

Exercises to “Standard Model of Particle Physics II”

Winter 2023/24

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Lecture webpage: <https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html>

Hand-in of solutions:

November 29, 2023 - 09:15, Phil. 12, kHS

Discussion of solutions:

November 29, 2023 - 11:15, Phil. 12, kHS

Problem 1: Custodial Symmetry and the ρ -Parameter [10 Points]

The 1-loop correction to the ρ -parameter is given by

$$\Delta\rho = \frac{3G_F}{8\pi^2 2\sqrt{2}} \left(m_t^2 + m_b^2 - 2 \frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \log \frac{m_t^2}{m_b^2} - \frac{11}{9} m_Z^2 \sin^2 \theta_W \log \frac{m_h^2}{m_Z^2} \right),$$

where m_t (m_b) is the top (bottom) quark mass, θ_W the Weinberg angle and m_h (m_Z) the mass of the Higgs (Z) boson.

- Convince yourself that $\Delta\rho = 0$ for $m_t = m_b$ and if the hypercharge gauge coupling is zero.
- As discussed in the lecture, the Higgs potential is invariant under $SU(2)_L \times SU(2)_R$. Define

$$L = \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad R = \begin{pmatrix} t \\ b \end{pmatrix}_R \quad \text{and} \quad \Phi = (\tilde{\phi}, \phi),$$

with ϕ the Higgs doublet, $\tilde{\phi} = i\sigma_2 \phi^*$, and σ_2 the second Pauli matrix. Show that the Lagrangian containing the top and bottom Yukawa couplings g_t and g_b is invariant under the $SU(2)_L \times SU(2)_R$ symmetry only if $g_t = g_b$.

- Show that the kinetic term for the Higgs boson is invariant under $SU(2)_L \times SU(2)_R$ symmetry, only if the hypercharge gauge coupling is zero. You should start by writing down the covariant derivative of Φ (taking into account the fact that ϕ and $\tilde{\phi}$ have hypercharges 1 and -1 respectively).

Problem 2: Integrating out the W -boson [10 Points]

Heavy particles decouple from a given theory when the relevant energy scale of a process is well below their mass. An important example are the W^\pm bosons of the SM, which we will consider in this exercise. The relevant part of the SM Lagrangian reads

$$\mathcal{L}_W = -\frac{1}{2}W^-_{\mu\nu}W^{+\mu\nu} + m_W^2 W^+_{\mu}W^{-\mu} + \frac{g}{\sqrt{2}}W^+_{\mu}J^{-\mu} + \frac{g}{\sqrt{2}}W^-_{\mu}J^{+\mu},$$

with the electroweak charged currents acting as source terms,

$$J^+_{\mu} = \bar{e}_L\gamma_{\mu}\nu_L, \quad J^-_{\mu} = \bar{\nu}_L\gamma_{\mu}e_L.$$

- a) Derive the classical equation of motion for the W^\pm bosons and expand to lowest order in $\frac{p}{m_W}$, where p is the momentum of the W (off-shell), i.e. $\partial_{\mu}W^{\pm}_{\nu} = -ip_{\mu}W^{\pm}_{\nu}$.
- b) Solve the equation to lowest order in $\frac{p}{m_W}$ ignoring non-linear interactions among the W 's.
- c) Show that this solution can be used to obtain the Lagrangian of Fermi's effective theory of weak interactions,

$$\mathcal{L}_{\text{Fermi}} = -2\sqrt{2}G_F J^+_{\mu}J^{-\mu}$$

and determine G_F in terms of the W mass, m_W , and the weak gauge coupling, g .