The principle of high-precision mass measurements for a test of fundamental constants can be illustrated in form of a scale. The atomic mass of a bound electron (right) is balanced by QED contributions in increasing order, playing the role of a precision mass set (left).

2.2 Fundamental Tests with Trapped Ions

Introduction

Over the past few years, novel ion trapping techniques and developments have not only pushed the accuracy of pivotal tests of the fundamental interactions and symmetries into previously unprecedented regimes, but have also laid the foundations of new and intriguing experiments. MPIK constitutes an ideal environment by combining on its site both experimental and theoretical expertise. MPIK offers uniquely advanced experimental opportunities, from highly charged ions (HCI) in electron beam ion traps (EBITs) to various other techniques for precision measurements using Penning traps. Complementing these efforts, the theory division specializes in calculations needed to analyze and interpret those results with ultimate precision. This symbiosis has enabled, among others, a stringent test of quantum electrodynamics (QED) in strong fields by a comparison of experimental g-factors of HCI determined with the highest accuracy and QED predictions. Furthermore, based on this work, the atomic mass of the electron has been determined with a 13-fold better accuracy than previously achieved. Advances in X-ray spectroscopy have yielded absolute values for transition energies in few-electron HCI. From these measurements, QED contributions can be extracted and probed. Sophisticated ion trapping and manipulation techniques are also at the center of the AEGIS experiment for the study of antimatter in gravitational fields. Its aims are to use antihydrogen created from trapped antiprotons and positrons in order to directly test the weak equivalence principle.

Calculation of Nuclear and QED Effects on the Bound Electron g-Factor

Electrons possess an intrinsic magnetic moment, characterized by a dimensionless quantity called the gyromagnetic factor (g-factor). This number describes how the quantum states of the electron behave in an externally applied magnetic field. Recent years have witnessed a remarkable improvement in the theoretical description of the g-factor of an electron bound in the attractive potential of an atomic nucleus. These studies are paralleled with and motivated by a quantum leap in the experimental accuracy in the investigation of the g-factor, especially in Penning trap measurements performed by the division headed by Klaus Blaum [1].

Measuring the electron’s g-factor in highly charged ions (HCI) provides an exciting possibility for testing fundamental theories. The strong Coulomb force of the nucleus renders the electron dynamics relativistic, which necessitates a description based on Dirac’s equation, and effects of strong-field quantum electrodynamics are increasingly relevant at higher and higher ionic charges. Experiments on the bound electron g-factor with ever-improving accuracy represent a continuing challenge for theory. Bound-state QED effects are scrutinized to the highest precision in recent Penning trap experiments, which have reached the \(10^{-11}\) level in terms of relative precision with hydrogenlike carbon and silicon ions.

These experiments will be extended to heavier and heavier ions in the near future. An essential motivation of such studies is that g-factor measurements with heavy ions are anticipated to yield a new value of the fine-structure constant, i.e., a fundamental constant defining the strength of any kind of electromagnetic interaction in the Universe. Besides
possess a non-zero spin. The most dominant nuclear structural effect, namely, the finite size effect is well understood. It can be calculated by means of perturbation theory, yielding easily evaluable analytical formulas, or numerically in an all-order fashion by including the potential corresponding to the extended nucleus into the usual Dirac equation. The comparison of theoretical and experimental g-factors may provide one with a new tool of determining nuclear radii: the accuracy is limited by the free wavelength of the Dirac equation only. With this technique we are able to extract the nuclear radius. This proof-of-principle study encourages one to further improve the accuracy and to extend the investigations to higher charges and to different isotopes of the same element.

In very recent studies, we have also considered the nuclear shape (or deformation) effect, i.e. the deviation of the nuclear shape from a perfectly spherical one [2]. While part of the nuclear deformation effect can be incorporated into the spherically averaged radius and thus can be understood as a constituent of the dominant finite size effect, some terms explicitly depending on the nuclear deformation parameters prevail and show a relative contribution to the g-factor that is visible for elements with an intermediate nuclear charge Z. Our calculations confirm that the effect is negligible for the elements experimentally studied so far, i.e., for hydrogen-like carbon and silicon ions \(Z = 6,14\), respectively, therefore, it does not influence the interpretation of those results. However, the nuclear shape contribution grows quickly with \(Z\) and it is visible e.g. for strontium isotopes with \(Z = 38\). A prominent example of the deformation effect becomes accepted for publication in Nature.

In summary, we can conclude that theoretical studies on the nuclear magnetic shielding were developed in a relativistic manner. In our very recent investigations, a more accurate theory of the nuclear magnetic shielding was developed, taking into account the self-energy and vacuum polarization QED corrections to the shielding parameter [3]. As we theoretically suggested, experiments extended to ions with nonzero nuclear spin may even deliver more accurate g-factor determinations of the bound electron with unprecedented uncertainties. In this regard, the g-factor of the bound electron in hydrogen-like silicon \(^{28}\text{Si}^{13+}\) has been determined with a relative statistical precision of \(4\times10^{-11}\) [4]. However, the final uncertainty of the g-factor was limited to \(4\times10^{-14}\) by the uncertainty of the electron mass, which is, as well as the ion mass, a required input parameter for the experimental g-factor. The experimental value is in excellent agreement with the theoretical prediction and constitutes the most stringent test of BS-QED calculations in strong fields [1].

Since the experimental uncertainty was limited by the electron mass, the measurement principle was inverted to determine the atomic mass of the electrons. To this end, hydrogen-like carbon was used, where the theoretical g-factor was calculated within the division of Christoph Keitel to a relative precision of \(3.5\times10^{-12}\) and the ion mass is known to \(1\times10^{-12}\). The required frequency ratio of Lamb frequency and free cyclotron frequency was measured with a precision of \(3\times10^{-14}\), resulting in an improved precision of the atomic mass of the electron by a factor of 13 (accepted for publication in Nature).

Highly charged ions (HCI) are magnifying glasses for relativistic, quantum electrodynamics, and nuclear size effects. The energy shifts and splittings caused by such effects scale at least with the fourth power of the atomic number \(Z\), what means a factor of seventy million across the periodic table of the elements. Extracting these features is by no means trivial, since they affect primarily the most deeply bound electrons in the K shell. In neutral atoms, directly addressing such electrons by some excitation processes leads to a complicated Auger process which removes a not well defined number of electrons in a matter of femtoseconds. The corresponding loss of resolution makes exploiting the full information from such experiments very difficult. A practicable solution is to work with HCI: they do not change their charge state under a wide range of collision conditions, and can be excited with Xrays without loss of integrity. Due to their inherent simplicity, hydrogen-like and helium-like ions are ideally suited for fundamental physics studies. For the two-body systems constituted by hydrogen-like ions, the largest part of the electronic energy can be calculated using the Dirac equation. Additional contributions from, e.g.,

\[ 2.2 \text{ Fundamental Tests with Trapped Ions} \]

**Experimentation Determination of the Bound Electron g-Factor in Highly Charged Ions**

A high-precision test of the standard model of physics, in this case especially bound-state quantum electrodynamics (BS-QED), requires a quantity which can be both calculated and measured with highest precision. Such a quantity is, e.g., the g-factor of the electron. By investigating the g-factor of an electron bound in highly charged ions, high electric field strengths at the position of the electron can be achieved, allowing to test the theory under extreme conditions, where possible deviations from known theoretical models are most likely to emerge.

Experimentally, the g-factor of the electron can be determined by measuring the spin-precession frequency (Larmor frequency) of the electron in a well-known magnetic field. To obtain the magnetic field at the position of the electron, the ion itself is utilized as a magnetic field probe by measuring its free cyclotron frequency. For a precision of \(5\times10^{-14}\) the g-factor of the recent Penning trap experiment with Si\(^{28+}\) has been determined with a relative statistical precision of \(4\times10^{-14}\) [4]. However, the final uncertainty of the g-factor was limited to \(4\times10^{-14}\) by the uncertainty of the electron mass, which is, as well as the ion mass, a required input parameter for the experimental g-factor. The experimental value is in excellent agreement with the theoretical prediction and constitutes the most stringent test of BS-QED calculations in strong fields [1].
QED are extracted by subtracting the theoretical value from the experimental results. We have developed a novel system for the absolute measurement of the Bragg angle in crystal spectrometers [6], a quantity which gives access to a direct and independent determination of the absolute X-ray wavelength and thus transition energy. The accuracy achieved reaches 1 ppm, the highest reported for X-ray studies in atomic systems. This level of accuracy does not allow for in-depth tests of the one-electron QED predictions, which would require at least a tenfold reduction of our experimental uncertainty. Nonetheless, our absolute results for hydrogenic ions perfectly agree with theory.

The situation is different, and more interesting, for heliumlike ions. Here, the bound-state QED can be tested in a system where two electrons are sensitive to the screening of their interaction with the vacuum fluctuations by the other electron. These interactions belong to a very general class of quantum-field-many-body interactions, and are the most accurately testable examples of them. For instance, Ar$^{19+}$ ions show a one-electron Lamb shift of 3.13 eV, and a two-electron contribution of 0.095 eV in their ground state. Our experiments (see Fig. 3) measure the transition energy from the first excited state $^3$P of $\text{Ar}^{16+}$ with an uncertainty of 5 meV. The uncertainty of our present result [6] is a factor of $\approx 2.0$ smaller than all of the two-electron QED corrections to the $^3S$ ground state energy and a factor of 2 smaller than the corrections caused by the exchange of two virtual photons (9 meV). Consequently, the experiment probes these QED contributions as well as the total one-electron contribution of the excited $^3$P state (6 meV), however it is not yet sensitive to its total two-electron term (3 meV). Future, improved measurements will probe QED in hydrogenlike ions in order to establish their Lyman-$\alpha$ lines as atomic X-ray standards at synchrotrons and free-electron lasers.

Another approach we are presently pursuing is the application of synchrotron radiation sources for this type of studies, as shown in Fig. 4. Modern undulators provide X-ray fluxes which have been shown to be sufficient for narrow-band excitation of electronic transitions in HCl by our experiments. We have developed an electron beam ion trap, FLASH-EBIT, to this end. An electron beam produces the ions but does not excite them. This is achieved by keeping the electron energy below a certain level. Under these conditions, the electronic transitions of interest are excited by overlapping an X-ray photon beam with the trapped ion ensemble. When the photon energy is exactly resonant with an electronic transition, the excitation follows, and subsequently also the emission of a fluorescence photon. Detection of such photons reveals the level energy. The great advantage of synchrotron radiation is the superior performance of beamline monochromators at such facilities, which results from the large linear dispersion achievable with narrowly collimated photon beams over distances of tens of meters. Such long observation distances would make weak signals impossible to detect in a spectroscopic setup collecting isotropically emitted radiation from an ion trap. With this technique, we are able to improve the resolution and quality of the signal. The data obtained for, e.g., Fe$^{24+}$ ions allows one to determine the natural width of the resonantly excited transition at 6.7 keV photon energy to 0.311(10) eV, and yields a calibrated energy value of 6706.549(5)/70(7) eV [7]. These results show an excellent agreement with state-of-the-art calculations based on the bound-state QED method and confirm that an apparent trend to a deviation from QED, which had been recently claimed, is related to the significantly larger uncertainties of older experimental results.

Our novel experimental method can still be further improved by using ultra-high resolution crystal monochromators in the photon beamline, and by cooling the trapped ions. It is planned to utilize nearly perfect crystals prepared by the German metrology institute PTB, and Mössbauer samples in combination with FLASH-EBIT in order to improve the accuracy by at least one order of magnitude in the near future. With the use of sympathetically cooled HCl, the perspective of reaching even further, into the regime of a few parts per billion seems to be within reach in the next years.

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A Test of the Weak Equivalence Principle with Antimatter

The weak equivalence principle (WEP) is one of the cornerstones of General Relativity (GR). It states that the trajectory of a falling test body in a gravitational field is independent of its composition. While the WEP has been very well tested with ordinary matter, a verification with antimatter has never been performed. Over the years, many arguments have been brought forward against an anomalous gravitational acceleration of antimatter. However, all of these arguments rely on assumptions, such as the validity of CPT or the relevance (and choice) of an absolute gravitational potential. Therefore, a deviation from WEP for antimatter cannot currently be ruled out. Ultimately, the question of antimatter gravity can only be answered by a dedicated, direct experiment.

Gravity is the only fundamental interaction which is not currently expressed as a quantum field theory. With a view to a possible unification of GR with the Standard Model of particle physics (SM), various quantum gravity theories are being discussed. In quantum gravity, the exchange of a tensor (spin-2) graviton corresponds to ordinary ‘Newtonian’ gravity. Additional exchange bosons with different coupling constants and ranges are conceivable. For instance, the existence of vector (spin-1) and scalar (spin-0) exchange bosons is allowed by current torsion balance measurements on ordinary matter under the condition that their coupling constants and ranges match precisely. The presence of the non-canceling vector and scalar contributions would cause an increased force between matter and antimatter.

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The possibility of anomalous antimatter gravity may also be considered within the Standard-Model Extension (SME), a general framework to study the implications of Lorentz violation on physical observables. Within the gravitational SME, Lorentz-violating terms arise from the coupling of a particle to a field with nonmetric interaction with gravity, or from a fixed background field which modifies the effective metric. Anomalous gravitational effects on antimatter may arise from the fact that some coefficients are CPT-odd and others are CPT-even. To discuss the implications of antimatter gravity, a toy model termed the ‘isotropic parachute model’ has been developed, which is constructed such that the effective classical Lagrangian can be expressed in terms of effective inertial and gravitational masses, where the latter differs between matter and antimatter. In the case of antimatter the effective gravitational mass, and accordingly the acceleration, is reduced, hence the name ‘parachute.’

Thanks to the production of copious amounts of cold, neutral antihydrogen H at CERN’s Antiproton Decelerator (AD) in the past few years, an antimatter gravity experiment has finally come within reach. The AEGIS collaboration (Antimatter Experiment: Gravity, Interferometry, Spectroscopy) intends to measure the local acceleration g of antihydrogen in the gravitational field of the Earth to (initially) 1% by detecting the vertical deflection of a cold, bunched, horizontal beam of H (see Fig. 5) [9]. The AEGIS experiment involves the following main experimental steps.

1. Production of positrons (e\(^+\)) from a source of beta-decaying radioisotope;
2. Capture and accumulation of antiprotons (\(\overline{p}\)) from the AD in a Penning trap;
3. Production of positronium (Ps) by implanting e\(^+\) into a nano-porous insulator material;
4. Excitation of Ps to a Rydberg state;
5. Formation of H by resonant charge exchange between excited Ps and cold \(\overline{p}\) according to the reaction Ps* + \(\overline{p}\) → 1\(^+\Pp + e\)
6. Formation of an H beam by Stark acceleration with inhomogeneous electric fields;
7. Measurement of g in a two-grating moiré deflectometer coupled with a position-sensitive detector.

A moiré deflectometer is the classical counterpart of a matter wave interferometer. Three identical material gratings are placed at equal distances f, one from each other. A particle beam passing through the first two gratings produces a shadow pattern on the third. The change in vertical position of the shadow pattern due to gravity is determined by recording the transmission as a function of the position of the third grating. Alternatively, a position-sensitive detector may be used to replace the third grating and detector. A three-grating moiré deflectometer has previously been used to measure the local gravitational acceleration of ordinary matter to a relative precision of 2×10\(^{-4}\). Under the influence of gravity, the shadow pattern is vertically displaced by a distance \(\Delta y = \frac{1}{2} gT\), where \(g\) is the local gravitational acceleration and \(T\) is the time of flight between each pair of gratings. The gravitational acceleration \(g\) is obtained from a quadratic fit to a plot of the vertical displacement against the mean time of flight for suitable classes of events with similar horizontal velocities.

Construction of the AEGIS experiment at the AD began in 2010. As of December 2013, the following main components are installed and operational: Positron source and accumulator; high-field (5 T) and low-field (1 T) superconducting magnets; e\(^+\) transfer line; \(\overline{p}\) capture trap; H recombination trap with Ps converter target holder, laser system for Ps excitation. A prototype of the moiré deflectometer was commissioned at the University of Heidelberg with metastable argon atoms. During beam times in June and December 2012, the capture and stacking of \(\overline{p}\) was demonstrated, and a \(\overline{p}\) lifetime in the capture trap of \(\approx10\min\) was observed. In addition, the feasibility of the moiré deflectometer technique was demonstrated by measuring the deviation of a horizontal \(\overline{p}\) beam due to residual electromagnetic fields in a reduced-size deflectometer setup.

During the 2013/2014 shutdown of the CERN accelerators, the design and construction of the remaining components of the AEGIS apparatus are being completed. In particular, the design of the deflectometer’s I\(_X\) detector, which will consist of a combination of nuclear emulsions [10] and Si strip detectors, is being finalized, and construction will begin shortly.

In summary, the gravitational interaction between matter and antimatter has never been studied experimentally. The AEGIS experiment will determine \(g\) for antimatter to initially 1% by measuring the vertical deflection of a horizontal H beam. Data taking is expected to begin in the second half of 2014.

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References