Exercises to "Standard Model of Particle Physics II"

Winter 2023/24

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions:
January 17, 2024 - 09:15, Phil. 12, kHS

Discussion of solutions:
January 17, 2024 - 11:15, Phil. 12, kHS

Problem 1: *Majorana neutrinos* **[10 Points]**

The Lagrangian for the coupling of a fermion pair f with the Z-boson is given by

$$\mathcal{L} = \frac{g}{2\cos\theta_W}\overline{f}\gamma^\mu(v_f - a_f\gamma_5)fZ_\mu \tag{1}$$

For neutrinos we have $a_\nu = v_\nu = 1/2$.

a) Calculate the decay width for $Z \to \overline{\nu}\nu$ in the Standard Model but keeping a possible neutrino mass in the expression.

Hint: The decay width for Dirac neutrinos is given by

$$\Gamma_{\mathrm{Dirac}} = \frac{|\vec{p}|}{32\pi^2 m_z^2}\int \mathrm{d}\Omega|\overline{\mathcal{A}}|^2\,, \tag{2}$$

where $\overline{\mathcal{A}}$ is the spin-averaged transition amplitude $\mathcal{A}$ for $Z \to \overline{\nu}\nu$, which may be calculated using the appropriate Feynman rules (i.e. the Lagrangian).

b) Neutrinos could also be Majorana particles, which obey the relation $\nu^C = \nu$. The superscript C denotes charge conjugation

$$\nu^C = C\overline{\nu}^T, \tag{3}$$

with $C = i\gamma_2\gamma_0$ in the Dirac basis. Show the following properties:

$$-C = C^T = C^{-1} = -C^* = C^\dagger \tag{4}$$

$$C^{-1}\gamma_\mu C = -\gamma_\mu^T \tag{5}$$

$$C^{-1}\gamma_5 C = \gamma_5^T \tag{6}$$

$$\overline{\Psi^C} = -\Psi^T C^{-1} \tag{7}$$

$$(\Psi_L)^C = \left(\Psi^C\right)_R\,, \tag{8}$$

where $\left(\Psi^C\right)_L = P_L\left(\Psi^C\right)$.

c) Show that for Majorana neutrinos the vector current $\overline{\nu}\gamma_\mu\nu$ vanishes. What happens with $\overline{\nu}\gamma_5\nu, \overline{\nu}\gamma_\mu\gamma_5\nu$ and $\overline{\nu}\left[\gamma_\mu, \gamma_\nu\right]\nu$?

d) Using the previous result calculate the decay width $Z \to \overline{\nu}\nu$ for Majorana neutrinos and compare with part a).

Hint 1: Due to the Majorana properties, the final state particles are indistinguishable from each other which results in a factor of 1/2.

Problem 2: *Galactic rotation curves* **[10 Points]**

NGC2998 is a spiral galaxy in Ursa Major. You can download its rotation curve data from `http://astroweb.case.edu/ssm/620f03/n2998.dat` (by Stacy McGaugh). Here, the first column gives the radius and the second column gives the observed/inferred circular velocity, with its 1σ uncertainty in the third column. The next few columns provide rotation curves that would arise from the stellar disk, gaseous disk, and the bulge alone.

a) Interpolate the data for the disk, gas, and bulge distribution of the circular velocity by plotting it against the distance from the center of the galaxy. Give a possible explanation for the discrepancy between the observed and the expected rotation curves.

b) Using Newtonian mechanics, derive the expression for the circular velocity of a star orbiting the central mass of a galaxy as a function of its distance from the center of the galaxy. Assume a circular orbit and that the central mass is a function of the distance with the density profile

$$\rho(r) = \frac{\rho_0}{\left(1 + \frac{r}{r_0}\right)^{\alpha}} \, . \tag{9}$$

c) Determine a set of parameters (ρ_0, r_0, and α) that fits the data for large r. What does this choice imply for the properties of the DM?

d) Modified Newtonian Dynamics (MOND) has been suggested to explain the rotation curves of disk galaxies without the need for DM. This theory postulates the following modified version of Newton's first law.

$$\mu(a/a_0)a(r) = \frac{M_\odot G_N}{r^2} \, , \qquad \text{where} \qquad \mu(a/a_0) = \begin{cases} \frac{a}{a_0} \, , & a \ll a_0 \\ 1 \, , & a \gg a_0 \end{cases} \tag{10}$$

Here, $a(r)$ is the gravitational acceleration, $\mu(x)$ is an interpolation function, and a_0 is a constant introduced by the MOND prescription that must be determined from experiment. Derive the circular velocity distribution of the galactic disk in MOND and find a good fit to the observed velocities.

Hint: You may assume we are far from the central bulge where the accelerations are small and treat the galaxy as a point mass with $M_\odot = 4 \times 10^{41}$ kg.