Exercises to "Standard Model of Particle Physics II"

Winter 2023/24

Prof. Dr. Manfred Lindner and PD Dr. Werner Rodejohann

Sheet 11 - January 17, 2024

Tutor: Juan Pablo Garces **e-mail:** juan.garces@mpi-hd.mpg.de

Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions:	**Discussion of solutions:**
January 24, 2024 - 09:15, Phil. 12, kHS	January 24, 2024 - 11:15, Phil. 12, kHS

Problem 1: *The WIMP miracle* [**8 Points**]

Dark Matter (DM) is often assumed to be a thermal relic which was in thermal equilibrium with the Standard Model particles only in the early phases of the Universe. Weakly interacting massive particles (WIMPs) are thermal relic DM candidates with masses $m_{\mathrm{DM}} \sim 100\,\mathrm{GeV}$ and couplings typical for electroweak physics. The fact that the observed relic density

$$\frac{\Omega_{\mathrm{DM}}}{0.2} \approx \frac{10^{-8}\,\mathrm{GeV}^{-2}}{\sigma} \tag{1}$$

can be explained by the existence of a WIMP is known as the *WIMP miracle*.

a) DM is generally believed to be cold, meaning that the temperature at which it thermally decoupled from the Standard Model is much lower than its mass. In this case, its number density is given by

$$n_{\mathrm{DM}} \sim (m_{\mathrm{DM}}T)^{3/2} e^{-m_{\mathrm{DM}}/T}\,, \quad \text{for} \quad m_{\mathrm{DM}} \gg T\,. \tag{2}$$

When the DM interaction rate Γ becomes comparable to the Hubble expansion rate H, the WIMP *freezes out*. In terms of the number density one can write $\Gamma = n_{\mathrm{DM}} \cdot v \cdot \sigma$, where v is the average velocity and σ is the interaction cross-section. In a radiation-dominated universe, the Friedmann equations give $H \sim T^2/M_{\mathrm{pl}}$ where $M_{\mathrm{pl}} = 10^{19}\,\mathrm{GeV}$ is the Planck scale. Use the freeze-out condition $\Gamma = H$ to show that this implies $m_{\mathrm{DM}} \cdot \sigma \cdot M_{\mathrm{pl}} > 1$.

b) Use the previous result to derive a lower bound on the DM mass using the cross section $\sigma = 10^{-8}\,\mathrm{GeV}^{-2}$, as suggested by the relic density.

c) Unitarity constraints provide an upper bound on the annihilation cross section. For an average velocity of $v = 0.3$, one finds

$$\sigma \lesssim \frac{4\pi}{m_{\mathrm{DM}}^2 v^2}\,. \tag{3}$$

Use this constraint to derive an upper bound on the DM mass.

Problem 2: *Left-right symmetric electroweak model* **[12 Points]**

The left-right symmetric model can be introduced by assuming right-handed fermion doublets in analogy to the left-handed ones. The quark and lepton spectra consist of

$$Q^i_{\mathrm{L,R}} = \begin{pmatrix} U^i_{\mathrm{L,R}} \\ D^i_{\mathrm{L,R}} \end{pmatrix} \qquad\qquad L^i_{\mathrm{L,R}} = \begin{pmatrix} \nu^i_{\mathrm{L,R}} \\ e^i_{\mathrm{L,R}} \end{pmatrix}, \tag{4}$$

with the following $\mathrm{SU}(2)_\mathrm{L}$, $\mathrm{SU}(2)_\mathrm{R}$, and $\mathrm{U}(1)_{B-L}$ transformation properties.

$$Q_\mathrm{L} : (2_\mathrm{L},\ 1_\mathrm{R},\ 1/3) \qquad\qquad Q_\mathrm{R} : (1_\mathrm{L},\ 2_\mathrm{R},\ 1/3) \tag{5}$$

$$L_\mathrm{L} : (2_\mathrm{L},\ 1_\mathrm{R},\ -1) \qquad\qquad L_\mathrm{R} : (1_\mathrm{L},\ 2_\mathrm{R},\ -1) \tag{6}$$

The Higgs sector contains a bi-doublet ϕ and two triplets Δ_L and Δ_R with the following transformation properties.

$$\phi : (2_\mathrm{L},\ 2_\mathrm{R},\ 0) \qquad\qquad \Delta_\mathrm{L} : (3_\mathrm{L},\ 1_\mathrm{R},\ 2) \qquad\qquad \Delta_\mathrm{R} : (1_\mathrm{L},\ 3_\mathrm{R},\ 2) \tag{7}$$

These scalars may be expressed in terms of the 2×2 matrices

$$\phi = \begin{pmatrix} \phi^0_1 & \phi^+_2 \\ \phi^-_1 & \phi^0_2 \end{pmatrix} \qquad\qquad \Delta_{\mathrm{L,\,R}} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}. \tag{8}$$

a) Why does this model not work with only the bi-doublet?

b) Construct the Lagrange density for the fermion-Higgs interactions $\mathcal{L}_{\mathrm{Yukawa}}$ (including all possible gauge singlets).

c) Use the assumption that the vacuum is electrically neutral after spontaneous symmetry breaking to derive the fermion mass terms in the broken phase.

d) *Optional:* Let us now modify the symmetry-breaking part of the model but leave the quark and lepton sector unchanged. In the Higgs sector we still have the bi-doublet ϕ, but instead of the triplets we introduce two scalar doublets $A_{\mathrm{L,R}}$ and a fermionic (Grassmann-valued) singlet χ with the following transformation properties under $\mathrm{SU}(2)_\mathrm{L}$, $\mathrm{SU}(2)_\mathrm{R}$, and $\mathrm{U}(1)_{\mathrm{B\text{-}L}}$.

$$A_\mathrm{L} : (2_\mathrm{L},\ 1_\mathrm{R},\ 1) \qquad\qquad A_\mathrm{R} : (1_\mathrm{L},\ 2_\mathrm{R},\ 1); \qquad\qquad \chi : (1_\mathrm{L},\ 1_\mathrm{R},\ 0) \tag{9}$$

Left-right symmetry then implies the invariance of the Lagrange density under the following transformations (where Ψ denotes any fermion field).

$$\Psi_\mathrm{L} \leftrightarrow \Psi_\mathrm{R} \qquad\qquad A_\mathrm{L} \leftrightarrow A_\mathrm{R} \qquad\qquad \phi \leftrightarrow \phi^\dagger \tag{10}$$

Construct the Lagrange density $\mathcal{L}_{\mathrm{Yukawa}}$ for the fermion masses in this model (you should again construct singlets under the whole gauge group).