

# Exercises to “Standard Model of Particle Physics II”

Winter 2015/16

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Sheet 6

November 18, 2015

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## Exercise 14: *Z*-decay and Majorana fields [15 Points]

The Lagrangian for the coupling of a fermion pair  $f$  with the  $Z$ -boson is

$$\mathcal{L} = \frac{g}{2 \cos \theta_W} \bar{f} \gamma^\mu (v_f - a_f \gamma_5) f Z_\mu,$$

For neutrinos we have that  $v_\nu = a_\nu = \frac{1}{2}$ .

- a) Calculate the decay width for  $Z \rightarrow \bar{\nu} \nu$ . Keep a possible neutrino mass in the expression.
- b) Neutrinos could be Majorana particles, which obey the relation  $\nu^c = \nu$ . The superscript  $c$  denotes charge conjugation,

$$\nu^c = C(\bar{\nu})^T,$$

with  $C = i\gamma_2\gamma_0$  in the Dirac basis.

Show the following properties

$$\begin{aligned} -C &= C^T = C^{-1} = C^* = C^\dagger, \\ C^{-1}\gamma_\mu C &= -\gamma_\mu^T, \quad C^{-1}\gamma_5 C = \gamma_5^T, \\ \bar{\psi}^c &= -\psi^T C^{-1}, \quad (\psi_L)^c = (\psi^c)_R. \end{aligned}$$

- c) Consider two spinor fields  $\psi$  and  $\chi$ . Show that

$$\bar{\psi}_L^c \chi_R^c = \bar{\chi}_L \psi_R,$$

where  $\psi_L^c := (\psi^c)_L = P_L(\psi^c)$  and so on.

- d) Show that the Lagrangian in the Majorana basis given by

$$\mathcal{L}_{\text{Maj}} = -\frac{1}{2} \begin{bmatrix} \bar{\psi}_L & \bar{\chi}_L \end{bmatrix} \begin{bmatrix} 0 & m \\ m & 0 \end{bmatrix} \begin{bmatrix} \psi_R^c \\ \chi_R^c \end{bmatrix} + \text{h.c.}$$

corresponds to a spinor field with Dirac mass term

$$\mathcal{L}_{\text{Dirac}} = -m \bar{\Psi} \Psi.$$

- e) Show that for Majorana neutrinos the vector current  $\bar{\nu} \gamma_\mu \nu$  vanishes. What happens with  $\bar{\nu} \gamma_5 \nu$ ,  $\bar{\nu} \gamma_\mu \gamma_5 \nu$  and  $\bar{\nu} [\gamma_\mu, \gamma_\nu] \nu$ ?
- f) Using the previous result calculate the decay width  $Z \rightarrow \nu \nu$  for Majorana neutrinos and compare with a).

### Tutor:

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Lecture webpage: [http://www.mpi-hd.mpg.de/manitop/StandardModel2/index\\_WS15.html](http://www.mpi-hd.mpg.de/manitop/StandardModel2/index_WS15.html)

### Hand-in and discussion of sheet:

Tuesday, 24.11.15, 16.15 am, INF 501 / CIP R. 103.