Flavor Models with Sterile Neutrinos

NuFact'11 Geneva, Aug, 2011

Contents:

- Sterile neutrinos in ν-osc. and $0\nu\beta\beta$ decays
- Mechanisms for light sterile neutrino masses
- Flavor symmetry with sterile neutrinos
- Realization in seesaw models

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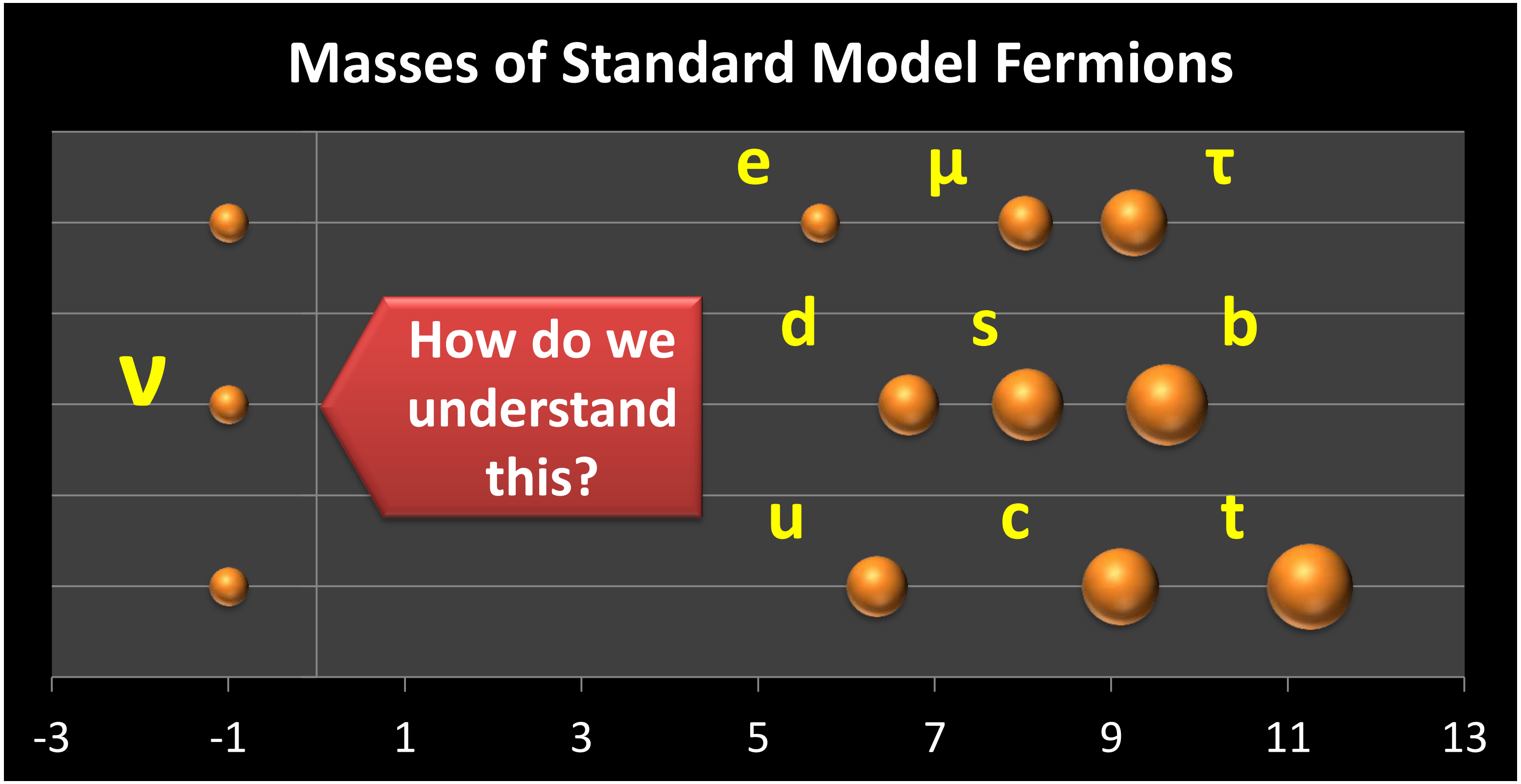
Based on JHEP07 (2011) 091

Neutrinos are massless in the SM as a result of the model's simple structure:

--- $SU(2)_L \times U(1)_Y$ gauge symmetry and Lorentz invariance;
Fundamentals of the model, mandatory for its consistency as a QFT.

--- Economical particle content:
No right-handed neutrinos --- a Dirac mass term is not allowed.
Only one Higgs doublet --- a Majorana mass term is not allowed.

--- Renormalizability:
No dimension ≥ 5 operators --- a Majorana mass term is forbidden.



Neutrino masses: **Seesaw**

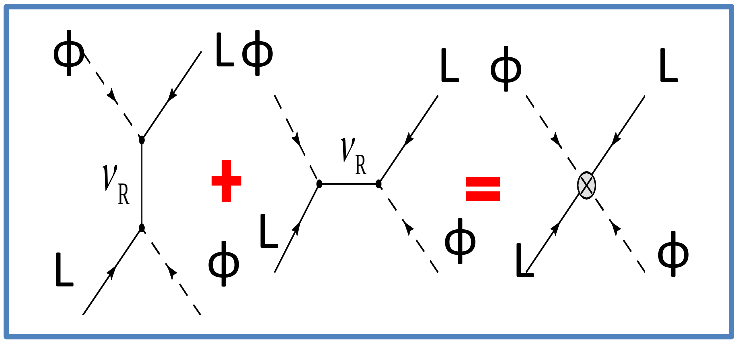
Neutrinos are Majorana particles

ν_R + Majorana & Dirac masses + seesaw

Natural description of the smallness of ν-masses

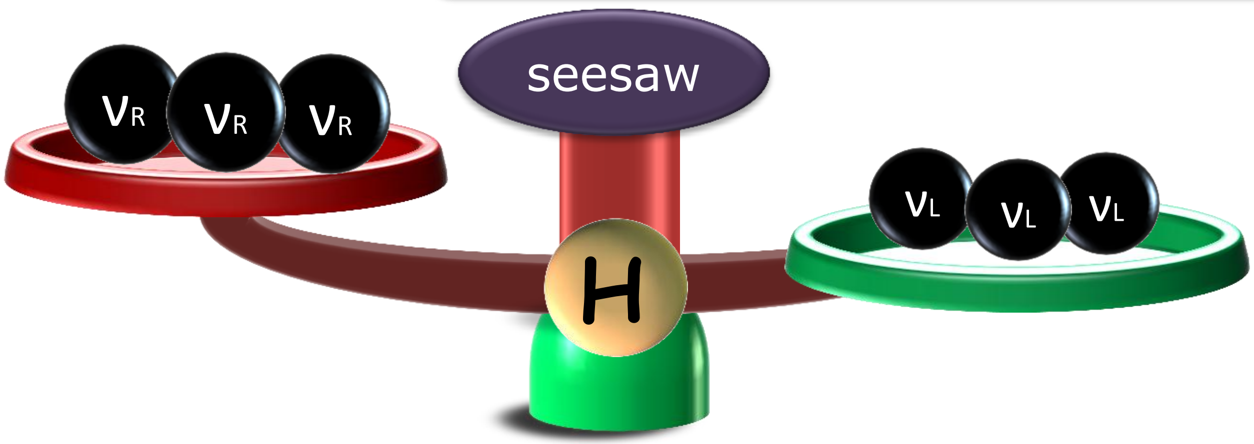
Integrate out right-handed neutrinos

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \left\{ Y \bar{l}_{\mathrm{L}} \nu_{\mathrm{R}} \tilde{\phi} + \boxed{\frac{1}{2} M_{\mathrm{R}} \bar{\nu}_{\mathrm{R}} \nu_{\mathrm{R}}^{\mathrm{C}}} + \mathrm{h.c.} \right\}$$



$$-iY^T \frac{\not{p}+M_R}{p^2-M_R^2} Y\left(\varepsilon_{cd}\varepsilon_{ba}+\varepsilon_{ca}\varepsilon_{bd}\right)P_L = i\kappa\left(\varepsilon_{cd}\varepsilon_{ba}+\varepsilon_{ca}\varepsilon_{bd}\right)P_L$$

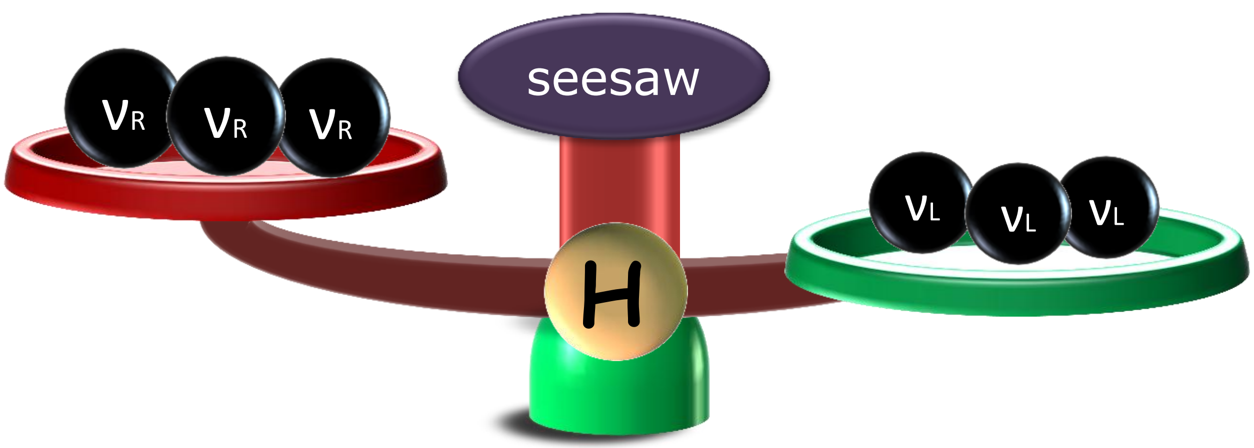
$$p^2 << M_R^2 \Rightarrow \boxed{Y^{\mathrm{T}} M_{\mathrm{R}}^{-1} Y = \kappa} \Rightarrow \boxed{m_\nu = -m_{\mathrm{D}}^{\mathrm{T}} M_{\mathrm{R}}^{-1} m_{\mathrm{D}}}$$



Neutrino masses: **Seesaw**

Typical choice of the seesaw scale:

$M_{\mathrm{R}} \sim \Lambda_{\mathrm{GUT}} \gg \Lambda_{\mathrm{EW}}$ & $M_{\mathrm{D}} \sim \Lambda_{\mathrm{EW}}$



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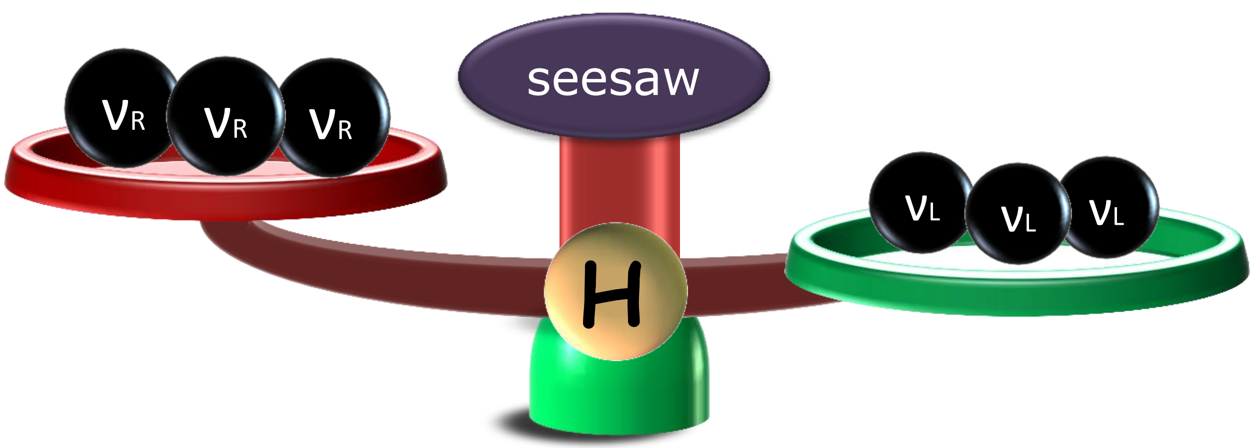
Alternatively, electroweak ($M_{\mathrm{R}} \sim \Lambda_{\mathrm{EW}}$) scale or eV scale seesaw ($M_{\mathrm{R}} \sim$ eV) could also be nature



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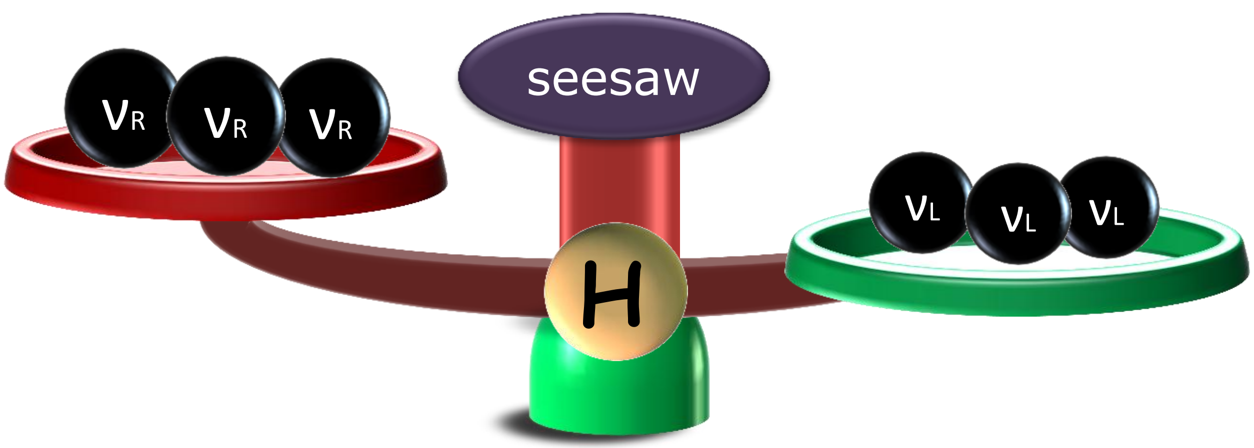
sterile neutrinos: ν_s



Neutrino mixing matrix:

4×4 case: $U = R_{34}\tilde{R}_{24}\tilde{R}_{14}R_{23}\tilde{R}_{13}R_{12}P$

$$R_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix} \qquad \tilde{R}_{14} = \begin{pmatrix} c_{14} & 0 & 0 & s_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14} & 0 & 0 & c_{14} \end{pmatrix}$$

$$P = \mathrm{diag}\left(1,\ e^{i\alpha/2},\ e^{i(\beta/2+\delta_{13})},\ e^{i(\gamma/2+\delta_{14})}\right)$$

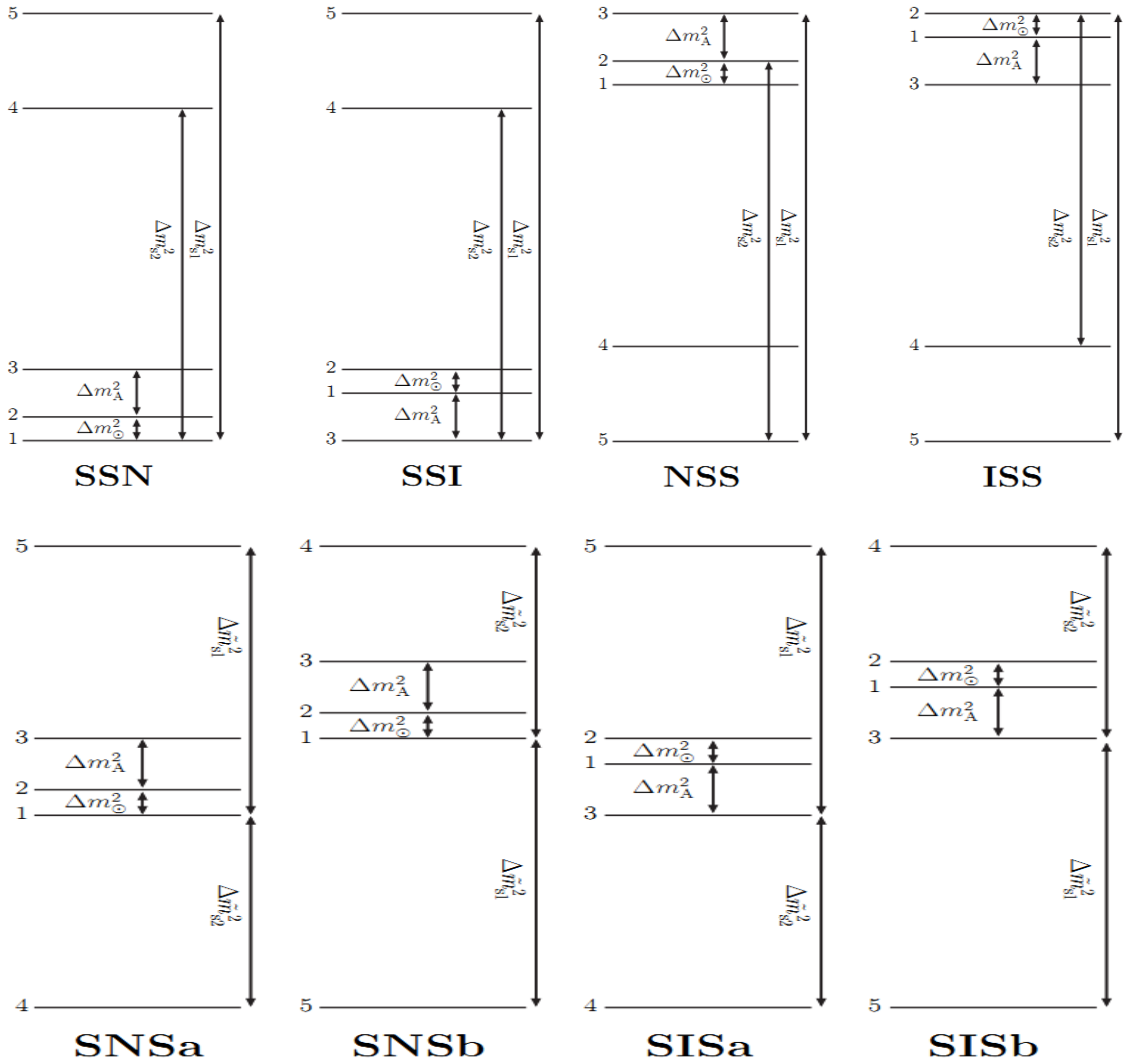
six mixing angles + 3 Dirac phases +3 Majorana phases

5×5 case: $U = \tilde{R}_{25}R_{34}R_{25}\tilde{R}_{24}R_{23}\tilde{R}_{15}\tilde{R}_{14}\tilde{R}_{13}R_{12}P$

Mass spectrum of five neutrinos

3+2 / 2+3

1+3+1



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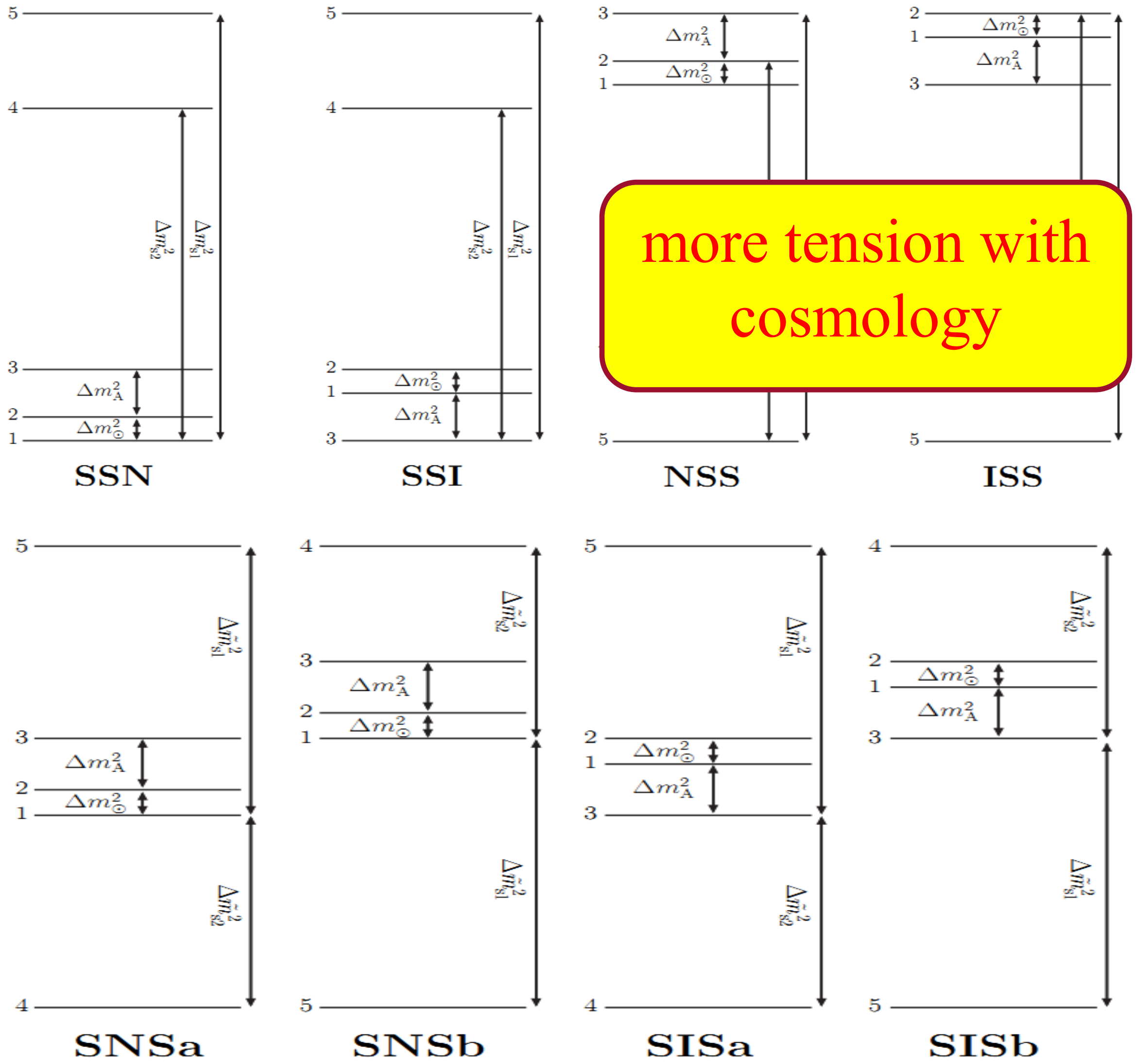
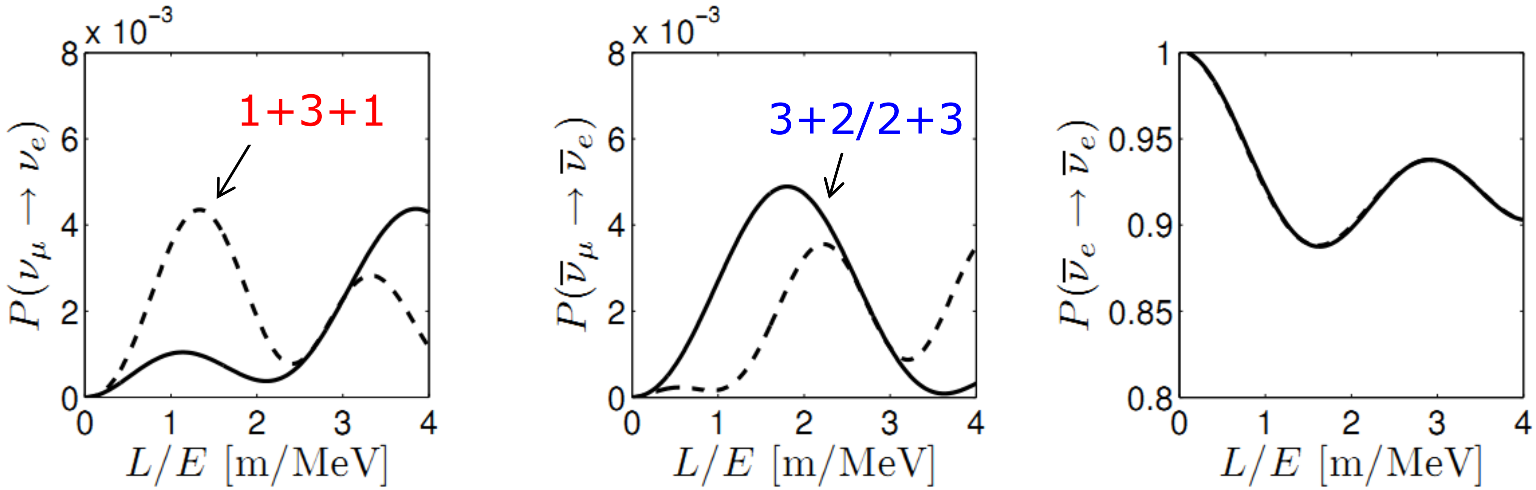


Best-fit and estimated 2σ values of the sterile neutrino parameters.

Kopp, Maltoni, Schwetz, 1103.4570

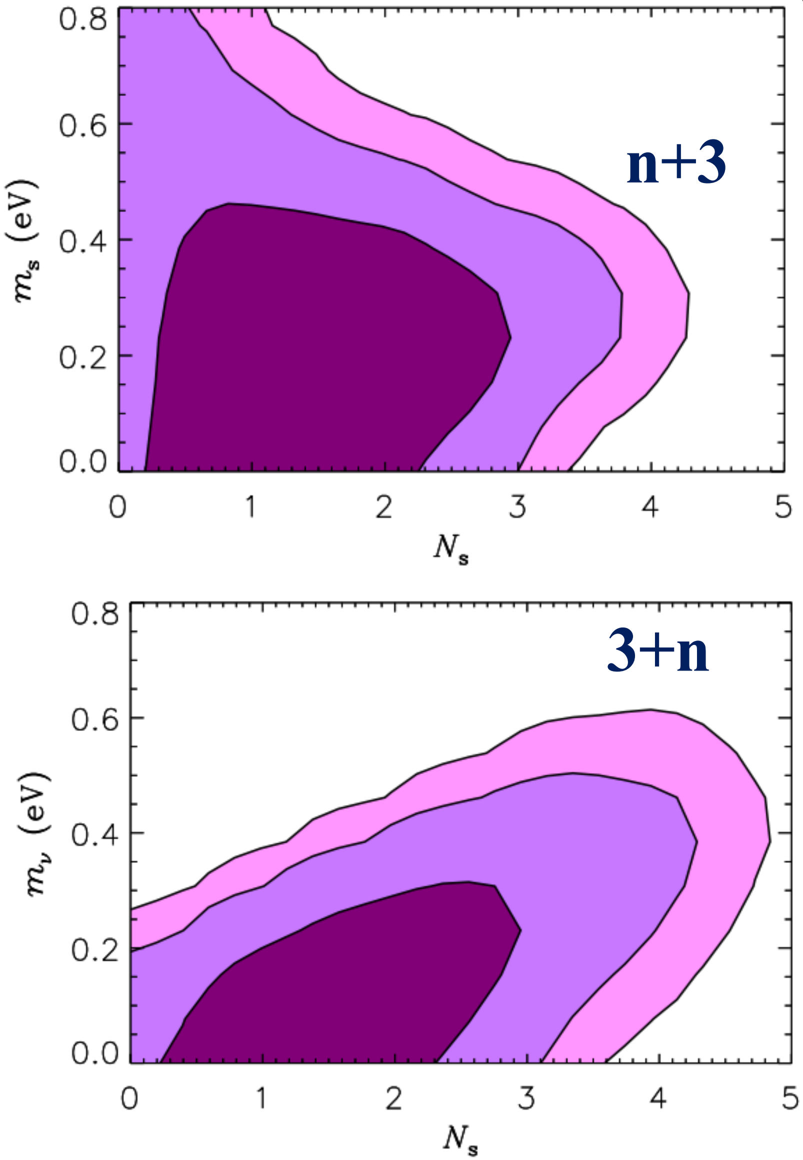
	parameter	Δm^2_{41} [eV]	$\|U_{e4}\|^2$	Δm^2_{51} [eV]	$\|U_{e5}\|^2$
3+1/1+3	best-fit	1.78	0.023		
	2σ	1.61–2.01	0.006–0.040		
3+2/2+3	best-fit	0.47	0.016	0.87	0.019
	2σ	0.42–0.52	0.004–0.029	0.77–0.97	0.005–0.033
1+3+1	best-fit	0.47	0.017	0.87	0.020
	2σ	0.42–0.52	0.004–0.029	0.77–0.97	0.005–0.035

Short-baseline neutrino oscillations: 3+2/2+3 vs. 1+3+1



Constraints from cosmology

CMB **J. Hamann et al, arXiv:1006.5276**



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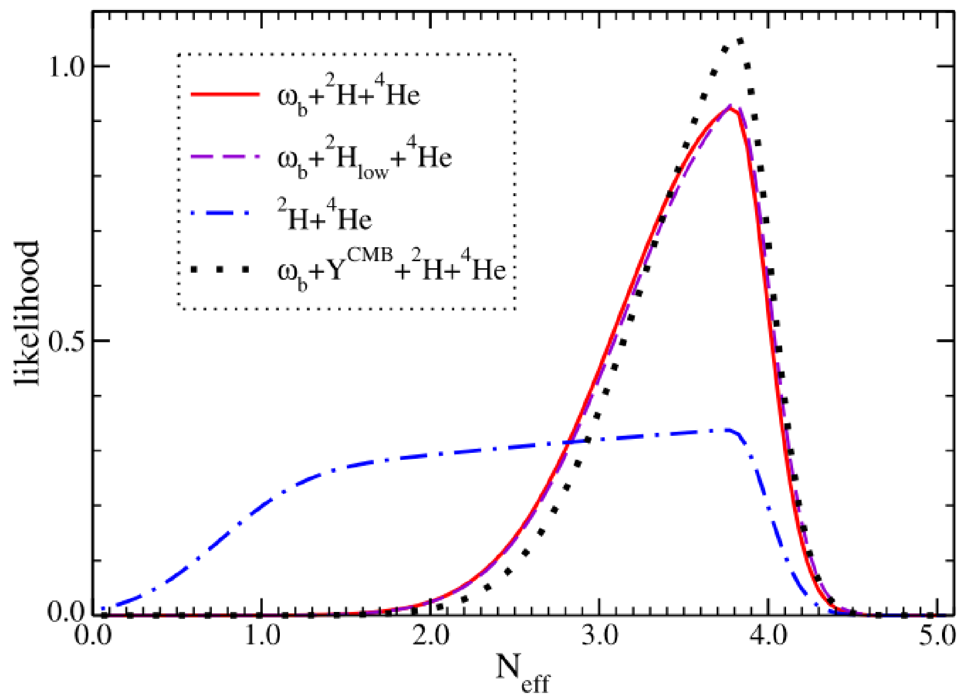
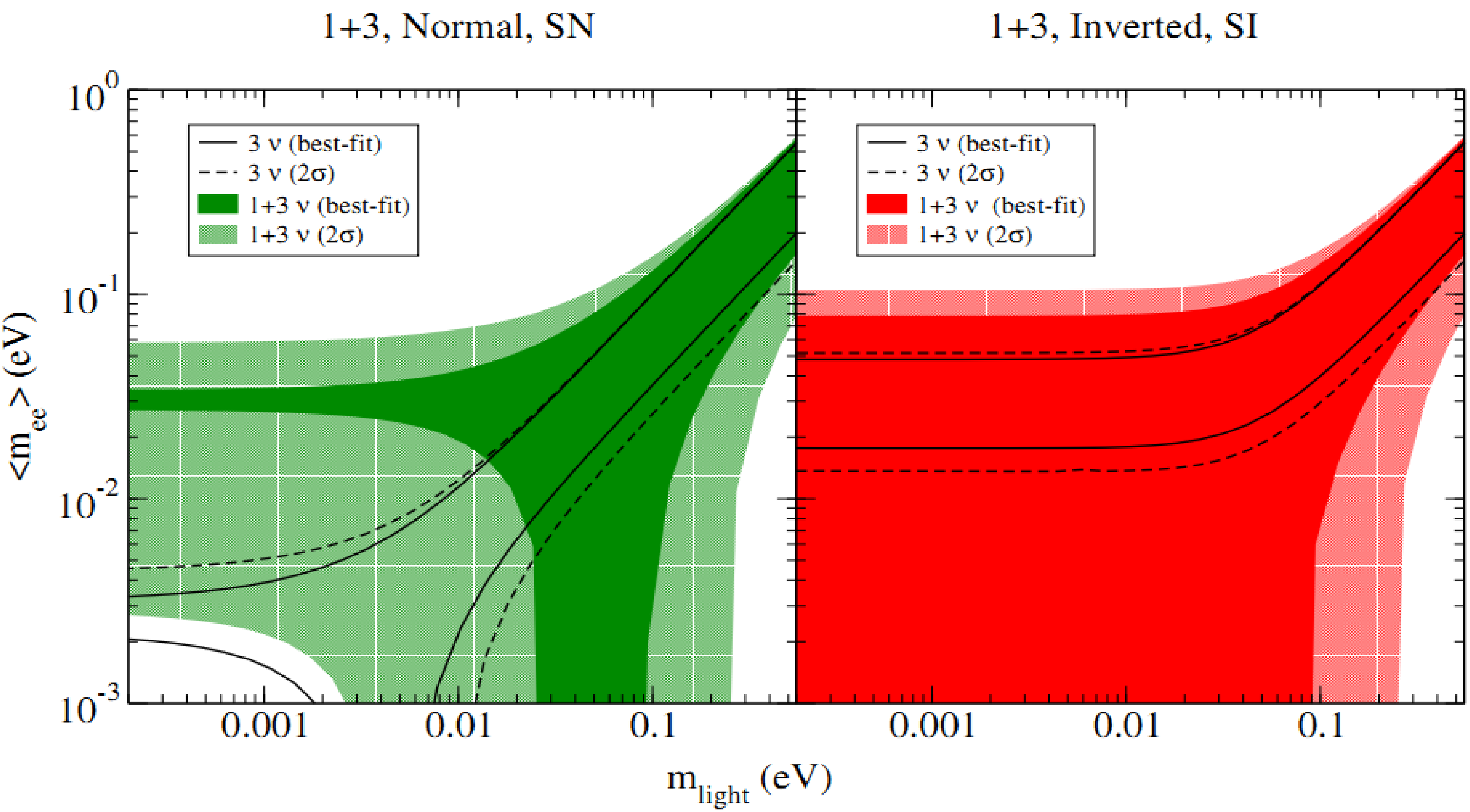


G. Mangano, P. Serpico, arXiv:1103.1261

Neutrino-less double beta decay

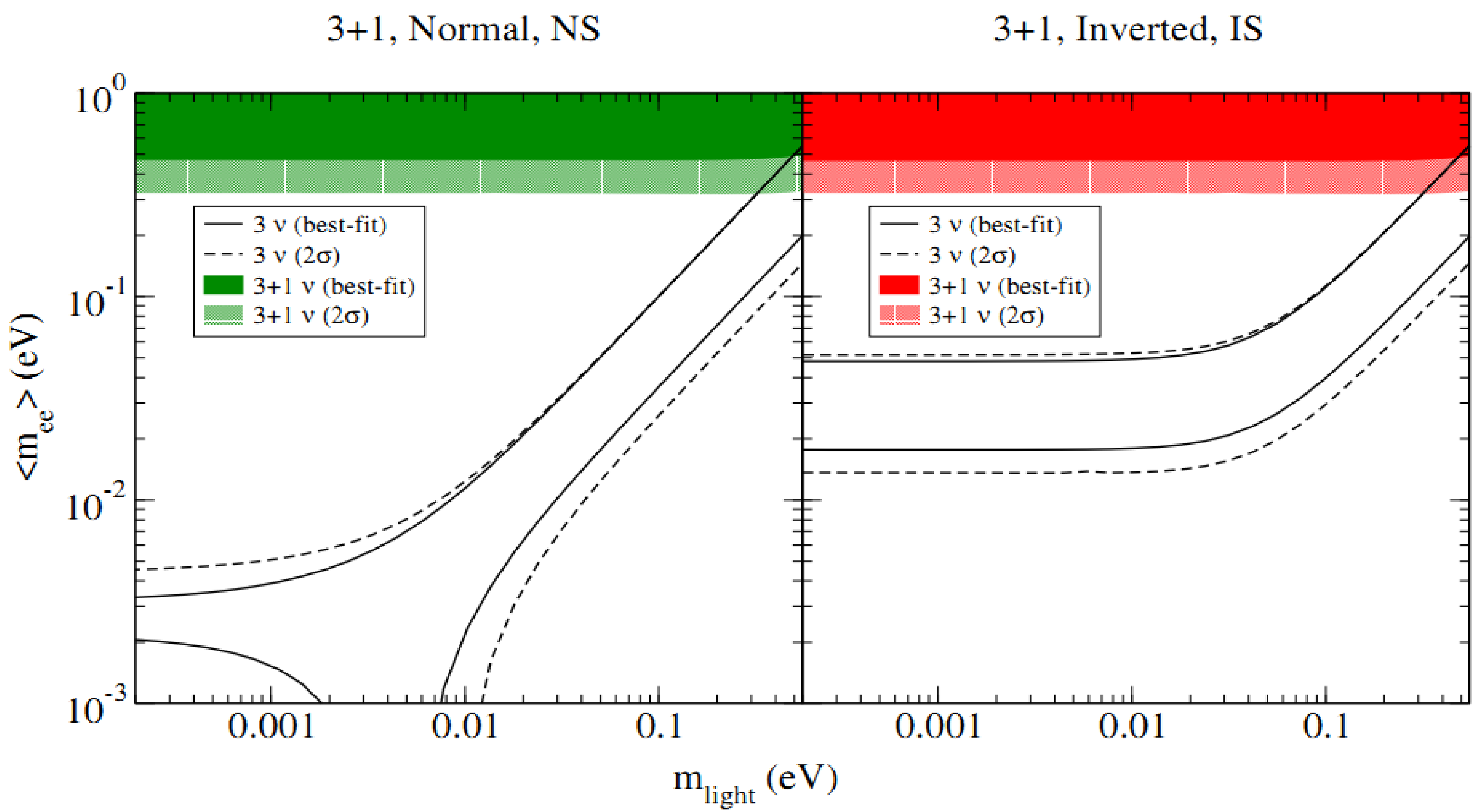
$$\langle m_{ee}\rangle_{(1+3)\nu} \simeq \left| c_{14}^2 \langle m_{ee}\rangle_{3\nu} + s_{14}^2 \sqrt{\Delta m_{41}^2}\, e^{i\gamma} \right|$$



The allowed ranges in the $\langle m_{ee}\rangle - m_{\text{light}}$ parameter space

Neutrino-less double beta decay

$$\langle m_{ee} \rangle_{(3+1)\nu} \simeq \sqrt{\Delta m^2_{41}} \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \alpha/2}$$



The allowed ranges in the $\langle m_{ee} \rangle - m_{\text{light}}$ parameter space

How to realize eV-scale ν_R

talk by Mavromatos

Extra dimension theories (Kusenko, Takahashi, Yanagida, **10**)

- Splitting between the SM brane and a hidden brane
- Effects of right-handed neutrinos are exponenally suppressed since they are located on the hidden brane

$$S = \int d^4x\, dy\, M \left(i\bar{\Psi}\Gamma^A \partial_A \Psi + m\bar{\Psi}\Psi \right)$$

zero mode with an exponential profile in the bulk

$$\Psi_R^{(0)}(y,x) = \sqrt{\frac{2m}{e^{2m\ell}-1}}\frac{1}{\sqrt{M}}e^{my}\psi_R^{(4D)}(x)$$

How to realize eV-scale ν_R

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- Splitting between the SM brane and a hidden brane
- Effects of right-handed neutrinos are exponenally suppressed since they are located on the hidden brane

$$S = \int d^4x\, dy \left\{ M \left(i\bar{\Psi}^{(0)}_{iR}\Gamma^A\partial_A\Psi^{(0)}_{iR} + m_i\bar{\Psi}^{(0)}_{iR}\Psi^{(0)}_{iR} \right) + \delta(y)\left(\frac{\kappa_i}{2} v_{\mathrm{B-L}}\bar{\Psi}^{(0)c}_{iR}\Psi^{(0)}_{iR} + \tilde{\lambda}_{i\alpha}\bar{\Psi}^{(0)}_{iR}L_\alpha\phi + \mathrm{h.c.}\right)\right\}$$

$$M_{Ri} = \kappa_i v_{\mathrm{B-L}}\frac{2m_i}{M(e^{2m_i\ell}-1)} \qquad \lambda_{i\alpha} = \frac{\tilde{\lambda}_{i\alpha}}{\sqrt{M}}\sqrt{\frac{2m_i}{e^{2m_i\ell}-1}} = \tilde{\lambda}_{i\alpha}\sqrt{\frac{M_{Ri}}{\kappa_i v_{\mathrm{B-L}}}}$$

$$\boxed{(m_\nu)_{\alpha\beta} = \left(\sum_i \frac{1}{\kappa_i}\tilde{\lambda}_{i\alpha}\tilde{\lambda}_{i\beta}\right)\frac{\langle\phi^0\rangle^2}{v_{\mathrm{B-L}}}}$$

How to realize eV-scale ν_R

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- Splitting between the SM brane and a hidden brane
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$$M_{Ri} = \kappa_i v_{\mathrm{B-L}} \frac{2m_i}{M(e^{2m_i \ell} - 1)} \qquad (m_\nu)_{\alpha\beta} = \left(\sum_i \frac{1}{\kappa_i} \tilde{\lambda}_{i\alpha} \tilde{\lambda}_{i\beta} \right) \frac{\langle \phi^0 \rangle^2}{v_{\mathrm{B-L}}}$$

Flavor symmetries (Lindner, Merle, Niro, **10**)

$L_e - L_\mu - L_\tau$ symmetry:

two heavy + one massless right-handed neutrinos

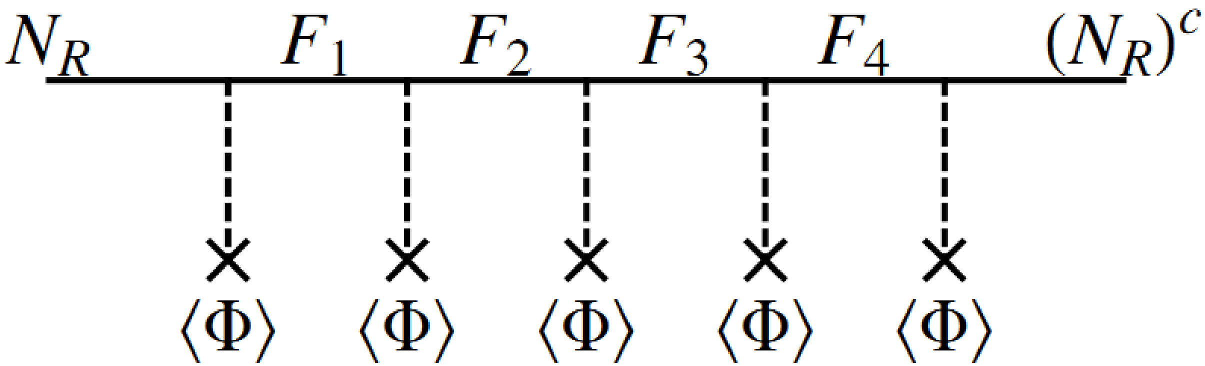
$$\mathcal{M}_\nu = \left(\begin{array}{ccc|ccc} 0 & m_L^{e\mu} & m_L^{e\tau} & m_D^{e1} & 0 & 0 \\ m_L^{e\mu} & 0 & 0 & 0 & m_D^{\mu 2} & m_D^{\mu 3} \\ m_L^{e\tau} & 0 & 0 & 0 & m_D^{\tau 2} & m_D^{\tau 3} \\ \hline m_D^{e1} & 0 & 0 & 0 & M_R^{12} & M_R^{13} \\ 0 & m_D^{\mu 2} & m_D^{\tau 2} & M_R^{12} & 0 & 0 \\ 0 & m_D^{\mu 3} & m_D^{\tau 3} & M_R^{13} & 0 & 0 \end{array} \right) \Rightarrow \begin{pmatrix} \lambda_+ & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_- & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Lambda_+ & 0 & 0 \\ 0 & 0 & 0 & 0 & \Lambda_- & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

light sterile neutrino from symmetry breaking

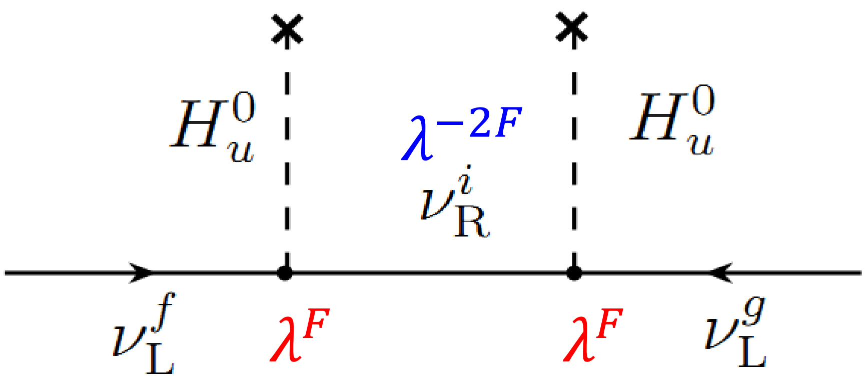
How to realize eV-scale ν_R

Froggatt-Nielsen mechanism

- Fermion flavors are differently charged under a $U(1)_{\text{FN}}$ symmetry
- Their masses receive a suppression factor $M \to M\lambda^F$ $\left(\lambda = \frac{\langle\phi\rangle}{\Lambda} < 1\right)$



- Neutrino masses are not affected by the FN charges



ν_s in flavor symmetry models: A_4 + FN mechanism

Field	L	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	ξ	ν_s
$\mathrm{SU}(2)_L$	2	1	1	1	2	1	1	1	1
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}''$	$\underline{1}'$	$\underline{1}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1
$\mathrm{U}(1)_{FN}$	$-$	3	1	0	$-$	$-$	$-$	$-$	6

Barry, Rodejohann, **HZ**, JHEP07(2011)091

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Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1
$U(1)_{FN}$	–	3	1	0	–	–	–	–	6

Barry, Rodejohann, **HZ**, JHEP07(2011)091

$$\mathcal{L}_{\rm Y} = \frac{y_e}{\Lambda} e^c (\varphi L) h_d + \frac{y_\mu}{\Lambda} \mu^c (\varphi L)' h_d + \frac{y_\tau}{\Lambda} \tau^c (\varphi L)'' h_d$$
$$+ \frac{x_a}{\Lambda^2} \xi (L h_u L h_u) + \frac{x_d}{\Lambda^2} (\varphi' L h_u L h_u) + \text{h.c.} + \dots$$
$$\mathcal{L}_{{\rm Y}_s} = \frac{x_e}{\Lambda^2} \xi (\varphi' L h_u) \nu_s + \frac{x_f}{\Lambda^2} (\varphi' \varphi' L h_u) \nu_s + m_s \nu_s^c \nu_s^c + \text{h.c.}$$

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$$\mathcal{L}_{\mathrm{Y}} = \frac{y_e}{\Lambda} e^c(\varphi L) h_d + \frac{y_\mu}{\Lambda} \mu^c(\varphi L)' h_d + \frac{y_\tau}{\Lambda} \tau^c(\varphi L)'' h_d + \frac{x_a}{\Lambda^2} \xi(L h_u L h_u) + \frac{x_d}{\Lambda^2} (\varphi' L h_u L h_u) + \text{h.c.} + \ldots$$

$$\mathcal{L}_{\mathrm{Y}_s} = \frac{x_e}{\Lambda^2} \xi(\varphi' L h_u) \nu_s + \frac{x_f}{\Lambda^2} (\varphi' \varphi' L h_u) \nu_s + m_s \nu_s^c \nu_s^c + \text{h.c.}$$

Assuming the flavon VEV alignments

$\langle\xi\rangle = u$

$\langle\varphi\rangle = (v, 0, 0)$

$\langle\varphi'\rangle = (v', v', v')$

$$\Rightarrow \quad M_\nu^{4\times4} = \begin{pmatrix} a + \frac{2d}{3} & -\frac{d}{3} & -\frac{d}{3} & e \\ \cdot & \frac{2d}{3} & a - \frac{d}{3} & e \\ \cdot & \cdot & \frac{2d}{3} & e \\ \cdot & \cdot & \cdot & m_s \end{pmatrix}$$

$$a = 2x_a \frac{u v_u^2}{\Lambda^2}, \quad d = 2x_d \frac{v' v_u^2}{\Lambda^2}, \quad e = \sqrt{2} x_e \frac{u v' v_u}{\Lambda^2}$$

active and sterile neutrino masses

$$m_1 = a + d\,, \qquad m_2 = a - \frac{3e^2}{m_s}\,, \qquad m_3 = -a + d\,, \qquad m_4 = m_s + \frac{3e^2}{m_s}$$

Numerical example: assuming Yukawa couplings are of order 1 and $\lambda = 10^{-1.5} \cong 0.03$

$$m_s \simeq 10^{0.5}\left(\frac{\lambda}{10^{-1.5}}\right)^{12}\left(\frac{v}{10^{11}\,\mathrm{GeV}}\right)^2\left(\frac{10^{12.5}\,\mathrm{GeV}}{\Lambda}\right)\mathrm{eV}$$

$$a \sim d \simeq 0.1\left(\frac{u}{10^{11}\,\mathrm{GeV}}\right)\left(\frac{v_u}{10^2\,\mathrm{GeV}}\right)^2\left(\frac{10^{12.5}\,\mathrm{GeV}}{\Lambda}\right)^2\mathrm{eV},$$

$$e \simeq 0.1\left(\frac{\lambda}{10^{-1.5}}\right)^6\left(\frac{u}{10^{11}\,\mathrm{GeV}}\right)\left(\frac{v'}{10^{11}\,\mathrm{GeV}}\right)\left(\frac{v_u}{10^2\,\mathrm{GeV}}\right)\left(\frac{10^{12.5}\,\mathrm{GeV}}{\Lambda}\right)^2\mathrm{eV}$$

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- Charged-lepton mass hierarchy: different FN changes of e_R, μ_R, τ_R
- Extension to the 3+2 case: simply add more singlet neutrinos

Neutrino mixing matrix

$$U \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & -\frac{\sqrt{3}e}{m_s} & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{3e^2}{2m_s^2} \end{pmatrix}$$

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Exact tri-bimaximal mixing pattern

Neutrino mixing matrix

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Exact tri-bimaximal mixing pattern + <u>sterile neutrino corrections</u>

Neutrino mixing matrix

$$U \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & -\frac{\sqrt{3}e}{m_s} & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{3e^2}{2m_s^2} \end{pmatrix}$$

Exact tri-bimaximal mixing pattern + sterile neutrino corrections

$$\sin^2\theta_{12} = \frac{|U_{e2}|^2}{1-|U_{e4}|^2} \simeq \frac{1}{3}\left[1-2\left(\frac{e}{m_s}\right)^2\right],$$

$$\sin^2\theta_{23} = \frac{|U_{\mu 3}|^2(1-|U_{e4}|^2)}{1-|U_{e4}|^2-|U_{\mu 4}|^2} \simeq \frac{1}{2}\left[1+\left(\frac{e}{m_s}\right)^2\right]$$

$$\sin^2\theta_{14} \simeq \sin^2\theta_{24} \simeq \sin^2\theta_{34} \simeq \left(\frac{e}{m_s}\right)^2 \simeq \frac{1}{2}(1-3\sin^2\theta_{12}) \simeq 2\sin^2\theta_{23} - 1$$

Neutrino mixing matrix

$$U \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & -\frac{\sqrt{3}e}{m_s} & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{3e^2}{2m_s^2} \end{pmatrix}$$

Exact tri-bimaximal mixing pattern + sterile neutrino corrections

$$\sin^2\theta_{12} = \frac{|U_{e2}|^2}{1-|U_{e4}|^2} \simeq \frac{1}{3}\left[1 - 2\left(\frac{e}{m_s}\right)^2\right],$$

$$\sin^2\theta_{23} = \frac{|U_{\mu 3}|^2(1-|U_{e4}|^2)}{1-|U_{e4}|^2-|U_{\mu 4}|^2} \simeq \frac{1}{2}\left[1 + \left(\frac{e}{m_s}\right)^2\right]$$

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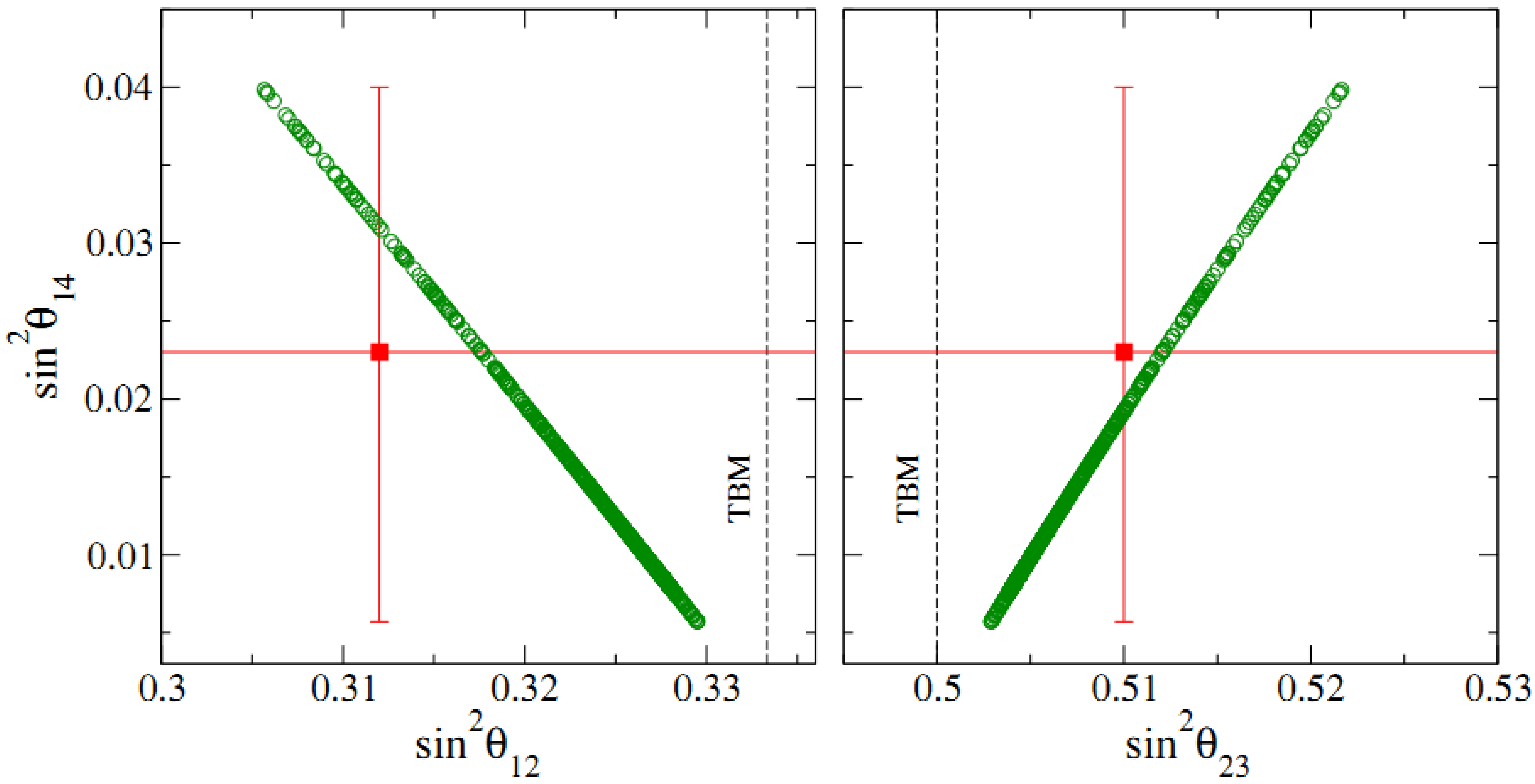
Connections between mixing angles



Light sterile neutrinos in seesaw models

sterile neutrinos from type-I seesaw

Model A: three eV-scale sterile neutrinos.

No neutrinoless double beta decay

More tension with cosmology

Model B: 1eV + 1keV + 1heavy sterile neutrinos

Neutrinoless double beta decay

Need to understand the mass splitting

Candidate for keV WDM

Model C: 1eV + 2heavy (>GeV) sterile neutrinos

Neutrinoless double beta decay

Successful leptogenesis

Model D: 1keV + 2heavy (>GeV) sterile neutrinos (*νMSM*)

Both baryon asymmetry and Warm Dark Matter puzzles can be solved

Failed in explaining the reactor anomaly

Minimal extended type-I seesaw

Barry, Rodejohann, **HZ**, 11

The model: SM + three right-handed neutrinos + one singlet *S*

$$-\mathcal{L}_m = \overline{\nu_L} M_D \nu_R + \overline{S^c} M_S \nu_R + \frac{1}{2}\overline{\nu^c_R} M_R \nu_R + \text{h.c.}$$

Minimal extended type-I seesaw

Barry, Rodejohann, **HZ**, 11

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$$-\mathcal{L}_m = \overline{\nu_L} M_D \nu_R + \overline{S^c} M_S \nu_R + \frac{1}{2}\overline{\nu_R^c} M_R \nu_R + \text{h.c.}$$

$$M_\nu^{7\times 7} = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & M_R & M_S^T \\ 0 & M_S & 0 \end{pmatrix}$$

- The full 7×7 neutrino mass matrix if of rank 6, and therefore, one active neutrino is massless.

$$m_\nu \simeq M_D M_R^{-1} M_S^T \left(M_S M_R^{-1} M_S^T\right)^{-1} M_S \left(M_R^{-1}\right)^T M_D^T - M_D M_R^{-1} M_D^T$$
$$m_s \simeq -M_S M_R^{-1} M_S^T$$

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$$m_s \simeq -M_S M_R^{-1} M_S^T$$

$M_D \sim 100$ GeV;
$M_S \sim 500$ GeV; $M_R \sim 2 \times 10^{14}$ GeV

$\Rightarrow$

$m_\nu \sim 0.05$ eV;
$m_s \sim 1.3$ eV; $U_{e4} \sim 0.2$

- No need to artificially insert small mass scales and tiny Yukawa couplings for light neutrino masses.
- Thermal leptogenesis works.
- Only one singlet S is allowed (minimal extension).

Minimal extended type-I seesaw

Barry, Rodejohann, **HZ**, **11**

The model: SM + three right-handed neutrinos + one singlet S

$$-\mathcal{L}_m = \overline{\nu_L} M_D \nu_R + \overline{S^c} M_S \nu_R + \frac{1}{2} \overline{\nu_R^c} M_R \nu_R + \text{h.c.}$$

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- The full 7×7 neutrino mass matrix if of rank 6, and therefore, one active neutrino is massless.

$$\boxed{\begin{aligned} m_\nu &\simeq M_D M_R^{-1} M_S^T \left(M_S M_R^{-1} M_S^T\right)^{-1} M_S \left(M_R^{-1}\right)^T M_D^T - M_D M_R^{-1} M_D^T \\ m_s &\simeq -M_S M_R^{-1} M_S^T \end{aligned}}$$

A similar idea was used with a sterile state of mass $\sim 10^{-3}$ eV introduced in order to explain the solar neutrino problem

(Chun, Joshipura, Smirnov, **95**)

Summary

1. The presence of light sterile neutrinos could significantly change both the short-baseline neutrino oscillation experiments and the effective mass measured in neutrino-less double beta decays.
2. We found that light sterile neutrinos can be naturally embedded into flavor symmetry models, e.g., A_4. In general, the admixture between active and sterile neutrinos leads to the deviation from the exact constant (e.g., tri-bimaximal) mixing pattern.
3. A minimal extended type-I seesaw model is presented, in which, without the need of introducing tiny Yukawa couplings, the smallness of sterile neutrino masses is ascribed to the existence of heavy singlet neutrinos, whereas the mixing between active and sterile neutrinos could still be sizable.

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Thanks