Exercises to "Standard Model of Particle Physics II"

Winter 2024/25

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions by: November 5, 2024 - 14:00

Discussion of solutions: November 5, 2024 – 14:00, INF 227, SR 1.404

Problem 1: *Electroweak symmetry breaking by a Higgs triplet* **[12 Points]**

a) Verify that

$$t_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad t_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad t_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

form a triplet representation of the SU(2) algebra $[t_a, t_b] = i\epsilon_{abc}t_c$.

b) Consider the complex triplet Higgs field $\Phi(x) = (\phi_1(x),\, \phi_2(x),\, \phi_3(x))^{\mathrm{T}}$ coupled covariantly to an $\mathrm{SU}(2)_{\mathrm{L}}$ gauge group with

$$D_\mu\Phi = \partial_\mu\Phi - igt_aW^a_\mu(x)\Phi,$$

where W^a_μ denote the gauge bosons of the $\mathrm{SU}(2)_{\mathrm{L}}$ group. We introduce a potential $V(\Phi)$ such that the gauge symmetry breaks spontaneously. The precise form of the potential is not interesting for the moment. As a result, the Higgs field acquires a vacuum expectation value (VEV), which can be written $\Phi_0 = (0,\, 0,\, v)^{\mathrm{T}}$. Derive the gauge boson mass eigenstates of the broken $\mathrm{SU}(2)_{\mathrm{L}}$ symmetry.

c) In the next step we extend the gauge group of the theory to $\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{U}(1)_{\mathrm{Y}}$ so that the covariant derivative now reads

$$D_\mu\Phi = \partial_\mu\Phi - igt_aW^a_\mu(x)\Phi - ig'\mathbb{1}B_\mu(x)\Phi,$$

where B_μ denotes the gauge boson of the $\mathrm{U}(1)_{\mathrm{Y}}$ group. Using the same VEV as in part b), derive the gauge boson mass eigenstates and the electroweak mixing angle. What are the consequences for the $\rho = m_W^2/(m_Z^2\cos^2\theta_{\mathrm{W}})$ parameter compared to the SM?

d) How does the particle content of part b) and c) differ from the Standard Model? How can these theories be distinguished in an experiment?

Problem 2: *Noether's theorem revised* [8 **Points**]

Consider a Lagrange density of the general form $\mathscr{L} = \mathscr{L}_0(u, \partial_\mu u)$, invariant under some symmetry in such a way that, according to the Noether's theorem, there is a conserved current

$$j^\mu = \frac{\partial \mathscr{L}}{\partial(\partial_\mu u)} \delta u \,.$$

Now consider to extend the theory, $\mathscr{L}_0 \to \mathscr{L} = \mathscr{L}_0 + \mathscr{L}_1$ to include $\mathscr{L}_1(u, \partial_\mu u)$ that depends on the same fields as $\mathscr{L}$ but is not invariant under the previously considered symmetry. The total current will not be conserved in this new case. Show that also the previously conserved Noether's current corresponding to the invariance of $\mathscr{L}$ will not be conserved anymore.