Exercises to "Standard Model of Particle Physics II"

Winter 2023/24

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Sheet 04 - November 15, 2023

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions:
November 22, 2023 - 09:15, Phil. 12, kHS

Discussion of solutions:
November 22, 2023 - 11:15, Phil. 12, R105

Problem 1: *Electroweak symmetry breaking by a Higgs triplet* **[10 Points]**

a) Verify that

$$t_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}, \qquad t_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & -i & 0\\ i & 0 & -i\\ 0 & i & 0\end{pmatrix}, \qquad t_3 = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$$

form a triplet representation of the SU(2) algebra $[t_a, t_b] = i\epsilon_{abc}t_c$.

b) Consider the complex triplet Higgs field $\Phi(x) = (\phi_1(x),\, \phi_2(x),\, \phi_3(x))^{\mathrm{T}}$ coupled covariantly to an $\mathrm{SU(2)_L}$ gauge group with

$$D_\mu\Phi = \partial_\mu\Phi - igt_a W_\mu^a(x)\Phi,$$

where W_μ^a denote the gauge bosons of the $\mathrm{SU(2)_L}$ group. We introduce a potential $V(\Phi)$ such that the gauge symmetry breaks spontaneously. The precise form of the potential is not interesting for the moment. As a result, the Higgs field acquires a vacuum expectation value (VEV), which can be written $\Phi_0 = (0,\, 0,\, v)^{\mathrm{T}}$. Derive the gauge boson mass eigenstates of the broken $\mathrm{SU(2)_L}$ symmetry.

c) In the next step we extend the gauge group of the theory to $\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$ so that the covariant derivative now reads

$$D_\mu\Phi = \partial_\mu\Phi - igt_a W_\mu^a(x)\Phi - ig'\mathbb{1}B_\mu(x)\Phi,$$

where B_μ denotes the gauge boson of the $\mathrm{U(1)_Y}$ group. Using the same VEV as in part b), derive the gauge boson mass eigenstates and the electroweak mixing angle. What are the consequences for the $\rho = m_W^2/(m_Z^2\cos^2\theta_{\mathrm{W}})$ parameter compared to the SM?

d) How does the particle content of part b) and c) differ from the Standard Model? How can these theories be distinguished in an experiment?

Problem 2: *The Rho-parameter* [**10 Points**]

We encountered the Rho-parameter defined by

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2(\theta_W)}$$

whose value predicted by the SM is 1. Now we generalize our findings while keeping the above definition.

a) From representation theory we know that in a n-dimensional representation (i.e. n-plet, or spin-$(n-1)/2$ representation), an eigenbasis to the generators t^3 of SU(2) can be found, such that t_3 acts like $t^3 = \mathrm{diag}(j, j-1, \ldots, -j)$, where $j = (n-1)/2$. Show that if we expand a Higgs n-plet Φ with a hypercharge of y in this basis its components will be eigenstates of $Q = t^3 + \hat{Y}$, where $\hat{Y}\Phi = y\mathbf{1}_{n\times n}\Phi = y\Phi$. What is the condition on y to ensure that there is a neutral component?

b) In this basis the raising and lowering operators defined from $t^{\pm} = (t^1 \pm it^2)$ can be descibed by their action on the orthonormal basis vectors e_m (labelled by their t^3 eigenvalues m):

$$t^+ e_m = \sqrt{j(j+1) - m(m+1)}e_{m+1}\,, \qquad t^- e_m = \sqrt{j(j+1) - m(m-1)}e_{m-1}\,.$$

Assume that the hypercharge is such that ϕ_m is neutral and that only this neutral component of Φ acquires a vev, $\vec{v} = ve_m$. Recall that the covariant derivative reads

$$D_\mu\Phi = (\partial_\mu - igt^iW_\mu^i - ig'\hat{Y}B_\mu)\Phi$$

and, show that

$$m_W^2 = \frac{g^2}{2}\vec{v}^\dagger(t^+t^- + t^-t^+)\vec{v}\,.$$

c) Show that this leads to

$$m_W^2 = g^2v^2(j(j+1) - m^2)\,.$$

d) With the standard mixing angle $\tan\theta_W = g'/g$, derive the mass of Z and use it to show that

$$\rho = \frac{v^2(j(j+1) - y^2)}{2v^2y^2}\,.$$

e) Check that the Higgs doublet with hypercharge 1/2 satisfies the condition that $\rho = 1$. What would be the next combination of weak isospin and (rational) hypercharge that can satisify this bound?