experimental problem:

T-violation cannot be directly experimentally tested, since one cannot change the direction of time.

"Solution":

Instead of studying neutrino oscillations "backward" in time, one can study them forward in time, but with initial and final flavors interchanged.

fundamental T violation (intrinsic) = due to non-vanishing $\{\delta_{CP}\}$

matter-induced T violation (extrinsic) = due to interchange of positions of source and detector (asymmetric matter density profile)

T violation in neutrino oscillations:

$$\Delta P^T_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha),$$

where $P(\nu_\alpha \to \nu_\beta)$ is the transition probability for $\nu_\alpha \to \nu_\beta$.

CP and CPT differences:

$$\Delta P^{CP}_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) - P(\bar\nu_\alpha \to \bar\nu_\beta)$$
$$\Delta P^{CPT}_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) - P(\bar\nu_\beta \to \bar\nu_\alpha)$$

Two neutrino flavors

There are no T-violating effects!

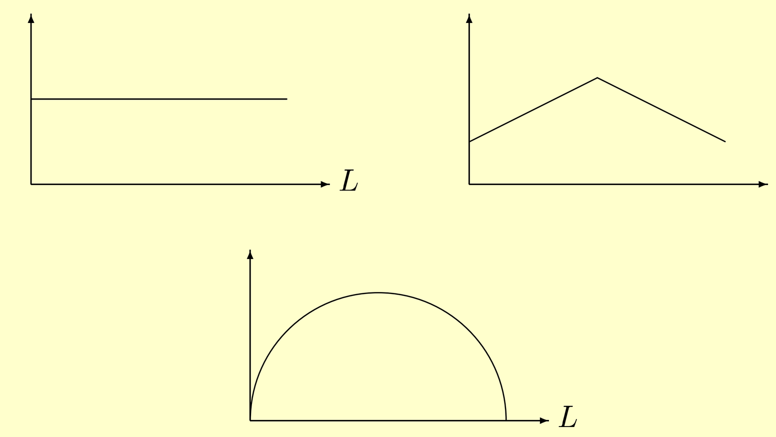
$P_{e\mu} = P_{\mu e} \quad \Rightarrow \quad \Delta P^T_{e\mu} = 0$

Three neutrino flavors

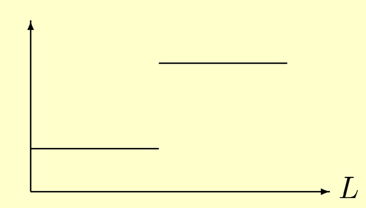
- In vacuum:
  CPT invariance $\Rightarrow$ T violation $\Leftrightarrow$ CP violation
- In matter:
  Matter is both CP and CPT-asymmetric, since it consists of particles (electrons and nucleons) and not of their antiparticles or, in general, of unequal numbers of particles and antiparticles.
  - Symmetric matter density profiles:
    Example: Constant matter density profiles
    If $\delta_{CP} = 0$, then $\Delta P^T_{\alpha\beta} = 0$.
  - Asymmetric matter density profiles:
    Example: Step function matter density profiles

Examples

Symmetric matter density profiles:



Asymmetric matter density profile:



The T-odd probability difference

An arbitrary matter density profile: θ_{13} and δ/Δ are small parameters!

$$\Delta P^T_{e\mu} \simeq -2 s_{23} c_{23}\, \mathrm{Im}\,[\beta^*(A - C^*)]$$
$$\simeq -2 s_{13} s_{23} c_{23} \left(\Delta - s^2_{12}\delta\right) \mathrm{Im}\left[e^{-i\delta_{CP}}\beta^*\left(A_a - C^*_a\right)\right]$$

Here:

$$s_{ij} \equiv \sin\theta_{ij}, \quad c_{ij} \equiv \cos\theta_{ij};$$

$$\delta \equiv \frac{\Delta m^2_{21}}{2E_\nu}, \quad \Delta \equiv \frac{\Delta m^2_{31}}{2E_\nu};$$

$$A_a \equiv \alpha \int_{t_0}^{t} \alpha^* f\, dt' + \beta \int_{t_0}^{t} \beta^* f\, dt', \quad C_a \equiv f \int_{t_0}^{t} \alpha f^*\, dt'.$$

$\alpha = \alpha(t, t_0)$ and $\beta = \beta(t, t_0)$ are to be determined from the solutions of the two flavor neutrino problem in the (1,2)-sector and $f = f(t, t_0) = \exp\left\{-i \int_{t_0}^{t} \left(\Delta - \frac{1}{2}[\delta + V(t')]\right) dt'\right\}$.

In addition: $\Delta P^T_{e\mu} = \Delta P^T_{\mu\tau} = \Delta P^T_{\tau e}$

$\Delta P^T_{e\mu}$ has been calculated for

1. matter consisting of two layers of constant density (layer widths L_1 and L_2, electron number densities N_1 and N_2, the corresponding matter-induced potentials V_1 and V_2, and the values of the mixing angle in the (1,2)-subsector in matter θ_1 and θ_2)[a] and
2. an arbitrary matter density profile in the adiabatic approximation.

In the low energy regime ($\delta = \Delta m^2_{21}/(2E_\nu) \gtrsim V_{1,2}$):

$$\Delta P^T_{e\mu} \simeq \cos\delta_{CP} \cdot 8 \underbrace{s_{12} c_{12} s_{13} s_{23} c_{23} \frac{\sin(2\theta_1 - 2\theta_2)}{\sin 2\theta_{12}}}_{J_{\rm eff} \text{ – effective Jarlskog invariant}}$$
$$\times \left\{\sin\omega_1 L_1 \sin\omega_2 L_2 \left[Y - \cos\left(\Delta_1 L_1 + \Delta_2 L_2\right)\right]\right\}$$
$$+ \sin\delta_{CP} \cdot 4 s_{13} s_{23} c_{23}$$
$$\times X_1 \left[Y - \cos\left(\Delta_1 L_1 + \Delta_2 L_2\right)\right]$$

$\cos\delta_{CP}$ term: matter-induced T violation

$\sin\delta_{CP}$ term: fundamental T violation

Here:

$$\omega_i \equiv \frac{1}{2}\sqrt{\left(\cos 2\theta_{12}\delta - V_i\right)^2 + \sin^2 2\theta_{12}\delta^2}, \quad (i = 1, 2);$$

$$Y = \cos\omega_1 L_1 \cos\omega_2 L_2 - \sin\omega_1 L_1 \sin\omega_2 L_2 \cos\left(2\theta_1 - 2\theta_2\right),$$
$$X_1 = \sin\omega_1 L_1 \cos\omega_2 L_2 \sin 2\theta_1 + \sin\omega_2 L_2 \cos\omega_1 L_1 \sin 2\theta_2;$$

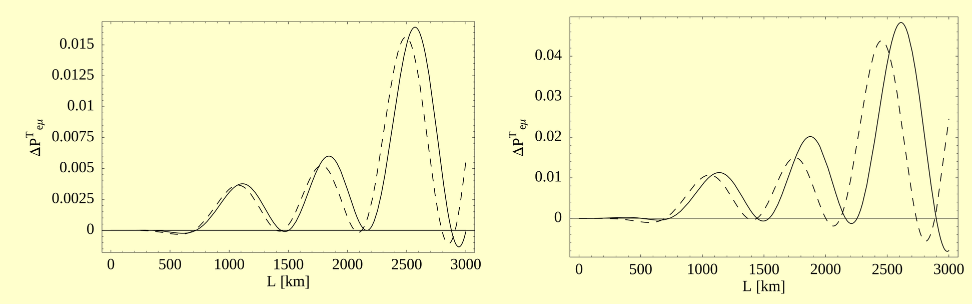
$$\Delta_i \equiv \Delta - \frac{1}{2}\left(\delta + V_i\right), \quad (i = 1, 2).$$

[a] If $N_1 = 0$ and $N_2 = 0$, then $\theta_1 = \theta_2 = \theta_{12}$.

Numerical analysis

Parameter values: $\theta_{12} = 0.56$, $\theta_{23} = \pi/4$, $\delta_{CP} = 0$, $\Delta m^2_{31} = 3.5 \cdot 10^{-3}\,\mathrm{eV}^2$

$\Delta P^T_{e\mu} = \Delta P^T_{e\mu}(L)$:

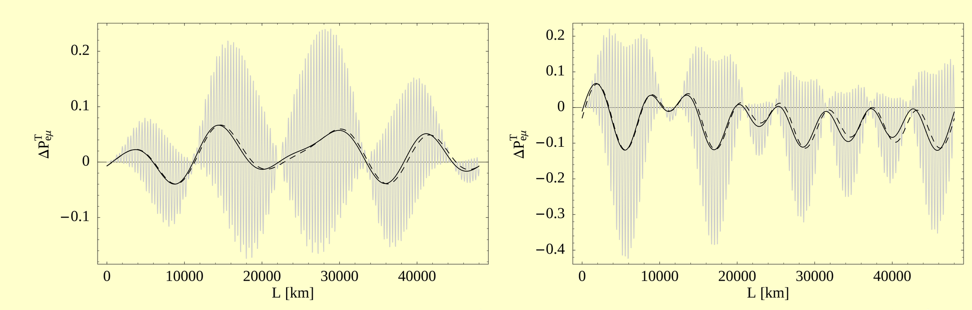


Left plot: $\theta_{13} = 0.1$, $\Delta m^2_{21} = 5 \cdot 10^{-5}\,\mathrm{eV}^2$ Right plot: $\theta_{13} = 0.16$, $\Delta m^2_{21} = 2 \cdot 10^{-4}\,\mathrm{eV}^2$

$L_1 = L_2 = L/2; \quad E_\nu = 1\,\mathrm{GeV}; \quad \rho_1 = 1\,\mathrm{g/cm^3}, \rho_2 = 3\,\mathrm{g/cm^3}$

solid curve - analytic result dashed curve - numerical result

$\Delta P^T_{e\mu} = \Delta P^T_{e\mu}(L)$:



Left plot: $\theta_{13} = 0.1$, $\Delta m^2_{21} = 5 \cdot 10^{-5}\,\mathrm{eV}^2$ Right plot: $\theta_{13} = 0.16$, $\Delta m^2_{21} = 2 \cdot 10^{-4}\,\mathrm{eV}^2$

$L_1 = L_2 = L/2; \quad E_\nu = 0.5\,\mathrm{GeV}; \quad \rho_1 = 0, \rho_2 = 6.4\,\mathrm{g/cm^3}$

grey curves - analytic results

black solid and dashed curves - result averaged over the fast oscillations of the analytic and numerical calculation, respectively

$\Rightarrow$ Oscillations governed by the large $\Delta m^2_{\rm atm} = \Delta m^2_{31}$ are very fast!

Left plot: Same parameter values as in Fig. 3a of P.M. Fishbane and P. Kaus, PLB **506**, 275 (2001), hep-ph/0012088.

Right plot: Larger values of θ_{13} and Δm^2_{21}.

Summary & Conclusions

- ✔ Complex interplay between fundamental and matter-induced T violation! In vacuum: T violation correlated to CP violation

$$\Delta P^{CP}_{\alpha\beta} + \Delta^T_{\bar\alpha\bar\beta} = \Delta P^{CPT}_{\alpha\beta} \equiv 0 \quad \Rightarrow \quad \Delta P^T_{\alpha\beta} = -\Delta P^{CP}_{\bar\alpha\bar\beta}$$

- ✔ Approximative analytical formulas for T-odd probability differences $\Delta P^T_{\alpha\beta}$ have been derived! Arbitrary matter density profile
- ✔ T-violating effects can be considered as a measure of genuine three-flavorness!
- ✔ For terrestrial experiments matter-induced T-violating effects can safely be ignored!
- ✔ Asymmetric matter effects cannot hinder the determination of the fundamental CP and T-violating phase δ_{CP} in long baseline experiments!

References

☞ E. Akhmedov, P. Huber, M. Lindner, and T. Ohlsson, Nucl. Phys. B **608**, 394 (2001), hep-ph/0105029.

☞ T. Ohlsson, J. High Energy Phys. PrHEP-hep2001/195, hep-ph/0108048.