

Exercises to “Standard Model of Particle Physics II”

Winter 2015/16

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Sheet 8

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Exercise 17: Neutrino oscillations [10 Points]

Consider the case of three neutrino flavours. The mixing matrix then reads

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

The oscillation has a CP-violating fraction $\Delta P_{\alpha\beta} \equiv P(\nu_\alpha \rightarrow \nu_\beta; t) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta; t)$.

- a) Consider the general expression for the transition probability

$$P(\nu_\alpha \rightarrow \nu_\beta; t) = \sum_j |U_{\alpha j}|^2 |U_{\beta j}|^2 + \sum_{j \neq k} U_{\alpha j} U_{\beta j}^* U_{\alpha k}^* U_{\beta k} e^{-i(E_j - E_k)t}.$$

Which result do you obtain for $\Delta P_{\alpha\alpha}$ and what does it imply?

- b) Use the unitarity of U to show that $\Im m[U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^*]$ is independent (up to a sign) of the choice of α and β ($\alpha \neq \beta$).

Note: The same invariant can be constructed for the CKM matrix and is a measure for CP-violation in the meson sector.

- c) Discuss your results and the influence of the mixing parameters on CP-violation.

Exercise 18: Neutrino oscillations in matter [10 Points]

The MSW effect describes oscillations of neutrinos in matter. Consider the case of two flavour oscillations in a realistic (non-constant) mass distribution.

The time evolution of the eigenfunctions of the Hamiltonian in matter is described by the following expression:

$$i \frac{d}{dt} \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} = \begin{pmatrix} E_A(t) & -i\dot{\theta}(t) \\ i\dot{\theta}(t) & E_B(t) \end{pmatrix} \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix}$$

The eigenstates in matter are $|\nu\rangle = (|\nu_A\rangle, |\nu_B\rangle)^T$ and to any given time they are connected to the flavor states via: $|\tilde{\nu}\rangle = \tilde{U}(t)|\nu\rangle$ with

$$|\tilde{\nu}\rangle = \begin{pmatrix} |\tilde{\nu}_e\rangle \\ |\tilde{\nu}_\mu\rangle \end{pmatrix} \quad \tilde{U} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

and the angle

$$\tan(2\theta) = \frac{\frac{\Delta m^2}{2E} \sin(2\theta_0)}{\frac{\Delta m^2}{2E} \cos(2\theta_0) - \sqrt{2} G_F N_e(t)}, \quad (1)$$

where θ_0 is constant in time, and $N_e(t)$ denotes the electron number density in matter.

The effective Hamiltonian is not diagonal, since matter eigenstates and thus the mixing angle θ are not constant. If, however, $\dot{\theta}$ is small, i.e. the change in θ occurs slowly, we obtain the adiabatic approximation, where we can neglect $\dot{\theta}$. Suppose that at the time $t = t_i$ an electron neutrino is produced,

$$|\tilde{\nu}_\alpha(t_i)\rangle \equiv |\tilde{\nu}_e\rangle = \cos\theta_i |\nu_A(t_i)\rangle + \sin\theta_i |\nu_B(t_i)\rangle.$$

- a) What is the expression for $|\tilde{\nu}_\alpha(t_f)\rangle$ in terms of $|\nu_A(t_i)\rangle$ and $|\nu_B(t_i)\rangle$ at some later time t_f in the adiabatic approximation?
- b) Show that in the adiabatic approximation the result for the transition probability is given by

$$P(\nu_e \rightarrow \nu_\mu) = \frac{1}{2} - \frac{1}{2}\cos(2\theta_i)\cos(2\theta_f) - \frac{1}{2}\sin(2\theta_i)\sin(2\theta_f)\cos\Phi_{AB}$$

with

$$\Phi_{IJ} \equiv \int_{t_i}^{t_f} [E_I(t) - E_J(t)] dt,$$

where $I, J \in \{A, B\}$.

- c) Use Eq. (1) to find an expression for $\sin^2(2\theta)$. The formula for $\sin^2(2\theta)$ describes a Breit-Wigner distribution. We define the resonance width at half height, Δr , as the width at which the curve of the distribution is larger than a half [$\sin^2(2\theta) > 1/2$]. Find an approximate expression for Δr .
- d) The adiabatic parameter at the MSW resonance is

$$\gamma_r \equiv \frac{\sin^2(2\theta_0)\Delta m^2}{\cos(2\theta_0)2E} L \quad .$$

The scale $L = |\dot{N}_e/N_e|_{\text{res}}^{-1}$ is the characteristic length scale where the electron density varies in the resonance region. Express γ_r in terms of Δr and the oscillation length given by

$$l_{\text{res}} = \frac{2\pi}{|E_A - E_B|_{\text{res}}} = \frac{4\pi E/\Delta m^2}{\sin(2\theta_0)}.$$

Discuss the adiabatic condition $\gamma_r \gtrsim 3$.

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Hand-in and discussion of sheet:

Tuesday, 08.12.15, 16.15 am, INF 501 / CIP R. 103.