Exercises to "Standard Model of Particle Physics II"

Winter 2024/25

Prof. Dr. Manfred Lindner and PD Dr. Werner Rodejohann

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Tutor: Juan Pablo Garcés **e-mail:** juan.garces@mpi-hd.mpg.de

Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions: November 26, 2024 - 14:00

Discussion of solutions: November 26, 2024 – 14:00, INF 227, SR 1.404

Problem 1: *Mass matrices and mixing angles* [**15 Points**]

A general (Dirac) mass term for fermions is given by

$$\mathscr{L}_M = \overline{\psi}_{i,L} M_{ij} \psi_{j,R} + \text{h.c.}$$

where M is hermitian and given by a $n \times n$ Yukawa coupling matrix Y times the Higgs vev.

a) Show that for an arbitrary $n \times n$ matrix M one can choose a bi-unitary transformation $UMV^\dagger$ to diagonalize M, such that no diagonal element $UMV^\dagger = D := diag(m_1, m_2, ..., m_n)$ is negative. The matrices U and V are unitary.

b) Show that for a real mass matrix M one can choose orthogonal diagonalization matrices.

c) As an example for calculable mixing angles consider a simple 2×2 mass matrix of the form

$$M = \begin{bmatrix} 0 & a \\ a^* & b \end{bmatrix}.$$

The unitary matrix that diagonalizes M can be described by a single parameter: a *mixing angle* θ. Show that the following relation between mixing angle and masses holds:

$$\tan\theta = \sqrt{\frac{m_1}{m_2}}.$$

Compare this with the Cabibbo angle and the down and strange quark masses.

d) A completely different situation holds for the symmetric mass matrix

$$M = \begin{bmatrix} a & b & b \\ b & \frac{1}{2}(a+b+d) & \frac{1}{2}(a+b-d) \\ b & \frac{1}{2}(a+b-d) & \frac{1}{2}(a+b+d) \end{bmatrix}.$$

Give the mixing matrix for this mass matrix (*Hint:* try first a 23-rotation).

Problem 2: *Mixing of leptons* [**5 Points**]

Show that a mixing matrix for charged leptons would have no physical effect if neutrinos were massless particles. In other words, charged lepton mixing in the Standard Model with massless neutrinos is redundant.