Exercises to "Standard Model of Particle Physics II"

Winter 2023/24

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions: December 13, 2023 - 09:15, Phil. 12, kHS

Discussion of solutions: December 13, 2023 - 11:15, Phil. 12, kHS

Problem 1: *Number of lepton flavors* [**8 Points**]

The total decay width of the Z boson is given by:

$$\Gamma_Z = \Gamma_e + \Gamma_\mu + \Gamma_\tau + \Gamma_{\text{had}} + \Gamma_{\text{inv}} \tag{1}$$

where Γ_{had} is the sum of all possible hadronic decays $\Gamma_{e,\mu,\tau}$ are the leptonic partial widths, and Γ_{inv} is the partial decay width of the Z boson to invisibles (i.e. into final states not detectable within colliders).

a) What decay channels in the Standard Model can contribute to the invisible decay width (at tree level)?

b) Assuming only neutrinos contribute to the invisible Z branching fraction, one can calculate the number of light neutrino generations using

$$N_\nu = \left(\frac{\Gamma_{\text{inv}}}{\Gamma_l}\right)_{\text{exp}} \left(\frac{\Gamma_{\text{l}}}{\Gamma_\nu}\right)_{\text{theory}} . \tag{2}$$

Calculate the theory prediction of $\left(\frac{\Gamma_{\text{l}}}{\Gamma_\nu}\right)_{\text{theory}}$ using the expression for the partial rate of the Z boson to fermions:

$$\Gamma_f = N_C^f \frac{\alpha m_Z}{12 \sin^2\theta_W \cos^2\theta_W} \left[(g_V^f)^2 + (g_A^f)^2\right] \tag{3}$$

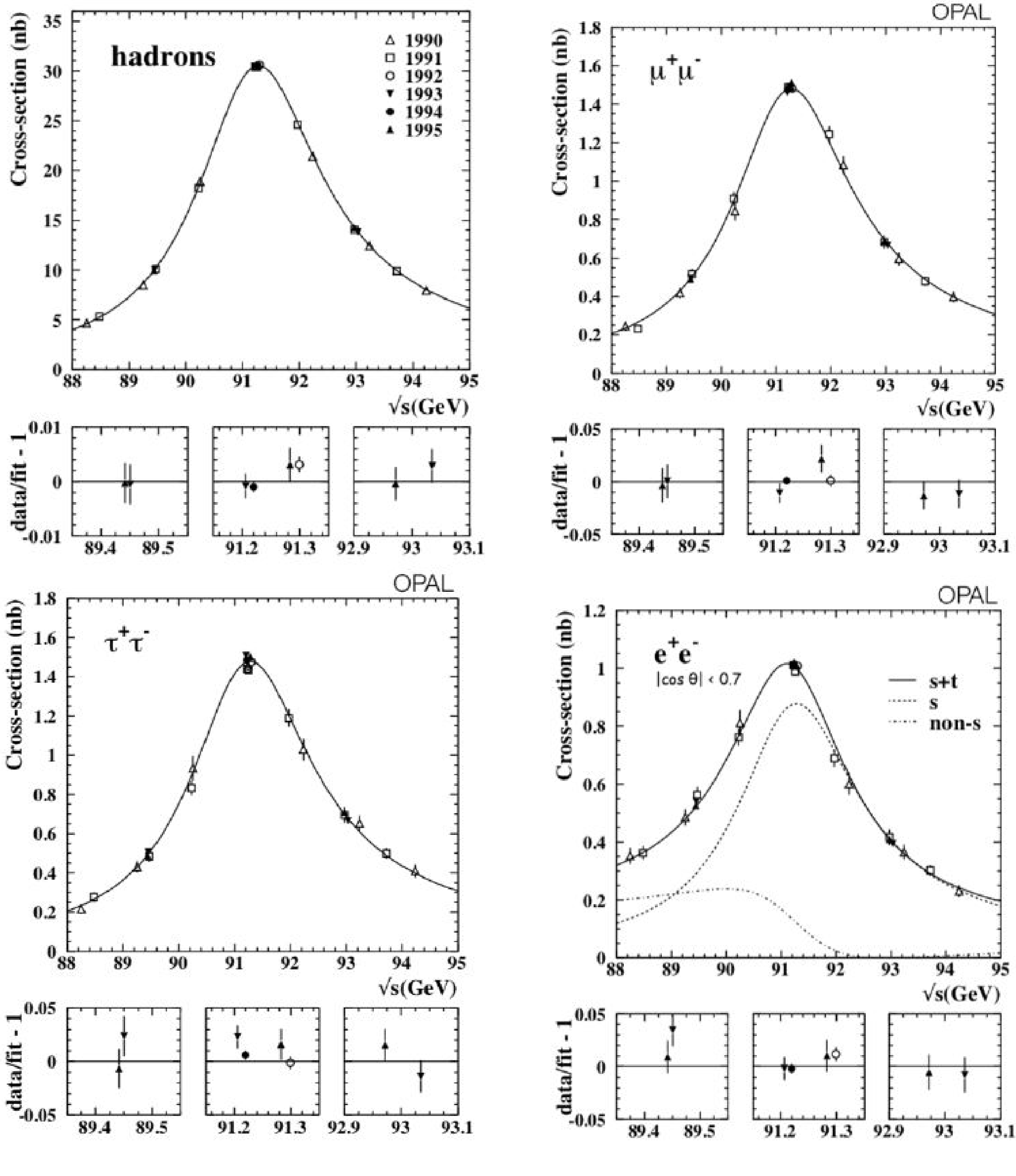
Keep in mind that you are interested in the ratio for **one** neutrino type.

c) The partial cross section at the peak of the distribution is given by

$$\sigma_{ff}^{\text{peak}} \simeq \frac{12\pi}{m_Z^2} \frac{\Gamma_e \Gamma_f}{\Gamma_Z^2} . \tag{4}$$

Using the plots provided on the back of this page, read off Γ_Z and calculate the partial width to hadrons and leptons.

d) Calculate the number of light neutrinos N_ν. What does *light* mean in this context? Are there any other ways to introduce a fourth neutrino into the Standard Model?



hadrons
1990
1991
1992
1993
1994
1995
Cross-section (nb)
√s(GeV)
data/fit - 1
OPAL
μ+μ-
τ+τ-
e+e-
|cos θ| < 0.7
s+t
s
non-s

Problem 2: *Mass matrices and mixing angles* [**8 Points**]

A general (Dirac) mass term for fermions is given by

$$\mathscr{L}_M = \overline{\psi}_{i,L} M_{ij} \psi_{j,R} + \text{h.c.}$$

where M is hermitian and given by a $n \times n$ Yukawa coupling matrix Y times the Higgs vev.

a) Show that for an arbitrary $n \times n$ matrix M one can choose a bi-unitary transformation $UMV^\dagger$ to diagonalize M, such that no diagonal element $UMV^\dagger = D := diag(m_1, m_2, ..., m_n)$ is negative. The matrices U and V are unitary.

b) Show that for a real mass matrix M one can choose orthogonal diagonalization matrices.

c) As an example for calculable mixing angles consider a simple 2×2 mass matrix of the form

$$M = \begin{bmatrix} 0 & a \\ a^* & b \end{bmatrix}.$$

The unitary matrix that diagonalizes M can be described by a single parameter: a *mixing angle* θ. Show that the following relation between mixing angle and masses holds:

$$\tan\theta = \sqrt{\frac{m_1}{m_2}}.$$

Compare this with the Cabibbo angle and the down and strange quark masses.

d) A completely different situation holds for the symmetric mass matrix

$$M = \begin{bmatrix} a & b & b \\ b & \frac{1}{2}(a+b+d) & \frac{1}{2}(a+b-d) \\ b & \frac{1}{2}(a+b-d) & \frac{1}{2}(a+b+d) \end{bmatrix}.$$

Give the mixing matrix for this mass matrix (*Hint:* try first a 23-rotation).

Problem 3: *Mixing of leptons* [**4 Points**]

Show that a mixing matrix for charged leptons would have no physical effect if neutrinos were massless particles. In other words, charged lepton mixing in the Standard Model with massless neutrinos is redundant.