Exercises to "Standard Model of Particle Physics II"

Winter 2015/16

Prof. Dr. Manfred Lindner and Dr. Werner Rodejohann Sheet 4 November 04, 2015

Exercise 8: *Noether's theorem revised* [5 **Points**]

Consider a Lagrange density of the general form $\mathscr{L} = \mathscr{L}_0(\psi,\ \partial_\mu\psi)$. From Noether's theorem we know that if the Lagrange density is left invariant by a symmetry transformation this leads to the conserved current

$$J^\mu = \frac{\partial \mathscr{L}_0}{\partial(\partial_\mu \psi)}\delta\psi.$$

Now imagine that we add a term $\mathscr{L}_1$ to the Lagrange density (so that $\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_1$) that does not respect the symmetry.
Show that in this case the current is not conserved any more, i.e.

$$\partial_\mu J^\mu \neq 0.$$

Exercise 9: *Higgs sector and gauge bosons* [5 **Points**]

Consider the kinetic term of the Standard Model Higgs doublet,

$$\mathscr{L} = (D_\mu\phi)^\dagger(D^\mu\phi),$$

with $D_\mu\phi = \partial_\mu\phi + igT^aW^a_\mu\phi + ig'\frac{Y}{2}B_\mu\phi$. In the covariant derivative, $T^a = \sigma^a/2$ are the generators of $\mathrm{SU(2)_L}$ and Y denotes hypercharge. With a suitable isospin rotation the Higgs doublet takes on the form

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}.$$

Derive the Feynman rules for the hW^+W^- and the hhW^+W^- couplings from the kinetic term of the Higgs.

Exercise 10: *Electroweak symmetry breaking by a Higgs triplet* [10 **Points**]

a) Verify that

$$t_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}, \qquad t_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & -\mathrm{i} & 0\\ \mathrm{i} & 0 & -\mathrm{i}\\ 0 & \mathrm{i} & 0\end{pmatrix}, \qquad t_3 = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$$

form a triplet representation of the SU(2) algebra $[t_a,\ t_b] = i\epsilon_{abc}t_c$.

b) Consider the complex triplet Higgs field $\mathbf{\Phi}(x) = (\phi_1(x),\ \phi_2(x),\ \phi_3(x))^{\mathrm{T}}$ coupled covariantly to an $\mathrm{SU}(2)_{\mathrm{L}}$ gauge group with

$$D_\mu \mathbf{\Phi} = \partial_\mu \mathbf{\Phi} - \mathrm{i} g t_a W_{a\mu}(x)\mathbf{\Phi},$$

where $W_{a\mu}$ denote the gauge bosons of the $\mathrm{SU}(2)_{\mathrm{L}}$ group. We introduce a potential $V(\mathbf{\Phi})$ such that the gauge symmetry breaks spontaneously. The precise form of the potential is not interesting for the moment. As a result, the Higgs field acquires a vacuum expectation value (vev), which can be written $\mathbf{\Phi}_0 = (0,\ 0,\ v)^{\mathrm{T}}$. Derive the gauge boson mass eigenstates of the broken $\mathrm{SU}(2)_{\mathrm{L}}$ symmetry.

c) In the next step we extend the gauge group of the theory to $\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{U}(1)_{\mathrm{Y}}$ so that the covariant derivative now reads

$$D_\mu \mathbf{\Phi} = \partial_\mu \mathbf{\Phi} - \mathrm{i} g t_a W_{a\mu}(x)\mathbf{\Phi} - \mathrm{i} g' \mathbb{1} B_\mu(x)\mathbf{\Phi},$$

where B_μ denotes the gauge boson of the $\mathrm{U}(1)_{\mathrm{Y}}$ group. Using the same vev as in part b), derive the gauge boson mass eigenstates and the electroweak mixing angle. What are the consequences for the $\rho = m_W^2/(m_Z^2 \cos^2\theta_{\mathrm{W}})$ parameter compared to the Standard Model (SM)?

d) How does the particle content of part b) and c) differ from the SM? How can this in principle be observed in an experiment?

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Lecture webpage: `http://www.mpi-hd.mpg.de/manitop/StandardModel2/index_WS15.html`

Hand-in and discussion of sheet:
Tuesday, 10.11.15, 16.15 am, INF 501 / CIP R. 103 (PCP).