Exercises to "Standard Model of Particle Physics II"

Winter 2022/23

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Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in of solutions:
January 18, 2023 - 11:15, Phil. 12, R105

Discussion of solutions:
January 18, 2023 - 11:15, Phil. 12, R105

Problem 1: *Number of lepton flavors* **[10 Points]**

The total decay width of the Z boson is given by:

$$\Gamma_Z = \Gamma_e + \Gamma_\mu + \Gamma_\tau + \Gamma_{\text{had}} + \Gamma_{\text{inv}} \tag{1}$$

where Γ_{had} is the sum of all possible hadronic decays $\Gamma_{e,\mu,\tau}$ are the leptonic partial widths, and Γ_{inv} is the partial decay width of the Z boson to invisibles (i.e. into final states not detectable within colliders).

a) What decay channels in the Standard Model can contribute to the invisible decay width (at tree level)?

b) Assuming only neutrinos contribute to the invisible Z branching fraction, one can calculate the number of light neutrino generations using

$$N_\nu = \left(\frac{\Gamma_{\text{inv}}}{\Gamma_l}\right)_{\text{exp}} \left(\frac{\Gamma_{\text{l}}}{\Gamma_\nu}\right)_{\text{theory}} . \tag{2}$$

Calculate the theory prediction of $\left(\frac{\Gamma_{\text{l}}}{\Gamma_\nu}\right)_{\text{theory}}$ using the expression for the partial rate of the Z boson to fermions:

$$\Gamma_f = N_C^f \frac{\alpha m_Z}{12 \sin^2\theta_W \cos^2\theta_W} \left[(g_V^f)^2 + (g_A^f)^2\right] \tag{3}$$

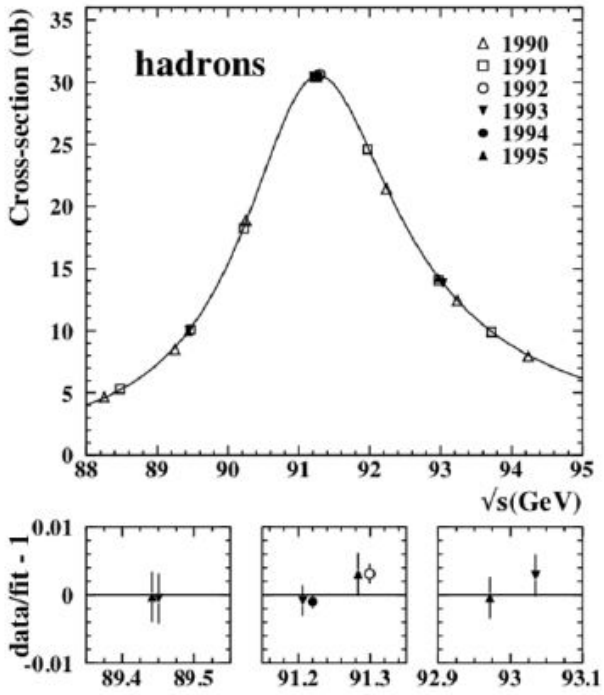
Keep in mind that you are interested in the ratio for **one** neutrino type.

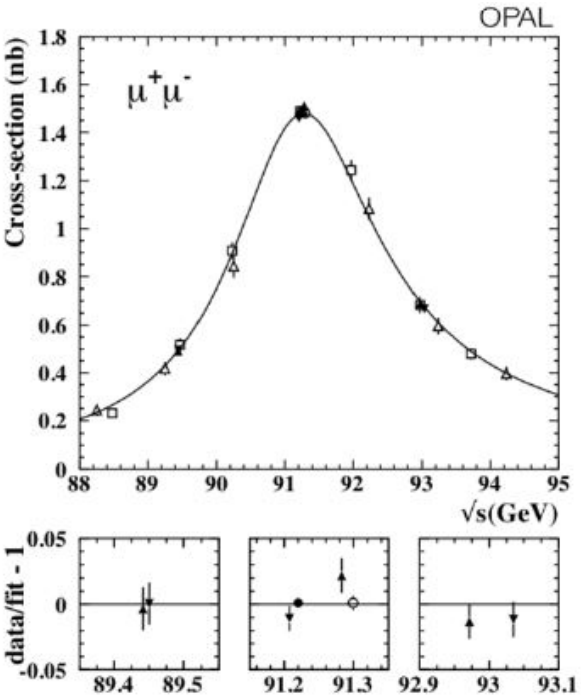
c) The partial cross section at the peak of the distribution is given by

$$\sigma_{ff}^{\text{peak}} \simeq \frac{12\pi}{m_Z^2} \frac{\Gamma_e \Gamma_f}{\Gamma_Z^2} . \tag{4}$$

Using the plots provided on the back of this page, read off Γ_Z and calculate the partial width to hadrons and leptons.

d) Calculate the number of light neutrinos N_ν. What does *light* mean in this context? Are there any other ways to introduce a fourth neutrino into the Standard Model?


hadrons
1990
1991
1992
1993
1994
1995
Cross-section (nb)
√s(GeV)
data/fit - 1


OPAL
μ+μ-
Cross-section (nb)
√s(GeV)
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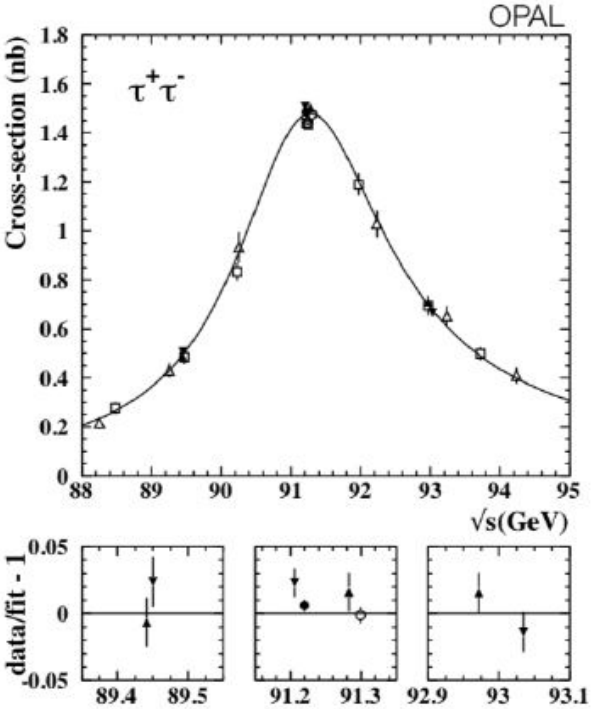
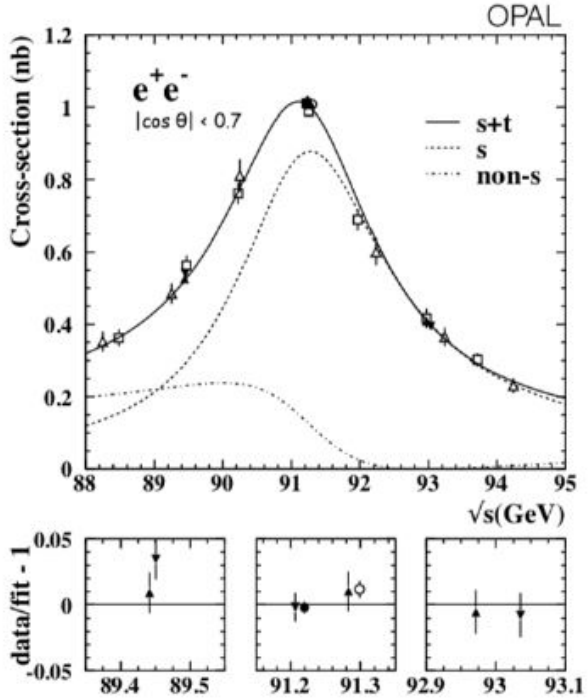

OPAL
τ+τ-
Cross-section (nb)
√s(GeV)
data/fit - 1


OPAL
e+e-
|cos θ| < 0.7
s+t
s
non-s
Cross-section (nb)
√s(GeV)
data/fit - 1

Problem 2: *Z' physics* **[10 Points]**

If one adds an additional $\mathrm{U}(1)'$ symmetry to the SM gauge group, the general effective Lagrange density after breaking the $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \times \mathrm{U}(1)'$ symmetry to $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{U}(1)_\mathrm{em}$ can be written as

$$\mathscr{L} = \mathscr{L}_\mathrm{SM} + \mathscr{L}_{Z'} + \mathscr{L}_\mathrm{mix}\,.$$

The relevant part of the Standard Model Lagrangian is given by

$$\mathscr{L}_\mathrm{SM} = -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{W}^a_{\mu\nu}\hat{W}^{\mu\nu,a} + \frac{1}{2}\hat{M}_Z^2\hat{Z}_\mu\hat{Z}^\mu - \frac{e}{c_W}j^\mu_B\hat{B}_\mu - \frac{e}{s_W}j^{\mu,a}_W\hat{W}^a_\mu$$

where the hats denote that the fields are not mass eigenstates. The Z' part reads

$$\mathscr{L}_{Z'} = -\frac{1}{4}\hat{Z}'_{\mu\nu}\hat{Z}'^{\mu\nu} + \frac{1}{2}\hat{M}^2_{Z'}\hat{Z}'_\mu\hat{Z}'^\mu - g' j'_\mu \hat{Z}'_\mu\,,$$

where g' denotes the $\mathrm{U}(1)'$ gauge coupling. Finally, the kinetic- and mass-mixing terms can be parameterized as

$$\mathscr{L}_\mathrm{mix} = -\frac{\sin\chi}{2}\hat{Z}'_{\mu\nu}\hat{B}^{\mu\nu} + \delta\hat{M}^2\hat{Z}'_\mu\hat{Z}^\mu\,.$$

a) Determine the mass eigenstates Z_1^μ and Z_2^μ and determine the couplings of $Z_{1,2}$ to the currents j_B, j_W and j'. You may set the kinetic mixing angle χ to zero for simplicity.
Hint: Reexpress $\hat{B}_\mu$ and $\hat{W}^3_\mu$ in terms of A_μ and Z_μ.

b) Since the mass of the physical Z boson changes compared to the SM, the ρ parameter is no longer equal to one (at tree-level). Use the current value $\rho = 1.0008^{+0.0017}_{-0.0007}$ to constrain the Z-Z' mixing. You may assume that $\hat{M}_{Z'} \gg \hat{M}_Z \gg \delta\hat{M}$.

c) A well-motivated extension of the SM is a gauged B-L symmetry (baryon minus lepton number). Write down the corresponding current for each of the SM fermions using the general formula,

$$j'_\mu = \sum_\psi \bar{\psi}\,\gamma_\mu \mathcal{Q}_{BL}\psi\,,$$

where $\mathcal{Q}_{BL}$ denotes the B-L charge operator.