Exercises to “Standard Model of Particle Physics II”

Winter 2022/23

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions: November 30, 2022 - 11:15, Phil. 12, R105

Discussion of solutions: November 30, 2022 - 11:15, Phil. 12, R105

Problem 1: *Custodial Symmetry and the ρ-Parameter* [**10 Points**]

The 1-loop correction to the ρ-parameter is given by

$$\Delta\rho = \frac{3G_F}{8\pi^2 2\sqrt{2}} \left(m_t^2 + m_b^2 - 2\frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \log \frac{m_t^2}{m_b^2} - \frac{11}{9} m_Z^2 \sin^2\theta_W \log \frac{m_h^2}{m_Z^2} \right),$$

where m_t (m_b) is the top (bottom) quark mass, θ_W the Weinberg angle and m_h (m_Z) the mass of the Higgs (Z) boson.

a) Convince yourself that $\Delta\rho = 0$ for $m_t = m_b$ and if the hypercharge gauge coupling is zero.

b) As discussed in the lecture, the Higgs potential is invariant under $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$. Define

$$L = \begin{pmatrix} t \\ b \end{pmatrix}_L , \quad R = \begin{pmatrix} t \\ b \end{pmatrix}_R \quad \text{and } \Phi = \left(\tilde{\phi}, \phi\right),$$

with ϕ the Higgs doublet, $\widetilde{\phi} = i\sigma_2\phi^*$, and σ_2 the second Pauli matrix. Show that the Lagrangian containing the top and bottom Yukawa couplings g_t and g_b is invariant under the $\mathrm{SU(2)_L}\times\mathrm{SU(2)_R}$ symmetry only if $g_t = g_b$.

c) Show that the kinetic term for the Higgs boson is invariant under $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$ symmetry, only if the hypercharge gauge coupling is zero. You should start by writing down the covariant derivative of Φ (taking into account the fact that ϕ and $\tilde{\phi}$ have hypercharges 1 and -1 respectively).

Problem 2: *Integrating out the W-boson* [**10 Points**]

Heavy particles decouple from a given theory when the relevant energy scale of a process is well below their mass. An important example are the $W^\pm$ bosons of the SM, which we will consider in this exercise. The relevant part of the SM Lagrangian reads

$$\mathcal{L}_W = -\frac{1}{2} W^-{}_{\mu\nu} W^{+\mu\nu} + m_W^2\, W^+{}_\mu W^{-\mu} + \frac{g}{\sqrt{2}} W^+{}_\mu J^{-\mu} + \frac{g}{\sqrt{2}} W^-{}_\mu J^{+\mu}\,,$$

with the electroweak charged currents acting as source terms,

$$J^+{}_\mu = \overline{e}_L \gamma_\mu \nu_L\,, \quad J^-{}_\mu = \overline{\nu}_L \gamma_\mu e_L\,.$$

a) Derive the classical equation of motion for the $W^\pm$ bosons and expand to lowest order in $\frac{p}{m_W}$, where p is the momentum of the W (off-shell), i.e. $\partial_\mu W^\pm{}_\nu = -ip_\mu W^\pm{}_\nu$.

b) Solve the equation to lowest order in $\frac{p}{m_W}$ ignoring non-linear interactions among the W's.

c) Show that this solution can be used to obtain the Lagrangian of Fermi's effective theory of weak interactions,

$$\mathcal{L}_{\text{Fermi}} = -2\sqrt{2} G_F J^+{}_\mu J^{-\mu}$$

and determine G_F in terms of the W mass, m_W, and the weak gauge coupling, g.