Exercises to "Standard Model of Particle Physics II"

Winter 2024/25

Prof. Dr. Manfred Lindner and PD Dr. Werner Rodejohann

Sheet 05 - November 13, 2024

Tutor: Juan Pablo Garces **e-mail:** juan.garces@mpi-hd.mpg.de
Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions by: November 19, 2024 - 14:00

Discussion of solutions: November 19, 2024 – 14:00, INF 227, SR 1.404

Problem 1: *Custodial Symmetry and the ρ-Parameter* **[10 Points]**

The 1-loop correction to the ρ-parameter is given by

$$\Delta\rho = \frac{3G_F}{8\pi^2 2\sqrt{2}}\left(m_t^2 + m_b^2 - 2\frac{m_t^2 m_b^2}{m_t^2 - m_b^2}\log\frac{m_t^2}{m_b^2} - \frac{11}{9}m_Z^2\sin^2\theta_W \log\frac{m_h^2}{m_Z^2}\right),$$

where m_t (m_b) is the top (bottom) quark mass, θ_W the Weinberg angle and m_h (m_Z) the mass of the Higgs (Z) boson.

a) Convince yourself that $\Delta\rho = 0$ for $m_t = m_b$ and if the hypercharge gauge coupling is zero.

b) As discussed in the lecture, the Higgs potential is invariant under $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$. Define

$$L = \begin{pmatrix} t \\ b \end{pmatrix}_L , \quad R = \begin{pmatrix} t \\ b \end{pmatrix}_R \quad \text{and } \Phi = \left(\tilde{\phi}, \phi\right),$$

with ϕ the Higgs doublet, $\widetilde{\phi} = i\sigma_2\phi^*$, and σ_2 the second Pauli matrix. Show that the Lagrangian containing the top and bottom Yukawa couplings g_t and g_b is invariant under the $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$ symmetry only if $g_t = g_b$.

c) Show that the kinetic term for the Higgs boson is invariant under $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$ symmetry, only if the hypercharge gauge coupling is zero. You should start by writing down the covariant derivative of Φ (taking into account the fact that ϕ and $\tilde{\phi}$ have hypercharges 1 and -1 respectively).

Problem 2: *Scalar electrodynamics* [**10 Points**]

Consider the Lagrange density of a scalar and a vector field given by:

$$\begin{aligned}\mathscr{L} &= (D_\mu\phi)^*(D^\mu\phi) - \mu^2(\phi^*\phi) - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &= (D_\mu\phi)^*(D^\mu\phi) - V(\phi^*\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.\end{aligned}$$

Local gauge symmetry couples the scalar field to the vector field via the covariant derivative

$$D_\mu\phi \equiv \partial_\mu\phi - ieA_\mu\phi.$$

a) Sketch the potential $V(\phi^*\phi)$, with $\mu^2 < 0$, $\lambda > 0$, and find the value of the scalar field that minimizes V.

b) Consider the case, where $\partial_0\vec{A} = 0$ and $A_0 = 0$. For this case, derive the equations of motion for $\vec{A}$ and find the current density $\vec{J}$, which acts as source for the gauge potential.

c) Show that, after electroweak symmetry breaking, where the scalar potential is minimal and ϕ develops the vacuum expectation value (vev) v, the current density is given by $\vec{J} = -2e^2v^2\vec{A}$, and that hence we have $\Delta\vec{B} = 2e^2v^2\vec{B}$.

d) The resistance R of a system is defined by $\vec{E} = R\vec{J}$. Show that, after electroweak symmetry breaking, the resistance vanishes ($R = 0$) and consequently the system is superconductive.