Exercises to "Standard Model of Particle Physics II"

Winter 2024/25

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Sheet 11 - January 15, 2025

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions: January 21, 2025 - 14:00

Discussion of solutions: January 21, 2025 – 14:00, INF 227, SR 1.404

Problem 1: Z' *physics* [**10 Points**]

If one adds an additional $\mathrm{U}(1)'$ symmetry to the SM gauge group, the general effective Lagrange density after breaking the $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \times \mathrm{U}(1)'$ symmetry to $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{U}(1)_\mathrm{em}$ can be written as

$$\mathscr{L} = \mathscr{L}_\mathrm{SM} + \mathscr{L}_{Z'} + \mathscr{L}_\mathrm{mix} \,.$$

The relevant part of the Standard Model Lagrangian is given by

$$\mathscr{L}_\mathrm{SM} = -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{W}^a_{\mu\nu}\hat{W}^{\mu\nu,a} + \frac{1}{2}\hat{M}_Z^2\hat{Z}_\mu\hat{Z}^\mu - \frac{e}{c_W}j_B^\mu\hat{B}_\mu - \frac{e}{s_W}j_W^{\mu,a}\hat{W}^a_\mu$$

where the hats denote that the fields are not mass eigenstates. The Z' part reads

$$\mathscr{L}_{Z'} = -\frac{1}{4}\hat{Z'}_{\mu\nu}\hat{Z'}^{\mu\nu} + \frac{1}{2}\hat{M}_{Z'}^2\hat{Z'}_\mu\hat{Z'}^\mu - g' j'_\mu \hat{Z}'_\mu \,,$$

where g' denotes the $\mathrm{U}(1)'$ gauge coupling. Finally, the kinetic- and mass-mixing terms can be parameterized as

$$\mathscr{L}_\mathrm{mix} = -\frac{\sin\chi}{2}\hat{Z'}_{\mu\nu}\hat{B}^{\mu\nu} + \delta\hat{M}^2\hat{Z}'_\mu\hat{Z}^\mu \,.$$

a) Determine the mass eigenstates Z_1^μ and Z_2^μ and determine the couplings of $Z_{1,2}$ to the currents j_B, j_W and j'. You may set the kinetic mixing angle χ to zero for simplicity.
Hint: Reexpress $\hat{B}_\mu$ and $\hat{W}^3_\mu$ in terms of A_μ and Z_μ.

b) Since the mass of the physical Z boson changes compared to the SM, the ρ parameter is no longer equal to one (at tree-level). Use the current value $\rho = 1.0008^{+0.0017}_{-0.0007}$ to constrain the Z-Z' mixing. You may assume that $\hat{M}_{Z'} \gg \hat{M}_Z \gg \delta\hat{M}$.

c) A well-motivated extension of the SM is a gauged B-L symmetry (baryon minus lepton number). Write down the corresponding current for each of the SM fermions using the general formula,

$$j'_\mu = \sum_\psi \bar{\psi}\,\gamma_\mu \mathcal{Q}_{BL}\psi \,,$$

where $\mathcal{Q}_{BL}$ denotes the B-L charge operator.

Problem 2: *Stückelberg Mechanism* [**10 Points**]

For a gauged abelian symmetry $U(1)'$ (it does not extend to non-abelian symmetries) there exists an interesting mechanism to generate a massive gauge boson, while retaining renormalizability. The method involves a real scalar field σ together with the Z'-boson associated to $U(1)'$.
Consider the Lagrangian

$$\mathscr{L} = -\frac{1}{4} Z'^{\mu\nu} Z'_{\mu\nu} + \frac{1}{2}(M_{Z'} Z'_\mu + \partial_\mu \sigma)(M_{Z'} Z'^\mu + \partial^\mu \sigma) + i\overline{\psi}\gamma^\mu(\partial_\mu - ig' Y' Z'_\mu)\psi - m\overline{\psi}\psi.$$

The gauge transformations for the Dirac fermion (with $U(1)'$ charge Y') and gauge boson are given by

$$\psi \to e^{-ig'Y'\theta(x)}\psi, \qquad Z'_\mu \to Z'_\mu - \partial_\mu \theta(x).$$

a) Calculate the gauge transformation of the real scalar σ that makes the Lagrangian invariant and show the invariance of the other terms with respect to the transformations of the three fields present in the Lagrangian.

b) Can you fix a gauge to eliminate σ from the theory? Show how it is possible. What happens to the number of degrees of freedom?