Exercises to "Standard Model of Particle Physics II"

Winter 2023/24

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions:
December 6, 2023 - 09:15, Phil. 12, kHS

Discussion of solutions:
December 6, 2023 - 11:15, Phil. 12, kHS

Problem 1: *Scalar electrodynamics* **[10 Points]**

Consider the Lagrange density of a scalar and a vector field given by:

$$\begin{aligned}\mathscr{L} &= (D_\mu\phi)^*(D^\mu\phi) - \mu^2(\phi^*\phi) - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &= (D_\mu\phi)^*(D^\mu\phi) - V(\phi^*\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.\end{aligned}$$

Local gauge symmetry couples the scalar field to the vector field via the covariant derivative

$$D_\mu\phi \equiv \partial_\mu\phi - ieA_\mu\phi.$$

a) Sketch the potential $V(\phi^*\phi)$, with $\mu^2 < 0$, $\lambda > 0$, and find the value of the scalar field that minimizes V.

b) Consider the case, where $\partial_0\vec{A} = 0$ and $A_0 = 0$. For this case, derive the equations of motion for $\vec{A}$ and find the current density $\vec{J}$, which acts as source for the gauge potential.

c) Show that, after electroweak symmetry breaking, where the scalar potential is minimal and ϕ develops the vacuum expectation value (vev) v, the current density is given by $\vec{J} = -2e^2v^2\vec{A}$, and that hence we have $\Delta\vec{B} = 2e^2v^2\vec{B}$.

d) The resistance R of a system is defined by $\vec{E} = R\vec{J}$. Show that, after electroweak symmetry breaking, the resistance vanishes ($R = 0$) and consequently the system is superconductive.

Problem 2: *Stückelberg Mechanism* [**5 Points**]

For a gauged abelian symmetry $U(1)'$ (it does not extend to non-abelian symmetries) there exists an interesting mechanism to generate a massive gauge boson, while retaining renormalizability. The method contains a real scalar field σ together with the Z'-boson associated to $U(1)'$.
Consider the Lagrangian

$$\mathscr{L} = -\frac{1}{4} Z'^{\mu\nu} Z'_{\mu\nu} + \frac{1}{2}(M_{Z'} Z'_\mu + \partial_\mu \sigma)(M_{Z'} Z'^\mu + \partial^\mu \sigma) + i\overline{\psi}\gamma^\mu(\partial_\mu - ig'Y'Z'_\mu)\psi - m\overline{\psi}\psi.$$

The gauge transformations for the Dirac fermion (with $U(1)'$ charge Y') and gauge boson are given by

$$\psi \to e^{-ig'Y'\theta(x)}\psi, \qquad Z'_\mu \to Z'_\mu - \partial_\mu \theta(x).$$

Calculate the gauge transformation of the real scalar σ that makes the Lagrangian invariant and show the invariance of the other terms. Can you fix a gauge to eliminate σ from the theory? Count degrees of freedom in both gauges.

Problem 3: *Uniqueness of the SM scalar potential* [**5 Points**]

The usual standard model scalar potential, invariant under $SU(2)_L \times U(1)_Y$, is given by

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$$

With ϕ the usual complex doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

a) Show that the quartic terms

$$V_1(\phi) = \lambda_1(\phi^\dagger \boldsymbol{\tau} \phi) \cdot (\phi^\dagger \boldsymbol{\tau} \phi) \qquad V_2(\phi) = \lambda_2 \sum_{a,b} (\phi^\dagger \tau^a \tau^b \phi)(\phi^\dagger \tau^a \tau^b \phi)$$

can be separately reduced to the quartic term in $V(\phi)$.

Hint: In the doublet representation of $SU(2)$ the following identity holds

$$\sum_a (\tau^a)_{ij}(\tau^a)_{kl} = 2(\delta_{jk}\delta_{il} - \frac{1}{2}\delta_{ij}\delta_{kl})$$