Exercises to "Standard Model of Particle Physics II"

Winter 2024/25

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions:
January 14, 2025 - 14:00

Discussion of solutions:
January 14, 2025 – 14:00, INF 227, SR 1.404

Problem 1: *DM direct detection* [**12 Points**]

Imagine you are an experimentalist in a collaboration working on the direct detection of dark matter. To detect possible WIMP scattering events, you are considering two different detector types. The first is based on a germanium target and the second has a xenon target. The characteristics of the two targets are listed below.

		Germanium	**Xenon**
Energy Threshold	E_t	$1\,\mathrm{keV/c^2}$	$5\,\mathrm{keV/c^2}$
Energy Interval	ΔE	$(1-40)\,\mathrm{keV/c^2}$	$(5-40)\,\mathrm{keV/c^2}$
Target Mass	M	1 kg	35 kg
Target Element Mass	m_A	$65\,\mathrm{GeV/c^2}$	$122\,\mathrm{GeV/c^2}$
Mass Number	A	73	131

Your goal is to provide the best possible limit on a theory predicting a

- light dark matter candidate $M_\chi = 5\,\mathrm{GeV/c^2}$,
- heavy dark matter candidate $M_\chi = 500\,\mathrm{GeV/c^2}$.

In both cases the commissioned runtime is T = 100 days.

a) Since the WIMP-nucleus relative speed is of order 100 km/s, elastic WIMP scattering occurs in the extreme non-relativistic limit. Direct detection experiments are limited by the nuclear recoil energy threshold of the target material E_t . In terms of the velocity v of the dark matter particle and the center of mass frame scattering angle θ, the recoil energy E is given by

$$E = v^2 \frac{\mu_N^2}{m_A}(1-\cos\theta), \tag{1}$$

in which the reduced mass is given by $\mu_N = \frac{m_A M_\chi}{m_A + M_\chi}$. Using Eq. 1, compute the minimal velocity v_{min} needed to generate a detectable energy deposit in the germanium and xenon detector, for the two dark matter masses given above (i.e. give four values of v). Note that v is given as a fraction of $c = 3\cdot 10^5$ km/s using the units in the table.

b) Assume that the dark matter velocity distribution is isotropic, spherically symmetric and follows a Maxwell-Boltzmann distribution

$$f(\mathbf{v}) = N e^{-\mathbf{v}^2/v_0^2} \tag{2}$$

with $v_0 = 220$ km/s being the circular velocity of the dark matter halo and $N = 1/(\sqrt{\pi}v_0)^3$. Integrate out the angular dependencies so that you can sketch the function with respect to v.

Indicate the values which you have derived in part a). Does it even make sense to consider very fast dark matter particles or should the velocity distribution be truncated at a certain speed v_{max}?

c) The expected rate for WIMP interactions can be expressed as

$$R \approx \frac{A^2}{2\mu_P^2 M_\chi}\sigma_0 \rho_\chi \int_{v_{min}}^{v_{max}} \frac{f(v)}{v} dv \cdot \Delta E, \tag{3}$$

in which $\mu_P = \frac{m_N M_\chi}{m_N + M_\chi}$ is the reduced mass of the dark matter and a nucleon (either proton or neutron) $m_N \approx 1\,\text{GeV/c}^2$. The local dark matter density $\rho_\chi = 0.3\,\text{GeV/cm}^3$ and the velocity distribution given above are astrophysical inputs. The mass of the dark matter candidate and the cross section $\sigma_0 = 1 \cdot 10^{-45}\,\text{cm}^2$ are quantities provided by your particle physics colleague. Compute the expected number of events $N = R \cdot T \cdot M$ for the two detectors and both the heavy and light dark matter candidate. (Make sure you are using consistent units!)

d) Explain the limiting features of the two detectors that were considered.

Problem 2: *The Tremaine-Gunn bound* [**8 Points**]

a) Presume neutrinos to be massive enough for them to be non-relativistic today. A gas of such neutrinos would not be homogeneous, but clustered around galaxies instead. Assume, that such a neutrino gas almost makes up the entire mass of a galaxy (i.e. other components are negligible). Measuring how fast a star rotates at a certain radius r relative to the galactic center allows to calculate the total mass contained within this radius. The number of neutrinos which makes up this mass is limited by the amount of available states in phase space, as they obey the Pauli exclusion principle. Use this fact, while assuming, for simplicity, that there exists only one kind of neutrino, to derive a lower limit on the neutrino mass m_ν (Tremaine-Gunn limit). You can postulate that all possible states are populated, with the number of states per unit phase space being given by:

$$n = \frac{g}{(2\pi\hbar)^3} \tag{4}$$

where g (= 2 in this case) denotes the number of relativistic degrees of freedom. Note that neutrinos can only be gravitationally bound if they are below escape velocity. Assume, in addition, spherical symmetry, that the escape velocity at r smaller than some radius R is the same as for $r = R$, and that a measurement of the galactic rotation curve at $r = 12\,\text{kpc}$ yields $v(r) = 220\,\text{km/s}$.

b) Compare your result for the neutrino mass limit to the currently known limits on neutrino masses. What is your conclusion on SM neutrinos as dark matter particles?