Exercises to "Standard Model of Particle Physics II"

Winter 2024/25

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Sheet 09 - December 18, 2024

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions:
January 7, 2025 - 14:00

Discussion of solutions:
January 7, 2025 – 14:00, INF 227, SR 1.404

Problem 1: *Galactic rotation curves* [**10 Points**]

NGC2998 is a spiral galaxy in Ursa Major. You can download its rotation curve data from `http://astroweb.case.edu/ssm/620f03/n2998.dat` (by Stacy McGaugh). Here, the first column gives the radius and the second column gives the observed/inferred circular velocity, with its 1σ uncertainty in the third column. The next few columns provide rotation curves that would arise from the stellar disk, gaseous disk, and the bulge alone.

a) Interpolate the data for the disk, gas, and bulge distribution of the circular velocity by plotting it against the distance from the center of the galaxy. Give a possible explanation for the discrepancy between the observed and the expected rotation curves.

b) Using Newtonian mechanics, derive the expression for the circular velocity of a star orbiting the central mass of a galaxy as a function of its distance from the center of the galaxy. Assume a circular orbit and that the central mass is a function of the distance with the density profile

$$\rho(r) = \frac{\rho_0}{\left(1 + \frac{r}{r_0}\right)^{\alpha}} \,. \tag{1}$$

c) Determine a set of parameters (ρ_0, r_0, and α) that fits the data for large r. What does this choice imply for the properties of the DM?

d) Modified Newtonian Dynamics (MOND) has been suggested to explain the rotation curves of disk galaxies without the need for DM. This theory postulates the following modified version of Newton's first law.

$$\mu(a/a_0)a(r) = \frac{M_\odot G_N}{r^2}\,, \qquad \text{where} \qquad \mu(a/a_0) = \begin{cases} \frac{a}{a_0}\,, & a \ll a_0 \\ 1\,, & a \gg a_0 \end{cases} \tag{2}$$

Here, $a(r)$ is the gravitational acceleration, $\mu(x)$ is an interpolation function, and a_0 is a constant introduced by the MOND prescription that must be determined from experiment. Derive the circular velocity distribution of the galactic disk in MOND and find a good fit to the observed velocities.

Hint: You may assume we are far from the central bulge where the accelerations are small and treat the galaxy as a point mass with $M_\odot = 4 \times 10^{41}$ kg.

Problem 2: *The WIMP miracle* [**10 Points**]

Dark Matter (DM) is often assumed to be a thermal relic which was in thermal equilibrium with the Standard Model particles only in the early phases of the Universe. Weakly interacting massive particles (WIMPs) are thermal relic DM candidates with masses $m_{\text{DM}} \sim 100\,\text{GeV}$ and couplings typical for electroweak physics. The fact that the observed relic density

$$\frac{\Omega_{\text{DM}}}{0.2} \approx \frac{10^{-8}\,\text{GeV}^{-2}}{\sigma} \tag{3}$$

can be explained by the existence of a WIMP is known as the *WIMP miracle*.

a) DM is generally believed to be cold, meaning that the temperature at which it thermally decoupled from the Standard Model is much lower than its mass. In this case, its number density is given by

$$n_{\text{DM}} \sim (m_{\text{DM}} T)^{3/2} e^{-m_{\text{DM}}/T}\,, \quad \text{for} \quad m_{\text{DM}} \gg T\,. \tag{4}$$

When the DM interaction rate Γ becomes comparable to the Hubble expansion rate H, the WIMP *freezes out*. In terms of the number density one can write $\Gamma = n_{\text{DM}} \cdot v \cdot \sigma$, where v is the average velocity and σ is the interaction cross-section. In a radiation-dominated universe, the Friedmann equations give $H \sim T^2/M_{\text{pl}}$ where $M_{\text{pl}} = 10^{19}\,\text{GeV}$ is the Planck scale. Use the freeze-out condition $\Gamma = H$ to show that this implies $m_{\text{DM}} \cdot \sigma \cdot M_{\text{pl}} > 1$.

b) Use the previous result to derive a lower bound on the DM mass using the cross section $\sigma = 10^{-8}\,\text{GeV}^{-2}$, as suggested by the relic density.

c) Unitarity constraints provide an upper bound on the annihilation cross section. For an average velocity of $v = 0.3$, one finds

$$\sigma \lesssim \frac{4\pi}{m_{\text{DM}}^2 v^2}\,. \tag{5}$$

Use this constraint to derive an upper bound on the DM mass.