Exercises to "Standard Model of Particle Physics II"

Winter 2024/25

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions:
January 28, 2025 - 14:00

Discussion of solutions:
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Problem 1: *Left-right symmetric electroweak model* [**20 Points**]

The left-right symmetric model can be introduced by assuming right-handed fermion doublets in analogy to the left-handed ones. The quark and lepton spectra consist of

$$Q^i_{\mathrm{L,R}} = \begin{pmatrix} U^i_{\mathrm{L,R}} \\ D^i_{\mathrm{L,R}} \end{pmatrix} \qquad L^i_{\mathrm{L,R}} = \begin{pmatrix} \nu^i_{\mathrm{L,R}} \\ e^i_{\mathrm{L,R}} \end{pmatrix}, \tag{1}$$

with the following $\mathrm{SU}(2)_\mathrm{L}$, $\mathrm{SU}(2)_\mathrm{R}$, and $\mathrm{U}(1)_{B-L}$ transformation properties.

$$Q_\mathrm{L} : (2_\mathrm{L},\ 1_\mathrm{R},\ 1/3) \qquad Q_\mathrm{R} : (1_\mathrm{L},\ 2_\mathrm{R},\ 1/3) \tag{2}$$

$$L_\mathrm{L} : (2_\mathrm{L},\ 1_\mathrm{R},\ -1) \qquad L_\mathrm{R} : (1_\mathrm{L},\ 2_\mathrm{R},\ -1) \tag{3}$$

The Higgs sector contains a bi-doublet ϕ and two triplets Δ_L and Δ_R with the following transformation properties.

$$\phi : (2_\mathrm{L},\ 2_\mathrm{R},\ 0) \qquad \Delta_\mathrm{L} : (3_\mathrm{L},\ 1_\mathrm{R},\ 2) \qquad \Delta_\mathrm{R} : (1_\mathrm{L},\ 3_\mathrm{R},\ 2) \tag{4}$$

These scalars may be expressed in terms of the 2×2 matrices

$$\phi = \begin{pmatrix} \phi^0_1 & \phi^+_2 \\ \phi^-_1 & \phi^0_2 \end{pmatrix} \qquad \Delta_{\mathrm{L,\,R}} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}. \tag{5}$$

a) Why does this model not work with only the bi-doublet?

b) Construct the Lagrange density for the fermion-Higgs interactions $\mathcal{L}_\mathrm{Yukawa}$ (including all possible gauge singlets).

c) Use the assumption that the vacuum is electrically neutral after spontaneous symmetry breaking to derive the fermion mass terms in the broken phase.

d) Let us now modify the symmetry-breaking part of the model but leave the quark and lepton sector unchanged. In the Higgs sector we still have the bi-doublet ϕ, but instead of the triplets we introduce two scalar doublets $A_\mathrm{L,R}$ and a fermionic (Grassmann-valued) singlet χ with the following transformation properties under $\mathrm{SU}(2)_\mathrm{L}$, $\mathrm{SU}(2)_\mathrm{R}$, and $\mathrm{U}(1)_\mathrm{B\text{-}L}$.

$$A_\mathrm{L} : (2_\mathrm{L},\ 1_\mathrm{R},\ 1) \qquad A_\mathrm{R} : (1_\mathrm{L},\ 2_\mathrm{R},\ 1); \qquad \chi : (1_\mathrm{L},\ 1_\mathrm{R},\ 0) \tag{6}$$

Left-right symmetry then implies the invariance of the Lagrange density under the following transformations (where Ψ denotes any fermion field).

$$\Psi_\mathrm{L} \leftrightarrow \Psi_\mathrm{R} \qquad A_\mathrm{L} \leftrightarrow A_\mathrm{R} \qquad \phi \leftrightarrow \phi^\dagger \tag{7}$$

Construct the Lagrange density $\mathcal{L}_\mathrm{Yukawa}$ for the fermion masses in this model (you should again construct singlets under the whole gauge group).