Exercises to "Standard Model of Particle Physics II"

Winter 2023/24

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions:
January 31, 2024 - 09:15, Phil. 12, kHS

Discussion of solutions:
January 31, 2024 - 11:15, Phil. 12, kHS

Problem 1: *The Boltzmann equation* [**10 Points**]

Consider a stable particle χ and its antiparticle $\bar{\chi}$ that participate in the annihilation process

$$\chi\,\bar{\chi} \leftrightarrow \psi\bar{\psi}\,, \tag{1}$$

where ψ denotes all possible final WIMP states. Assuming a co-moving volume whose expansion is governed by the standard Friedmann equations, the number density n of ψ in that volume is governed by the Boltzmann equations

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -3Hn - \langle\sigma_{\mathrm{ann}}v\rangle(n^2 - n_{\mathrm{eq}}^2) \tag{2}$$

$$\frac{\mathrm{d}s}{\mathrm{d}t} = -3Hs\,, \tag{3}$$

where t is time, s is the entropy density, H is the Hubble parameter, n_{eq} is the "WIMP equilibrium number", and $\langle\sigma_{\mathrm{ann}}v\rangle$ is the thermally-averaged total annihilation cross-section.

a) It is convenient to combine the two Boltzmann equations above into one differential equation by defining the new variables $Y = n/s$ and $x = m/T$ where m is the mass of ψ and T is the temperature. Use these definitions to derive the following relation.

$$\frac{\mathrm{d}Y}{\mathrm{d}x} = \frac{1}{3H}\frac{\mathrm{d}s}{\mathrm{d}x}\langle\sigma_{\mathrm{ann}}v\rangle(Y^2 - Y_{\mathrm{eq}}^2) \tag{4}$$

b) The Hubble parameter is related to the energy density ρ according to

$$H^2 = \frac{8\pi}{3M_{\mathrm{pl}}^2}\rho\,, \tag{5}$$

while the energy density and entropy density may be expressed in terms of their respective effective degrees of freedom, $g_{\mathrm{eff}}(T)$ and $h_{\mathrm{eff}}(T)$, as

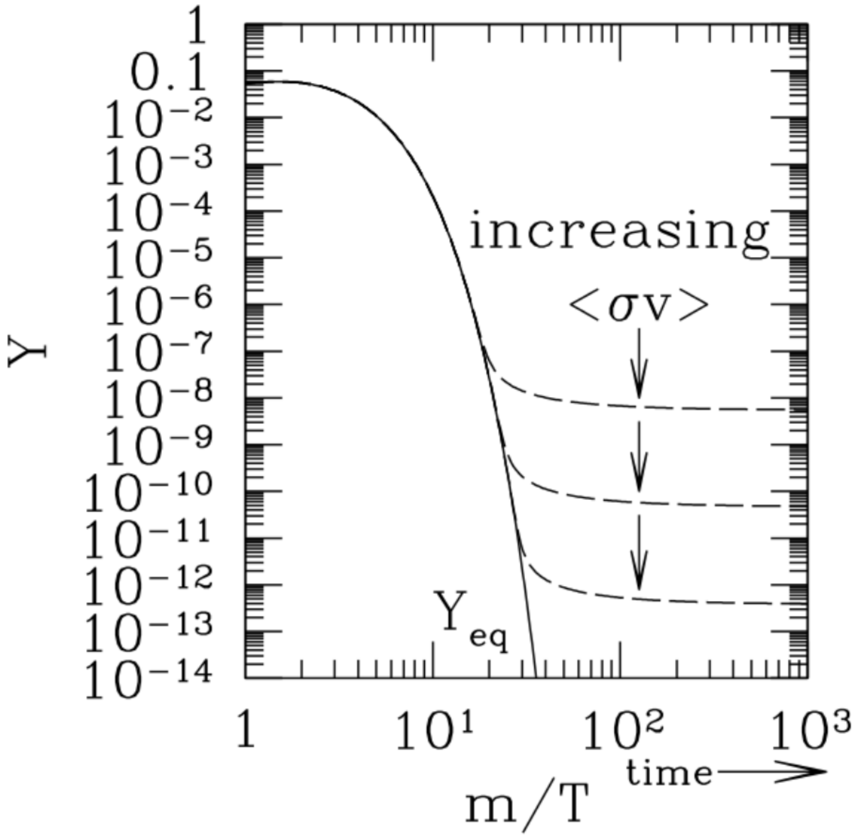
$$\rho(T) = \frac{\pi^2}{30}T^4 g_{\mathrm{eff}}(T)\,, \qquad s(T) = \frac{2\pi^2}{45}T^3 h_{\mathrm{eff}}(T)\,. \tag{6}$$

Use these relations to find an expression for $\mathrm{d}s/\mathrm{d}x$, then plug this into the previous result to derive

$$\frac{\mathrm{d}Y}{\mathrm{d}x} = -\left(\frac{45}{\pi M_{\mathrm{pl}}^2}\right)^{-1/2}\frac{g_*^{1/2}m}{x^2}\langle\sigma_{\mathrm{ann}}v\rangle(Y^2 - Y_{\mathrm{eq}}^2)\,, \tag{7}$$

where

$$g_*^{1/2} = \frac{h_{\mathrm{eff}}}{g_{\mathrm{eff}}^{1/2}}\left(1 + \frac{T}{3h_{\mathrm{eff}}}\frac{\mathrm{d}h_{\mathrm{eff}}}{\mathrm{d}T}\right)\,. \tag{8}$$



c) One may solve (7) numerically which yields the famous plot on the following page. Give a brief explanation of what is happening in this figure.

Problem 2: *Z' physics* **[10 Points]**

If one adds an additional $\mathrm{U}(1)'$ symmetry to the SM gauge group, the general effective Lagrange density after breaking the $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \times \mathrm{U}(1)'$ symmetry to $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{U}(1)_\mathrm{em}$ can be written as

$$\mathscr{L} = \mathscr{L}_\mathrm{SM} + \mathscr{L}_{Z'} + \mathscr{L}_\mathrm{mix}\,.$$

The relevant part of the Standard Model Lagrangian is given by

$$\mathscr{L}_\mathrm{SM} = -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{W}^a_{\mu\nu}\hat{W}^{\mu\nu,a} + \frac{1}{2}\hat{M}_Z^2\hat{Z}_\mu\hat{Z}^\mu - \frac{e}{c_W}j_B^\mu\hat{B}_\mu - \frac{e}{s_W}j_W^{\mu,a}\hat{W}^a_\mu$$

where the hats denote that the fields are not mass eigenstates. The Z' part reads

$$\mathscr{L}_{Z'} = -\frac{1}{4}\hat{Z'}_{\mu\nu}\hat{Z'}^{\mu\nu} + \frac{1}{2}\hat{M}_{Z'}^2\hat{Z'}_\mu\hat{Z'}^\mu - g' j'_\mu \hat{Z}'_\mu\,,$$

where g' denotes the $\mathrm{U}(1)'$ gauge coupling. Finally, the kinetic- and mass-mixing terms can be parameterized as

$$\mathscr{L}_\mathrm{mix} = -\frac{\sin\chi}{2}\hat{Z'}_{\mu\nu}\hat{B}^{\mu\nu} + \delta\hat{M}^2\hat{Z}'_\mu\hat{Z}^\mu\,.$$

a) Determine the mass eigenstates Z_1^μ and Z_2^μ and determine the couplings of $Z_{1,2}$ to the currents j_B, j_W and j'. You may set the kinetic mixing angle χ to zero for simplicity.
Hint: Reexpress $\hat{B}_\mu$ and $\hat{W}^3_\mu$ in terms of A_μ and Z_μ.

b) Since the mass of the physical Z boson changes compared to the SM, the ρ parameter is no longer equal to one (at tree-level). Use the current value $\rho = 1.0008^{+0.0017}_{-0.0007}$ to constrain the Z-Z' mixing. You may assume that $\hat{M}_{Z'} \gg \hat{M}_Z \gg \delta\hat{M}$.

c) A well-motivated extension of the SM is a gauged B-L symmetry (baryon minus lepton number). Write down the corresponding current for each of the SM fermions using the general formula,

$$j'_\mu = \sum_\psi \bar{\psi}\gamma_\mu \mathcal{Q}_{BL}\psi\,,$$

where $\mathcal{Q}_{BL}$ denotes the B-L charge operator.