Lecture:

Standard Model of Particle Physics

Heidelberg SS 2012

Weak Interactions I

Spinors and Helicity States

$$u_R = |\vec{p},\ \lambda = +1/2\rangle \quad (u_1)$$
$$u_L = |\vec{p},\ \lambda = -1/2\rangle \quad (u_2)$$
$$v_L = |\vec{p},\ \lambda = -1/2\rangle \quad (v_1)$$
$$v_R = |\vec{p},\ \lambda = +1/2\rangle \quad (v_2)$$

or more general: (u_1), (u_2), (v_1), (v_2)

fermions:

$$u_R = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \frac{|\vec{p}|}{E+m} \\ 0 \end{pmatrix} \qquad u_L = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-|\vec{p}|}{E+m} \end{pmatrix}$$

anti-fermions:

$$v_L = \sqrt{E+m}\begin{pmatrix} \frac{|\vec{p}|}{E+m} \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad v_R = \sqrt{E+m}\begin{pmatrix} 0 \\ \frac{-|\vec{p}|}{E+m} \\ 0 \\ 1 \end{pmatrix}$$

limit $p \to \infty$

$u_R \to v_L$

$u_L \to v_R$

Chirality Operator

limit: $m \to 0, p \to \infty$

$$u_R \sim v_L \sim \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad u_L \sim v_R \sim \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

operator: $$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \underline{1} \\ \underline{1} & 0 \end{pmatrix}$$

right chirality

$$\gamma_5 u_R = u_R$$
$$\gamma_5 v_L = v_L$$

left chirality

$$\gamma_5 u_L = -u_L$$
$$\gamma_5 v_R = -v_R$$

eigenvalues ± 1

left-handed (chiral) particles: -1

right-handed (chiral) particles: +1

note: a right-handed chiral anti-particle has a left-handed helicity

Projection Operator

<u>Definition:</u> $$\Pi^{\pm} = \frac{1 \pm \gamma_5}{2}$$

in the limit of |E| → infinity

fermions

$$\Pi^{+} u_R = u_R \qquad \Pi^{+} u_L = 0$$
$$\Pi^{-} u_L = u_L \qquad \Pi^{-} u_R = 0$$

anti-fermions

$$\Pi^{+} v_L = v_L \qquad \Pi^{+} v_R = 0$$
$$\Pi^{-} v_R = v_R \qquad \Pi^{-} v_L = 0$$

can reformulate Dirac Equation:

$$i\gamma^{\mu}\partial_{\mu} u_R = m\, u_L \qquad i\gamma^{\mu}\partial_{\mu} u_L = m\, u_R$$

note: massive fermions must have left-handed and right handed components

Recap

General Four Fermion Lagrangian

$$L = \frac{G}{\sqrt{2}} \left(\bar{f}\ \Gamma f\,' \right) \left(\overline{f\,''}\ \tilde{\Gamma} f\,''' \right)$$

Vector Current:

$$j_V^\mu = \bar{\psi}\gamma^\mu\psi$$

scalar coupling:

$$\lambda = \bar{\psi}\psi$$

Axial-vector Current:

$$j_A^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$$

pseudoscalar coupling:

$$\lambda = \bar{\psi}\gamma^5\psi$$

Tensor Coupling

$$\sigma_A^{\mu\nu} = \bar{\psi}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)\psi$$

Vector, Axial and Scalar Currents

Vector Current:

$$j_V^\mu = \bar{u}\gamma^\mu u$$

in QED: $\partial_\mu j_V^\mu = 0$

$$\bar{u}\gamma^\mu u = \bar{u}_L\gamma^\mu u_L + \bar{u}_R\gamma^\mu u_R$$

(no helicity flip)

Axial-vector Current:

$$j_A^\mu = \bar{u}\gamma^\mu\gamma^5 u$$

note: $\gamma^\mu\gamma^5 = -\gamma^5\gamma^\mu$

$$\bar{u}\gamma^\mu\gamma^5 u = \bar{u}_L\gamma^\mu\gamma^5 u_L + \bar{u}_R\gamma^\mu\gamma^5 u_R$$

(no helicity flip)

Scalar Coupling

$$\bar{u}u = \bar{u}_R u_L + \bar{u}_L u_R$$

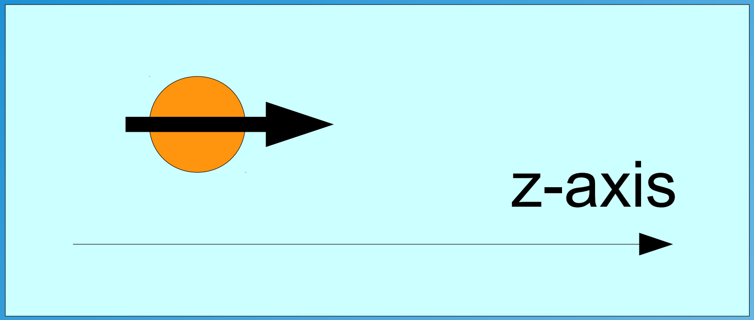
(helicity flip!)

Relations:

$$j_L^\mu = 1/2\,(j_V^\mu - j_A^\mu) \qquad j_R^\mu = 1/2\,(j_V^\mu + j_A^\mu)$$

Helicity Example

Particle at almost rest ($0 < p \rightarrow 0$) with spin orientation in +z direction:



polarisation: $\lambda = \vec{j}\,\vec{z} = \frac{N_{+1/2} - N_{-1/2}}{N_{+1/2} + N_{-1/2}} = 1$

helicity: $H = \frac{\vec{j}\,\vec{p}}{|\vec{p}|} = 1$ (classical)

Helicity Example

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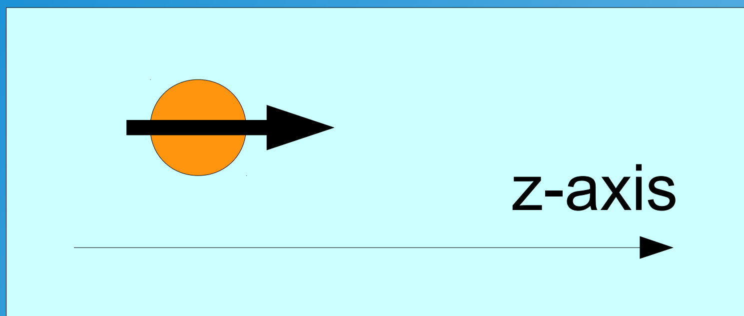

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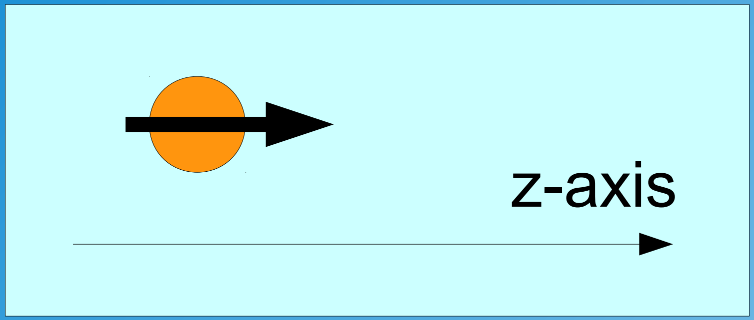
- In a classical approach states with defined helicities ($H = \pm 1$) can be prepared.
- In Quantum mechanics the spin and momentum are to be replaced by operators.
- However, in classical quantum mechanics for massive particles the helicity is not Lorentz invariant.
- This is easily seen in the above example:

For $p \rightarrow p' = -p$ the helicity makes a flip: $H \rightarrow H' = -1$

Helicity Example

Particle at almost rest ($0 < p \rightarrow 0$) with spin orientation in +z direction:



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In a Lorentz invariant theory, particle interactions can not be described by non-Lorentz invariant quantities!

- The solution to this problem in provided by the Dirac equations

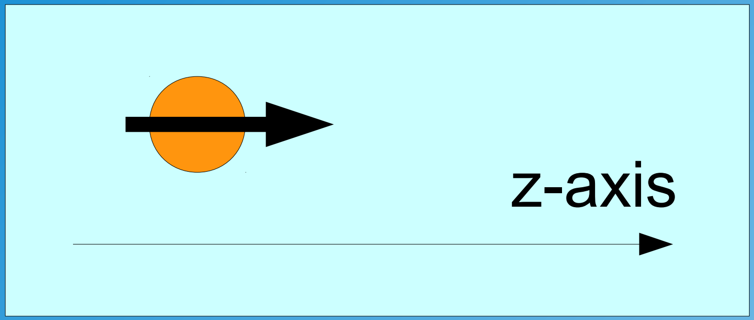
Dirac spinors: $u = u_L + u_R$

only the chiral states in the limit $p \rightarrow \infty$ are Lorentz invariant

$$u_R = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad u_L = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

Helicity Example

Particle at almost rest ($0 < p \rightarrow 0$) with spin orientation in +z direction:



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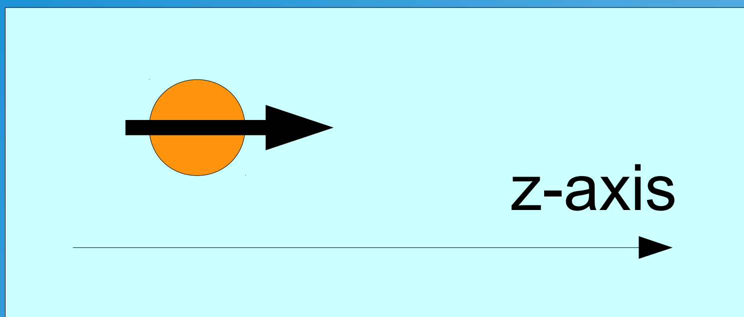
helicity: $H = \frac{\vec{j}\,\vec{p}}{|\vec{p}|} = 1$ (classical)

$$u = u_L + u_R = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-|\vec{p}|}{E+m} \end{pmatrix} + \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \frac{|\vec{p}|}{E+m} \\ 0 \end{pmatrix} = \sqrt{2\mathrm{m}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \sqrt{2\mathrm{m}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Helicity Example

Particle at almost rest ($0 < p \to 0$) with spin orientation in +z direction:



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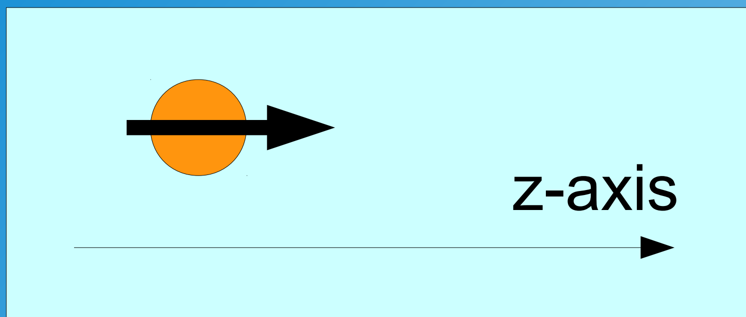
Projection of right helicity state

$$\Pi^+ u = \frac{1+\gamma_5}{2} u = \frac{\sqrt{2m}}{\boxed{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} + \frac{\sqrt{2m}}{\boxed{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad \Pi^- u = \frac{1-\gamma_5}{2} u = \frac{\sqrt{2m}}{\boxed{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} + \frac{\sqrt{2m}}{\boxed{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

Helicity Example

Particle at almost rest ($0 < p \to 0$) with spin orientation in +z direction:



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Projection of right helicity state

$$\Pi^+ u = \frac{1+\gamma_5}{2} u = \frac{\sqrt{2m}}{2}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} + \boxed{\frac{\sqrt{2m}}{2}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}}$$

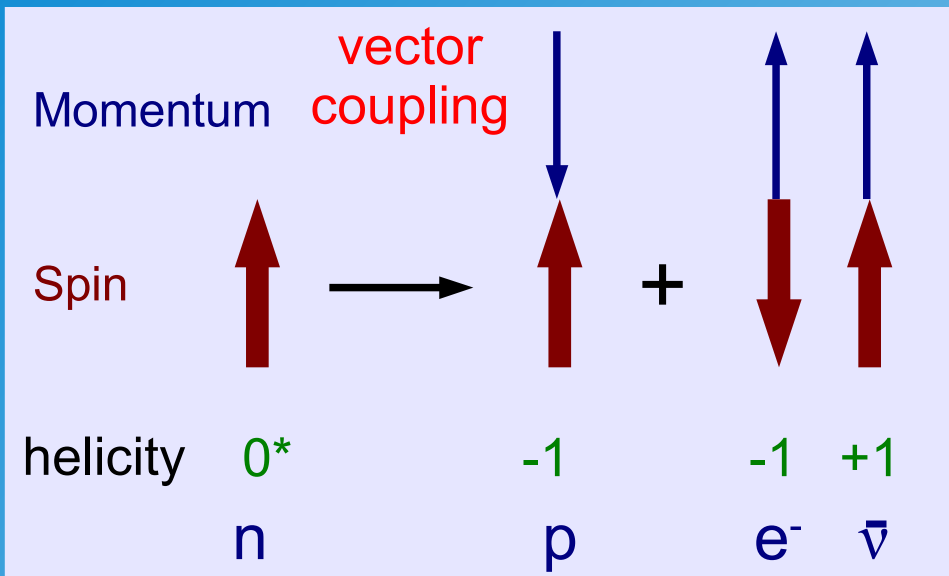
$$\Pi^- u = \frac{1-\gamma_5}{2} u = \boxed{\frac{\sqrt{2m}}{2}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}} + \frac{\sqrt{2m}}{2}\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

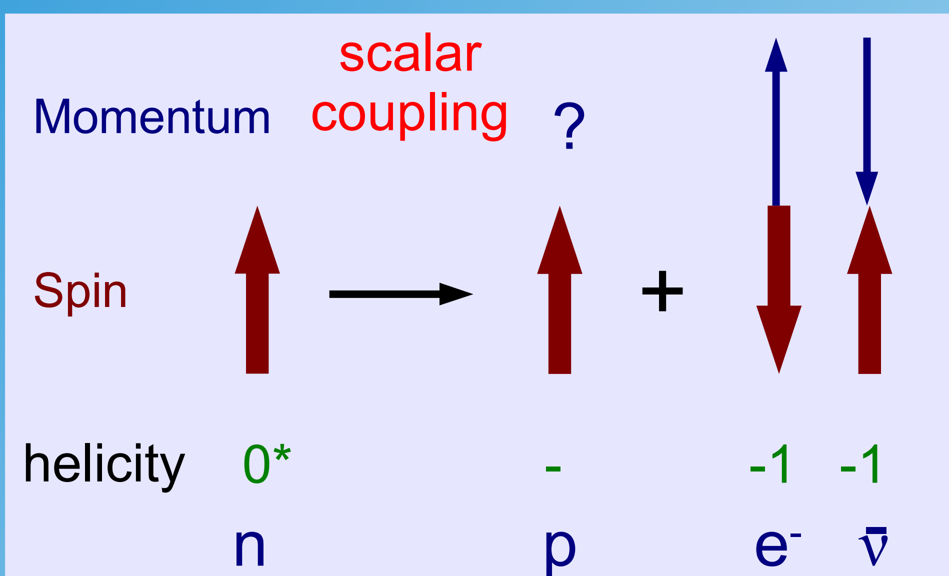
50% **right** chiral state

50% **left** chiral state

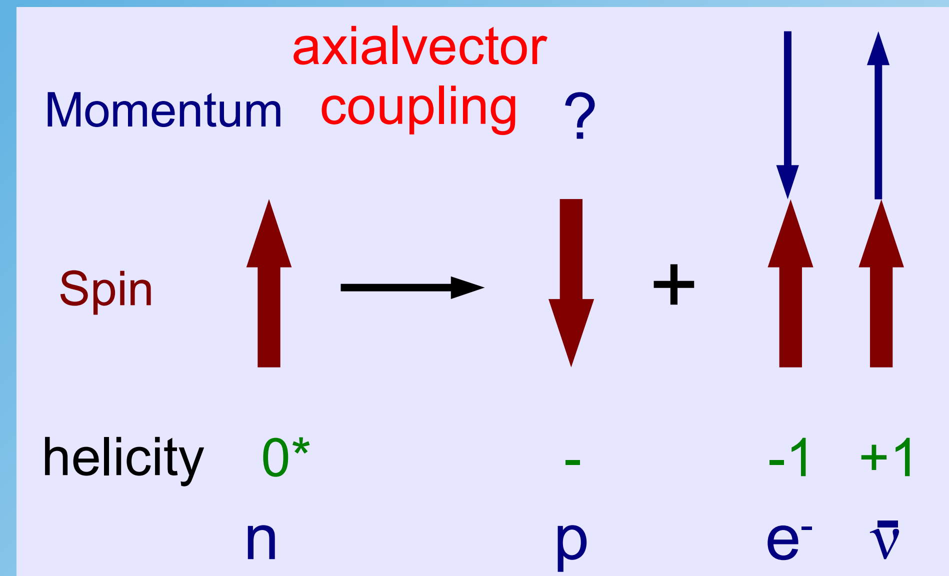
Test of Lorentz Structure?

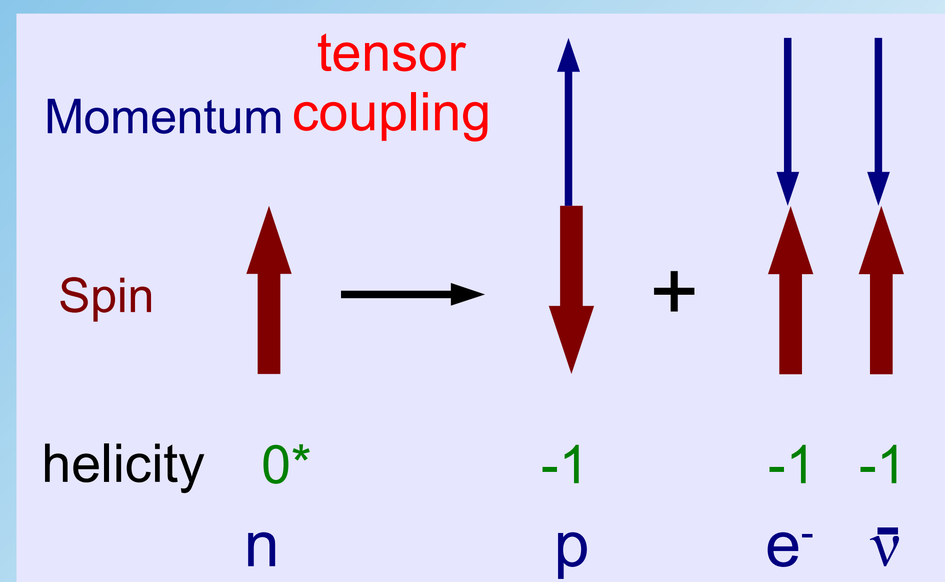
Fermi transition





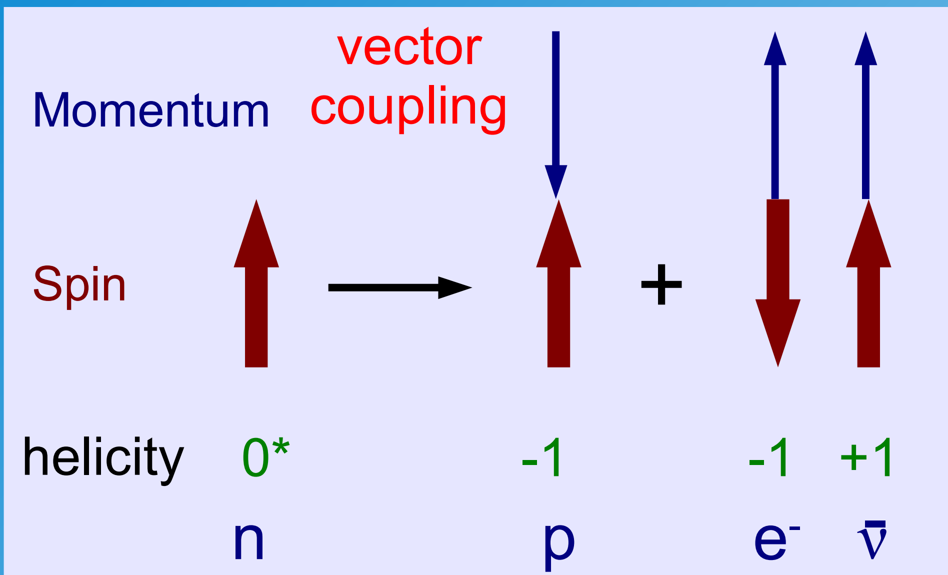
Gamov Teller transition





Test of Lorentz Structure?

Fermi transition



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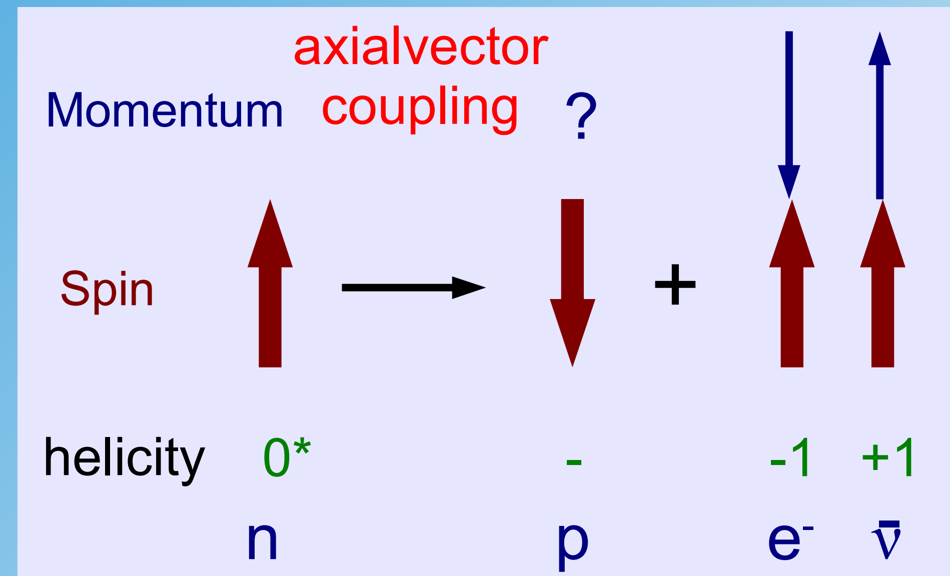
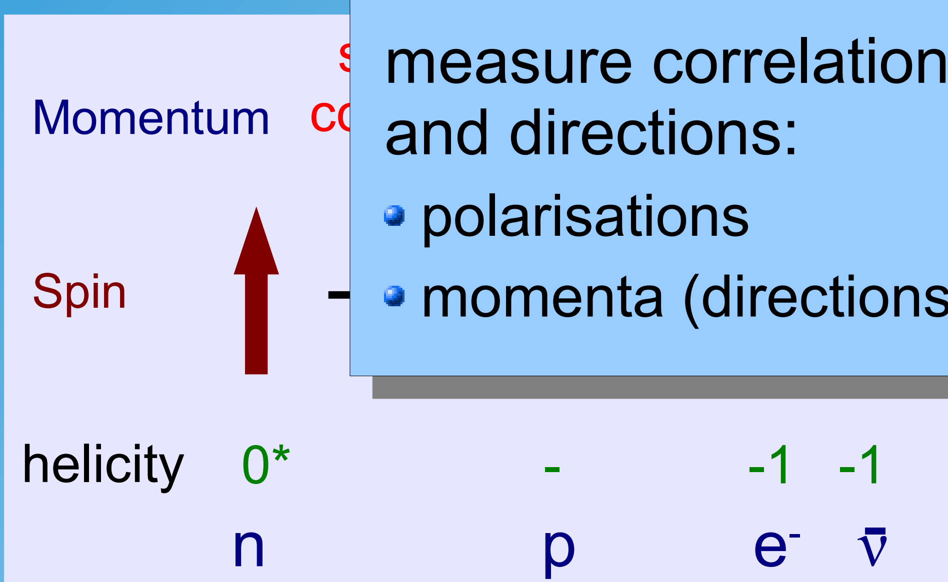
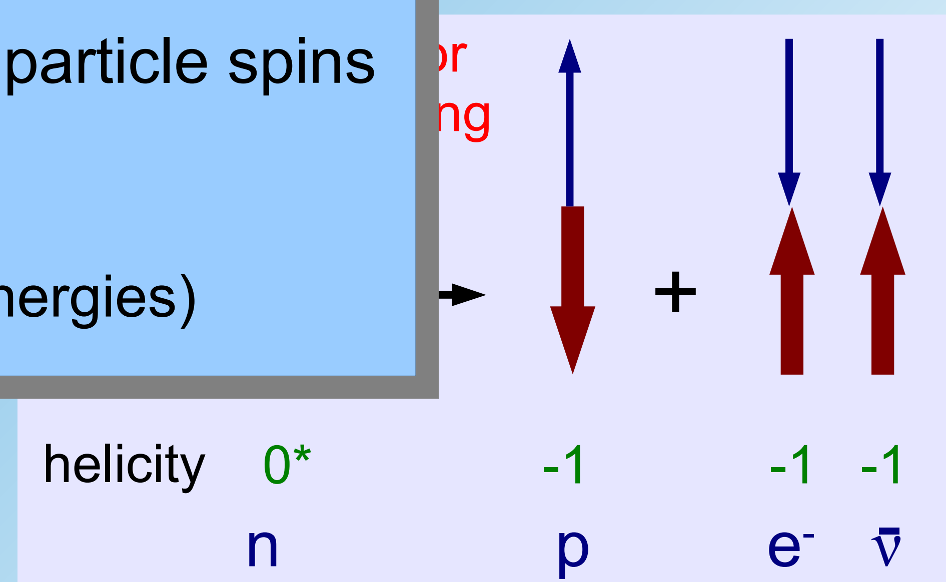






measure correlations of particle spins and directions:

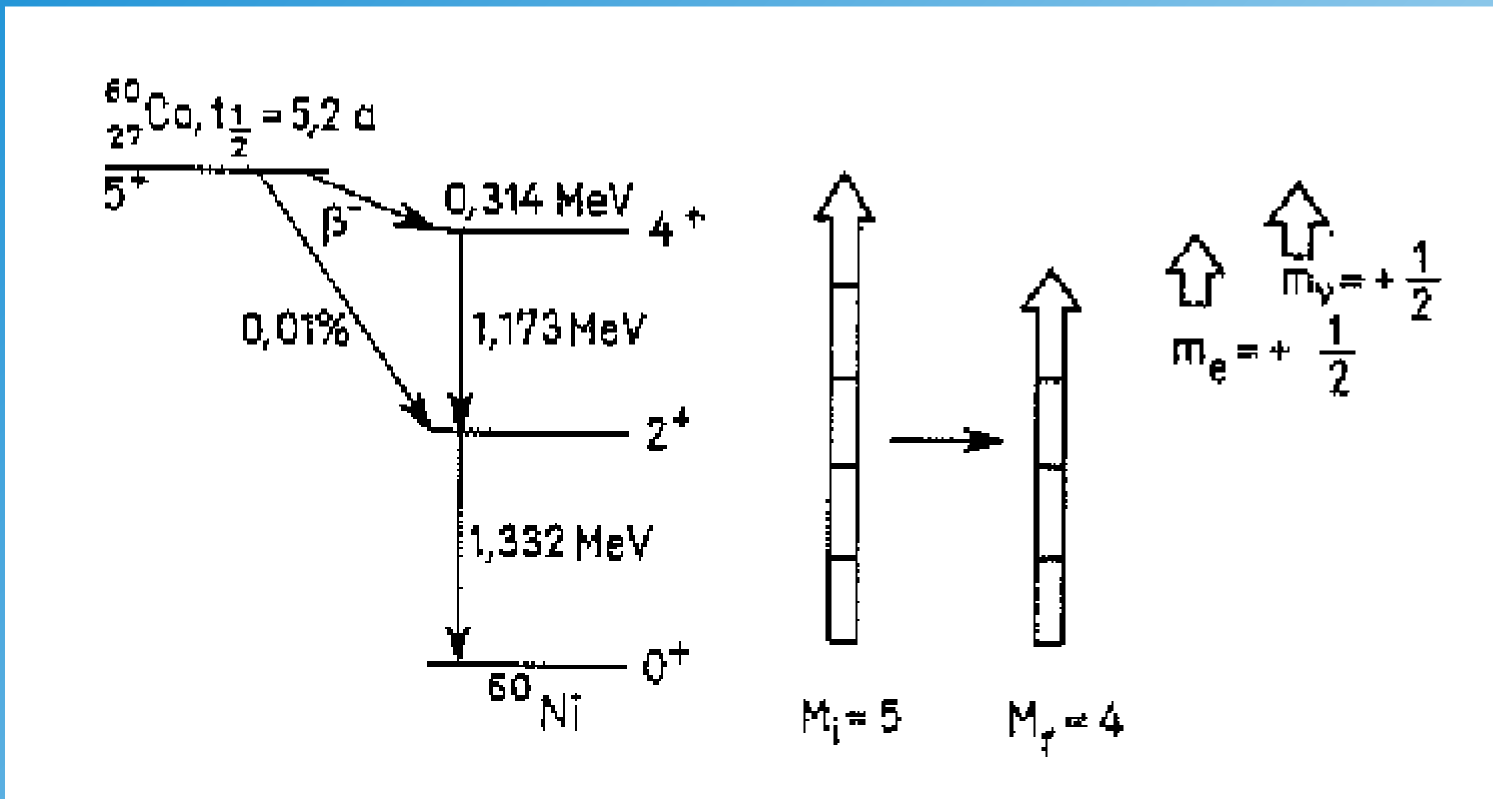
- polarisations
- momenta (directions + energies)

Important Experiments

- Wu-Experiment (1957): radioactive decay of Co^{60}
- Goldhaber-Experiment (1958): radioactive decay of Eu^{152}
- Muon Decay: Michel spectrum
- Pion Decay: branching ratios
- Neutrino Nucleon Scattering: neutrino-antineutrino
- Nuclear Beta Decays

Idea of the Wu-Experiment

$^{60}Co \rightarrow {}^{60}Ni + e^- + \bar{\nu}$

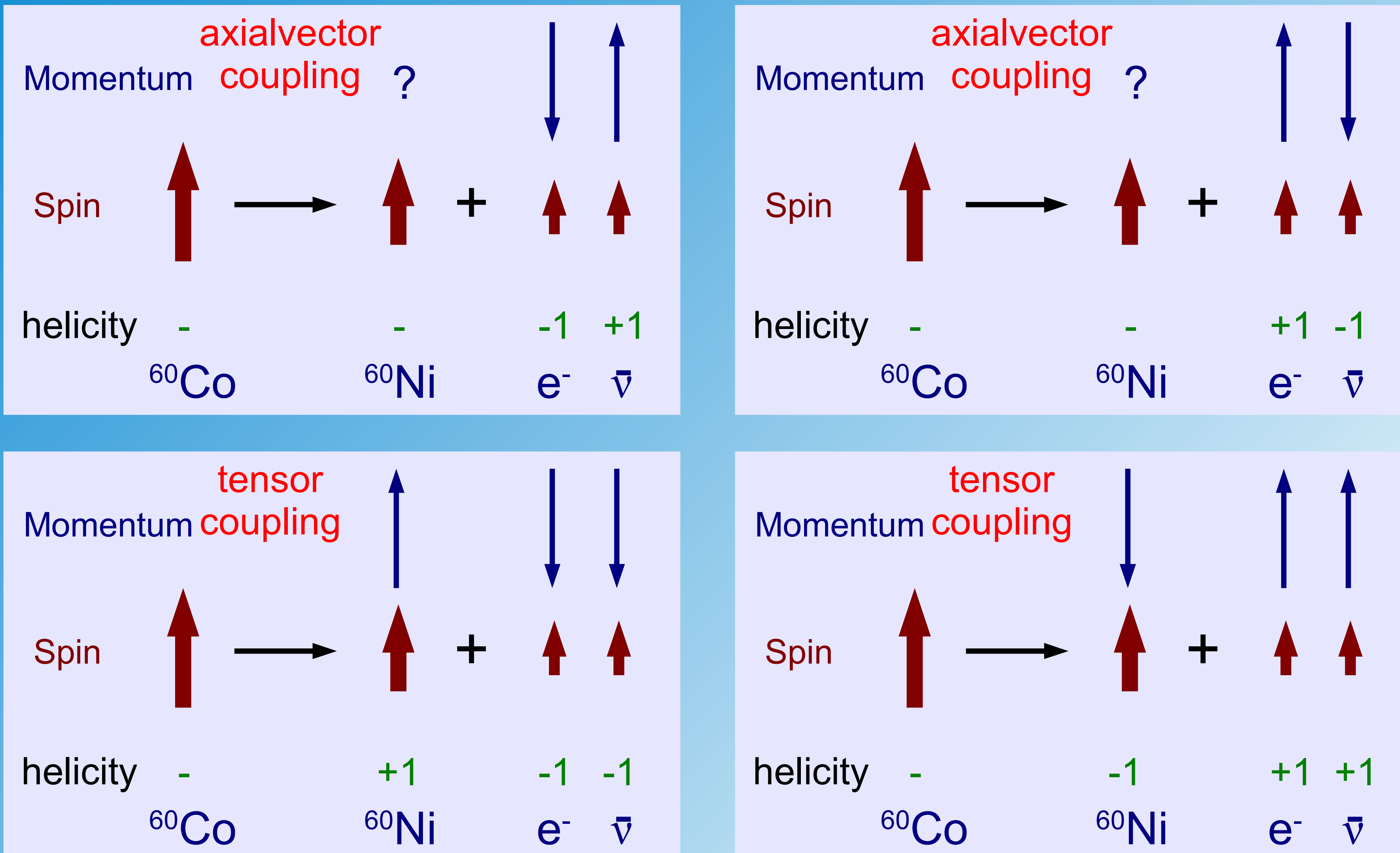


- High Spin of Cobalt leads to Gamov-Teller transition
- Polarisation of electron (neutrino)?

$$\lambda\left(e^-\right) = ?$$

Test of Lorentz Structure in ^{60}Co

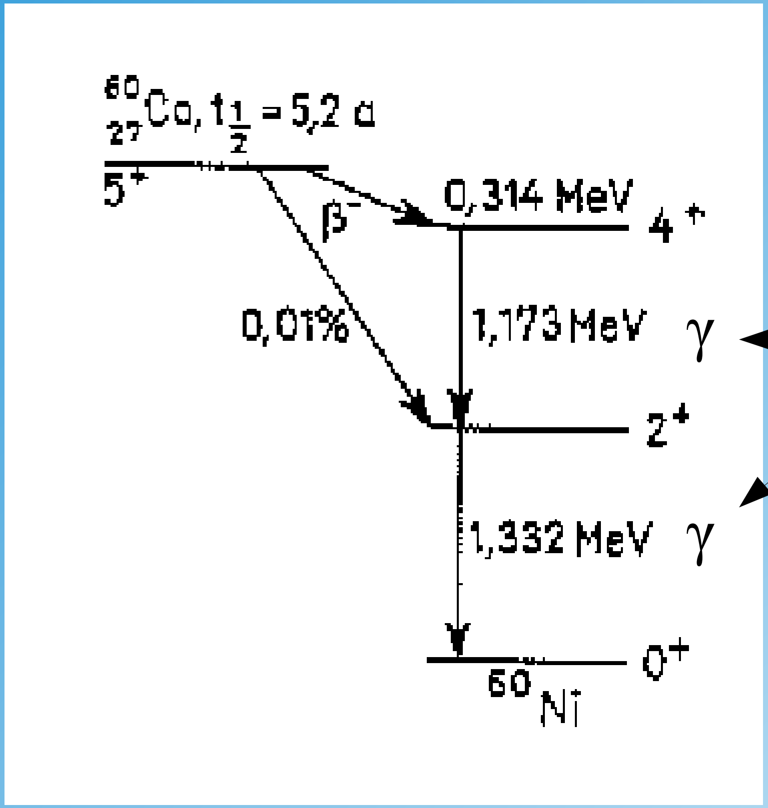
Gamov Teller transitions



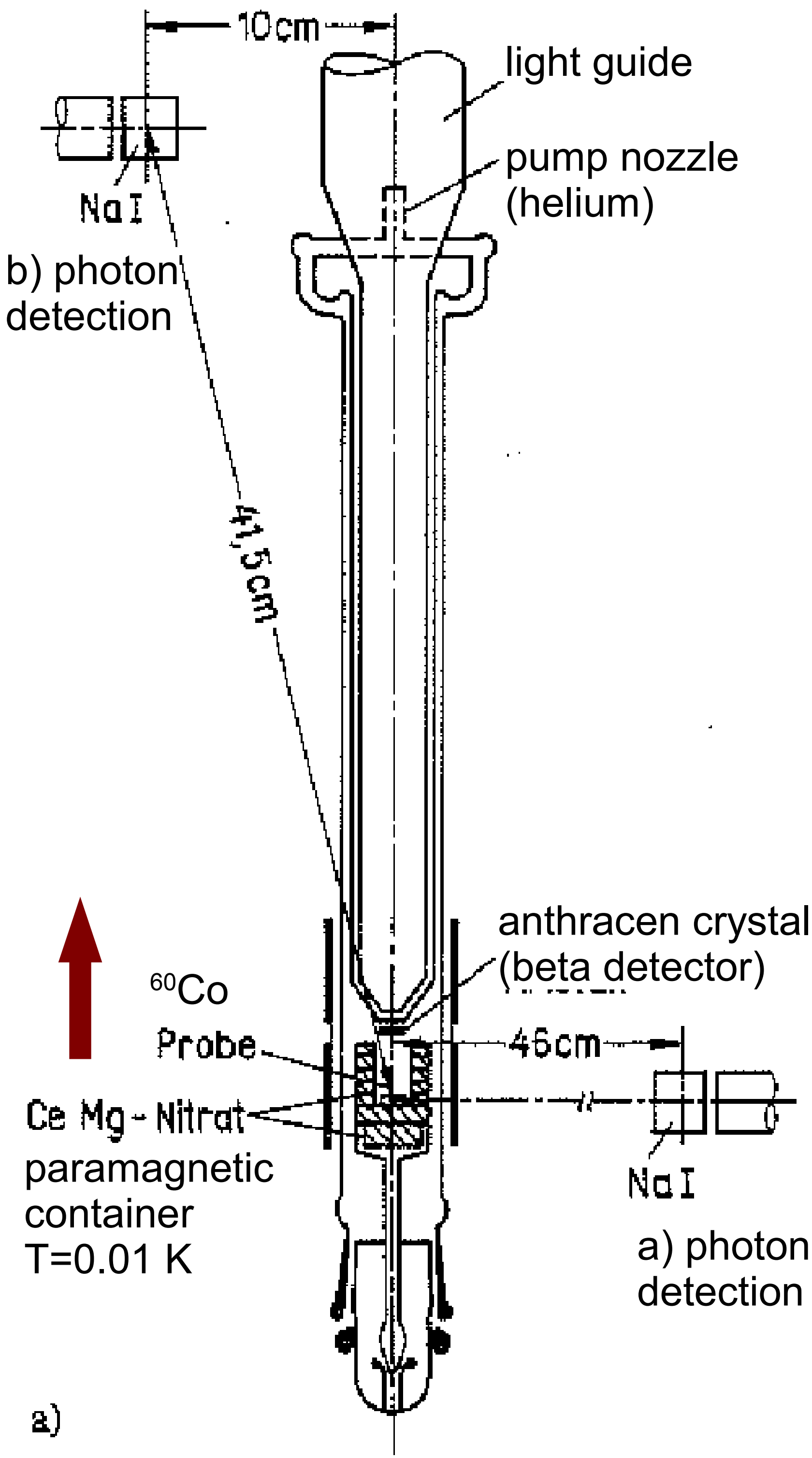
Measurement of the Ni*-Polarisation

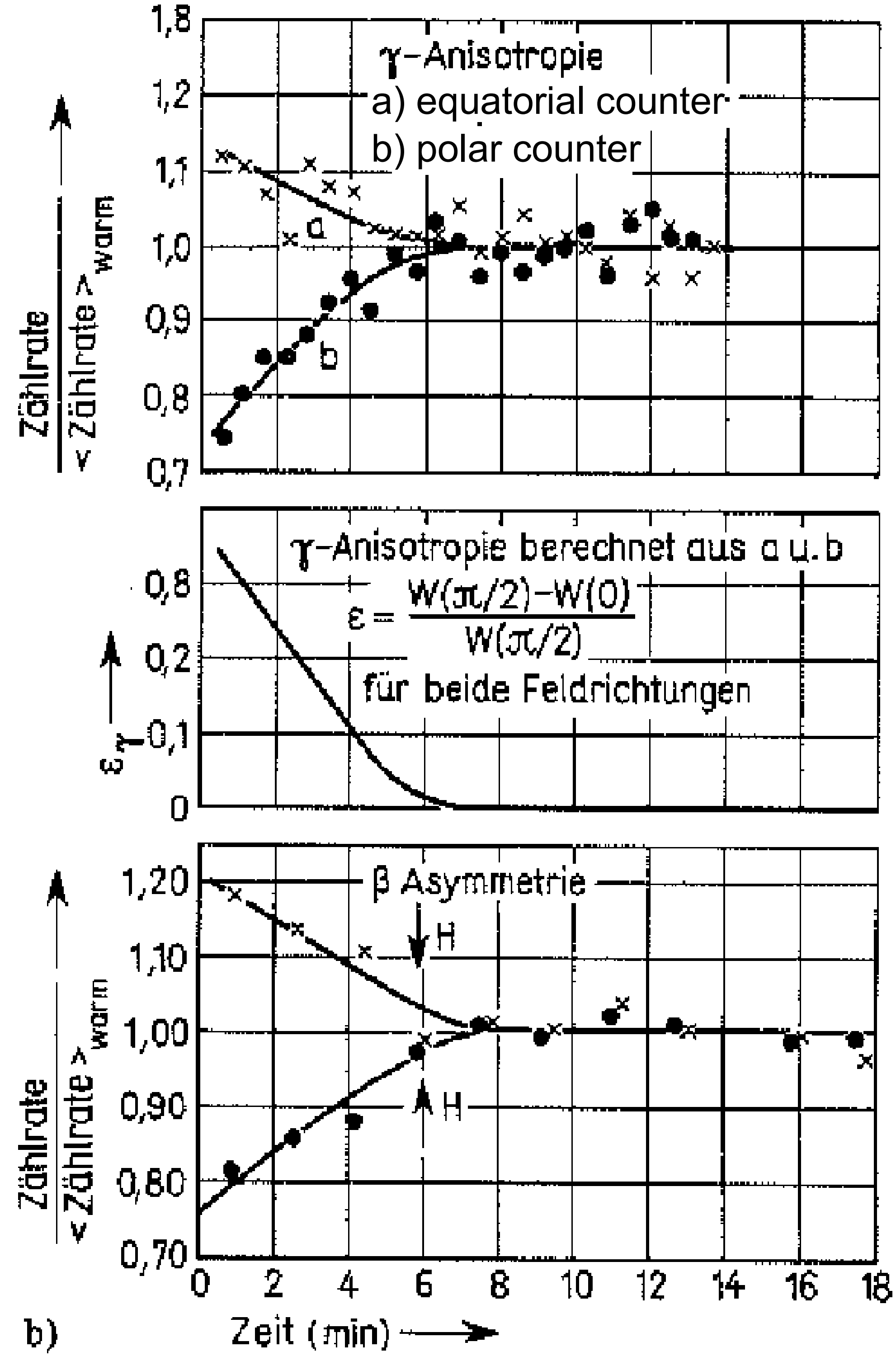
- ^{60}Ni*(J=4) is produced in an excited state!
- Beta Decay followed by photo-nuclear decay:

$$^{60}\text{Ni}^* \rightarrow {}^{60}\text{Ni}^* \, \gamma$$



Photons are polarised and oriented (symmetrically) in direction of the Ni (Co) polarisation axis. Maximum is orthogonal to Ni (Co) polarisation


10cm
light guide
pump nozzle
(helium)
NaI
b) photon
detection
41,5cm
anthracen crystal
(beta detector)
^{60}Co
Probe
46cm
Ce Mg - Nitrat
paramagnetic
container
T=0.01 K
NaI
a) photon
detection
a)


γ-Anisotropie
a) equatorial counter
b) polar counter
Zählrate / < Zählrate >warm
γ-Anisotropie berechnet aus a u. b
ε = W(π/2)−W(0) / W(π/2)
für beide Feldrichtungen
ε_γ
β Asymmetrie
H
Zeit (min)
b)

Conclusion Wu-Experiment

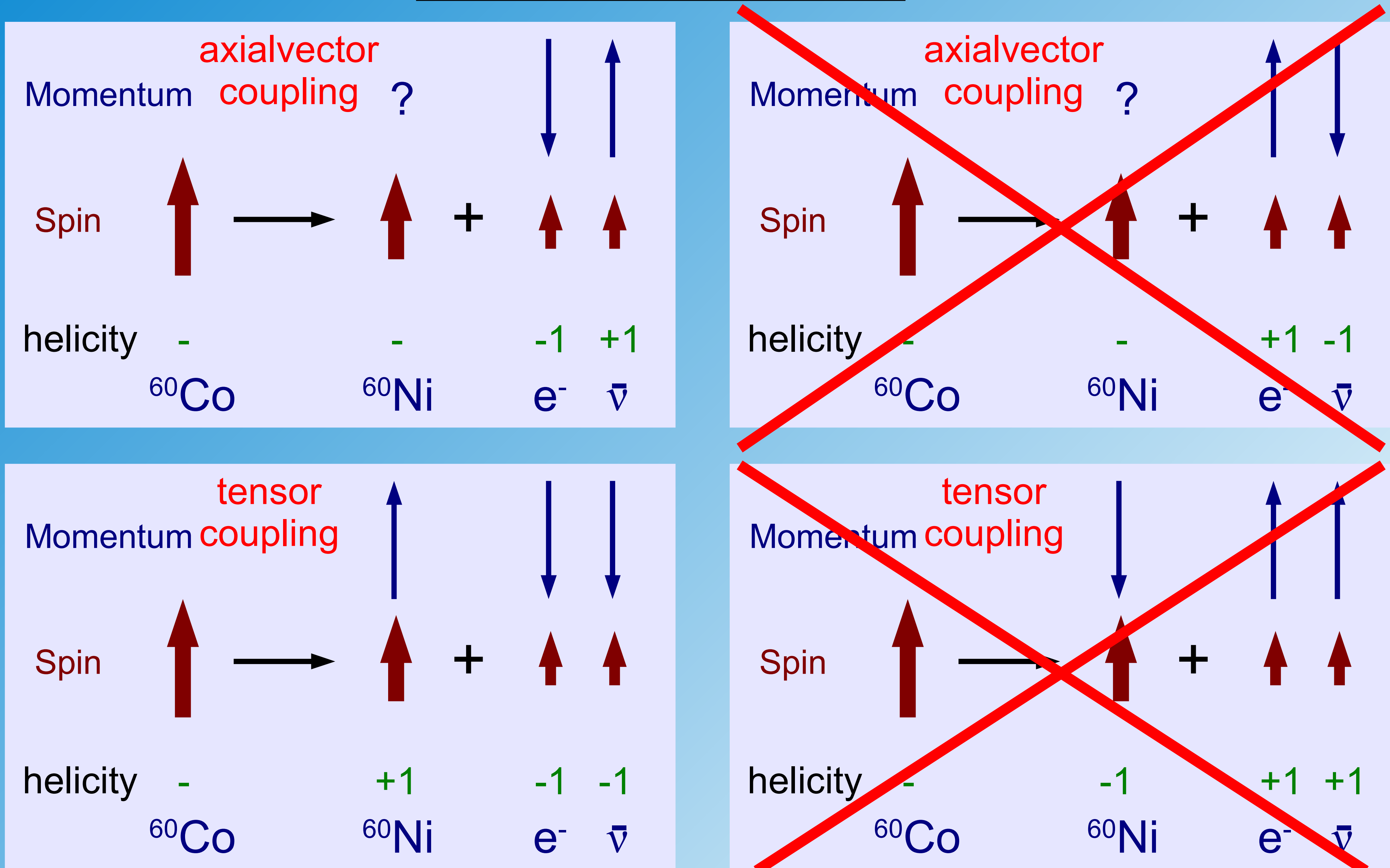
- Electrons are dominantly emitted opposite to Co spin direction
- Product **J · p / |p|** (helicity) is non-zero!
- Helicity is negative!
- Discrete-Parity symmetry is violated (initial state had H=0)

Note: the angular distribution of the electrons is given by:

$$\frac{dN}{d\cos\theta} \propto 1 + A\cos\theta$$

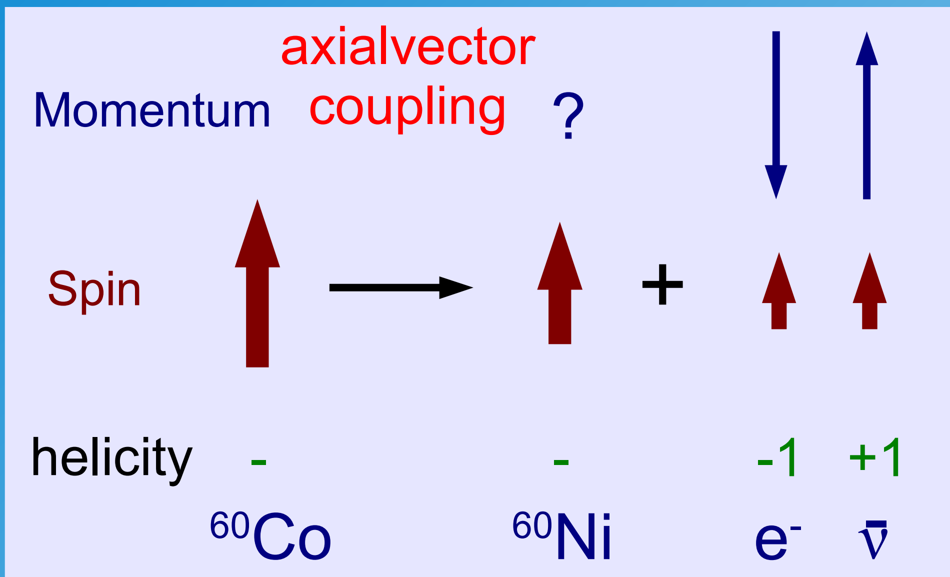
Test of Lorentz Structure in ^{60}Co

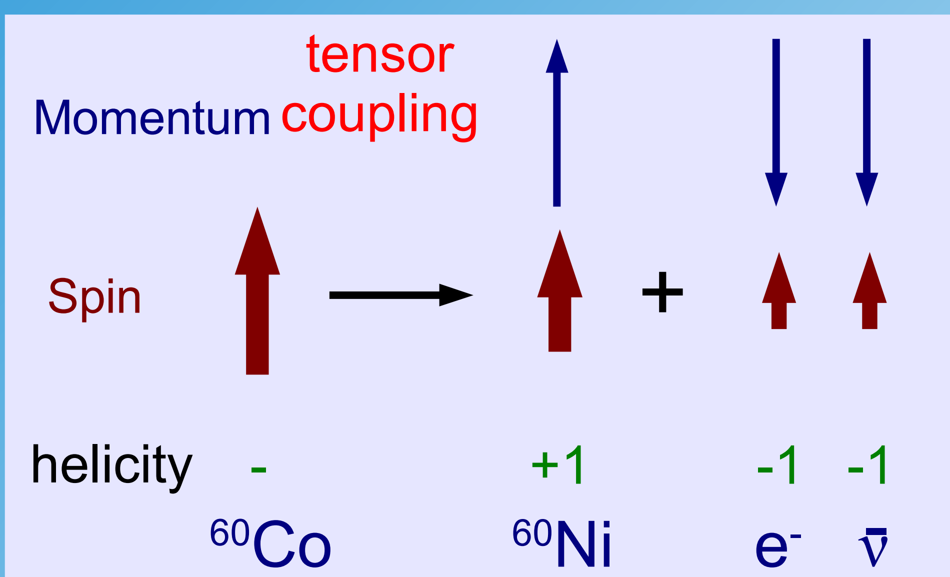
Gamov Teller transitions



Test of Lorentz Structure in ^{60}Co

Gamov Teller transitions

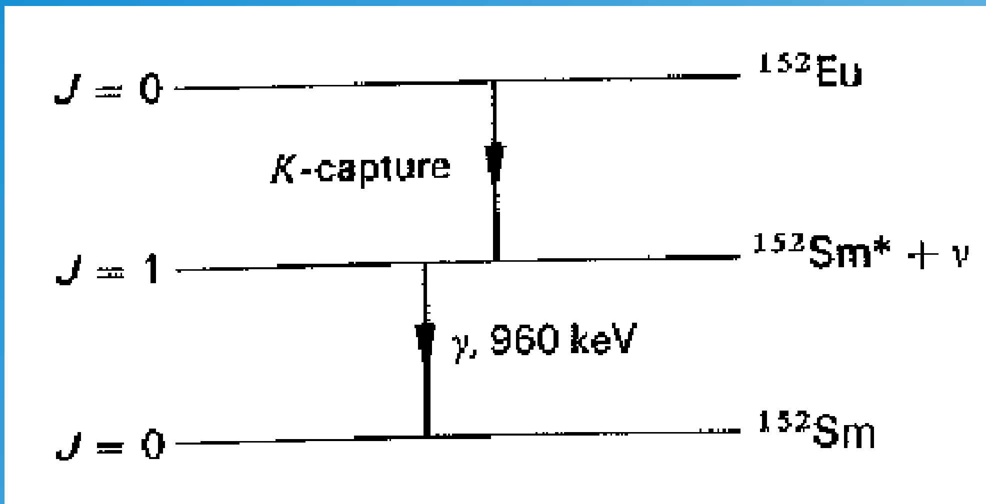


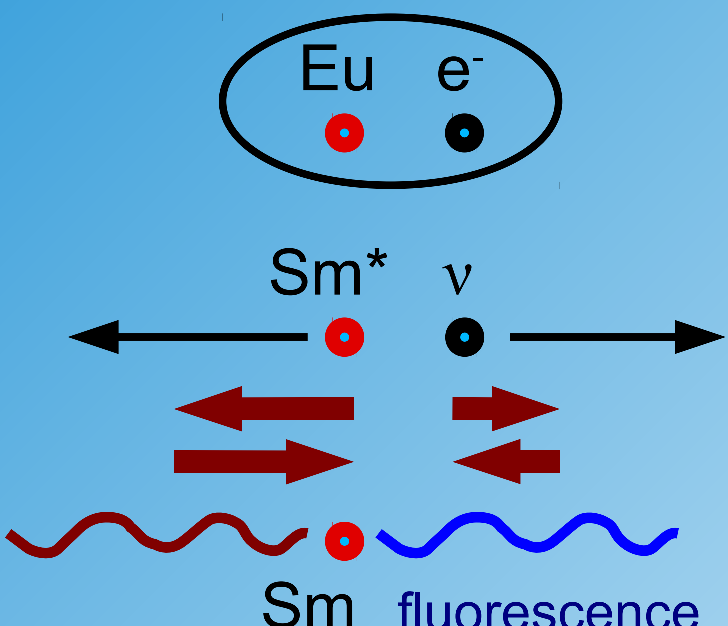


What about the neutrino helicity?

Europium Decay Chain

Lifetime ^{152}Eu = 13.5 a





$^{152}\text{Eu} + e^- \rightarrow {}^{152}\text{Sm}^* + \nu$

- Europium has no nuclear spin
- Polarisation of neutrino and Sm* are opposite!
- The neutrino polarisation can be measured by determining the Sm* polarisation
- Luckily Sm* decays further:
  $\text{Sm}^* \rightarrow {}^{152}\text{Sm} + \gamma$
- photon is of low energy if emitted opposite to Sm* flight direction
  - → fluorescence
- exploit Compton scattering to determine photon polarisation

Goldhaber Experiement

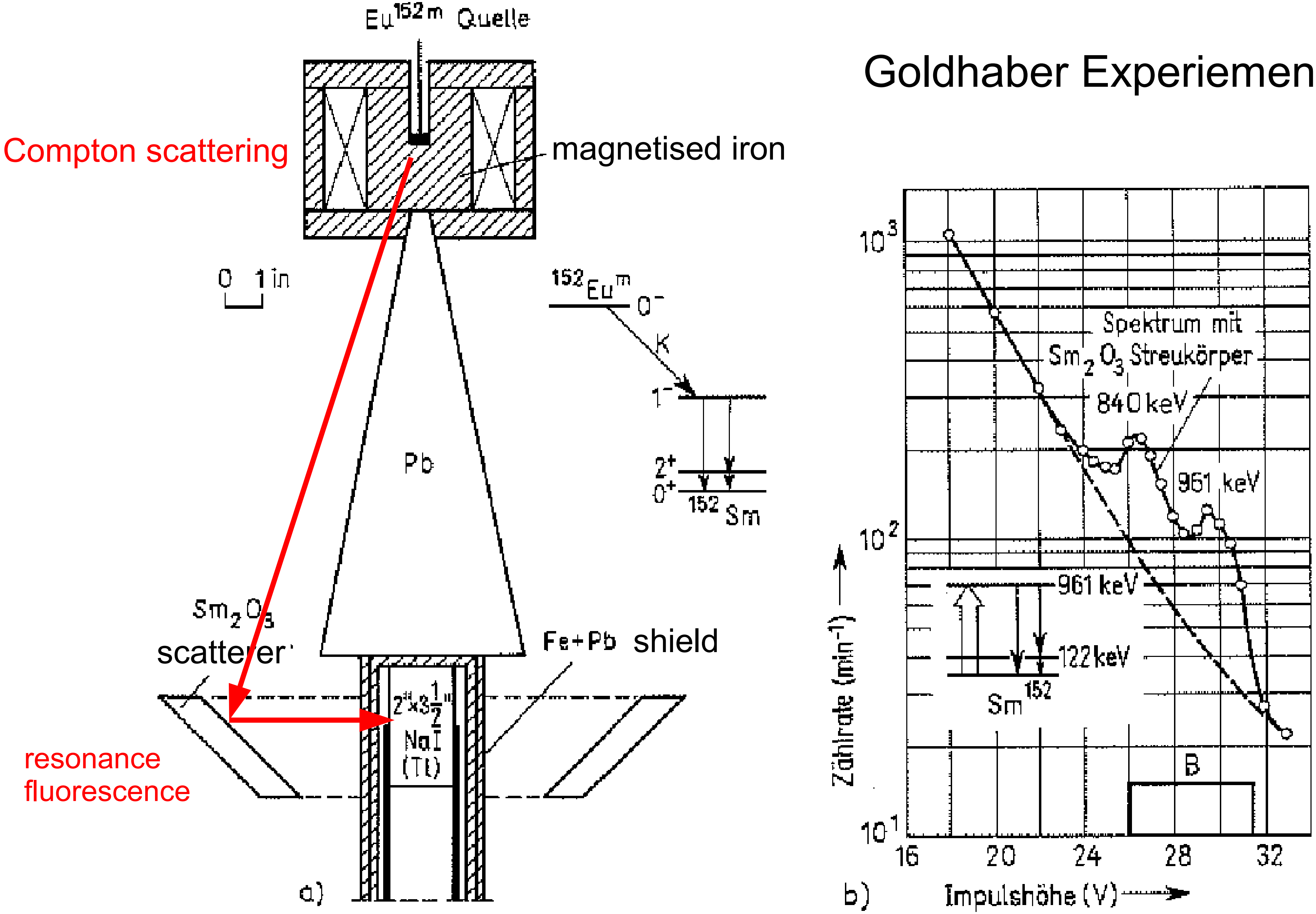


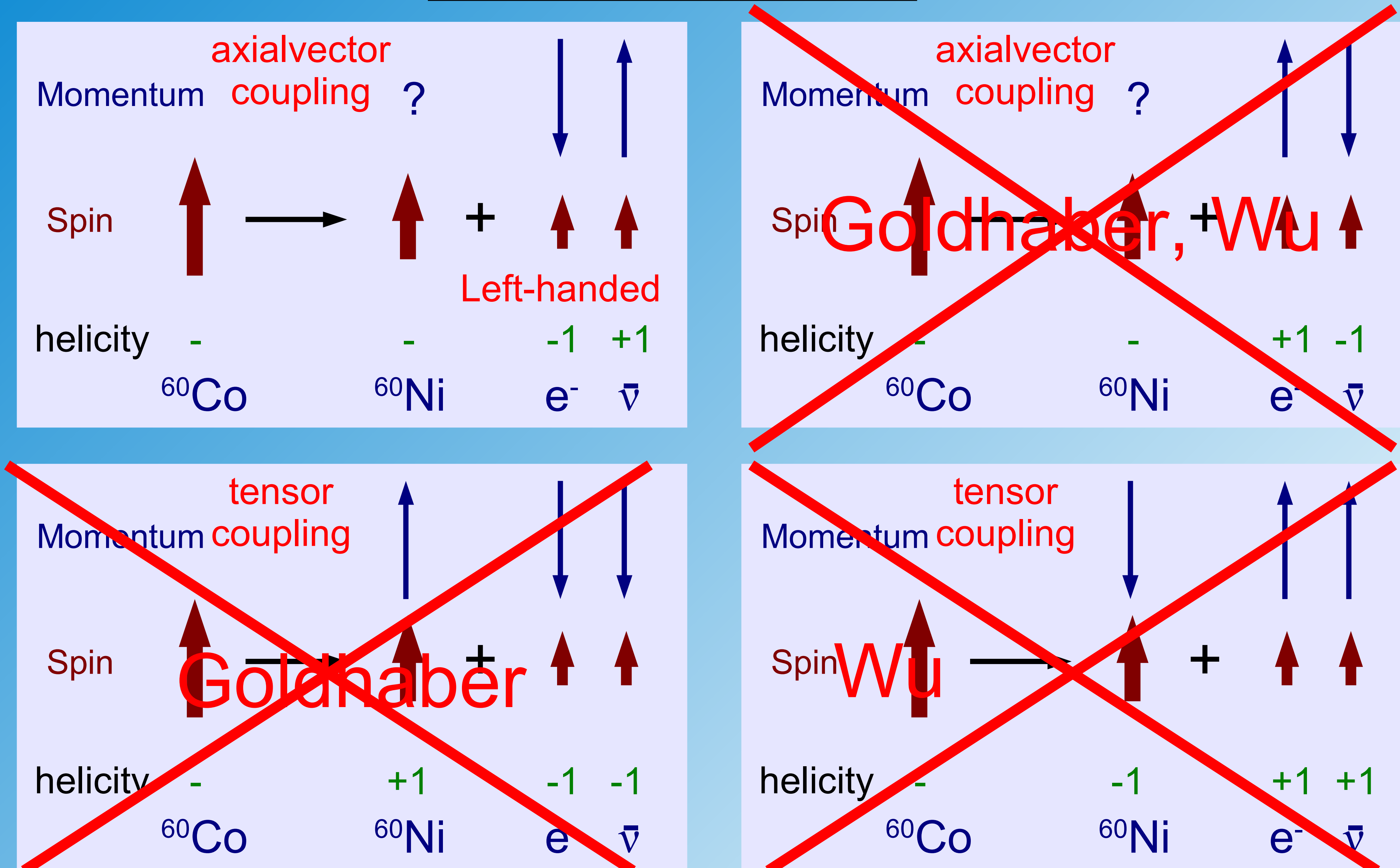
Fig. 144 a) Anordnung zur Messung der Helizität des Neutrinos (Goldhaber u. Mitarbeiter; b) Impulsverteilung für das γ-Streuspektrum. Gestrichelt: nicht-resonanter Untergrund; nach [Gol 58]

Goldhaber: Result + Conclusion

- Due to the ingenious construction of the experiment the polarisation of the photon corresponds to the polarisation of the neutrino
- The photon helicity was measured to be left-handed!
- As a result the helicity of the neutrino has to be left handed
- Left-handed chiral (neutrino!) fermion couplings can be described either by V-A coupling or T(P)-S couplings.
- T(P)-S couplings excluded by Wu- and Goldhaber-experiments

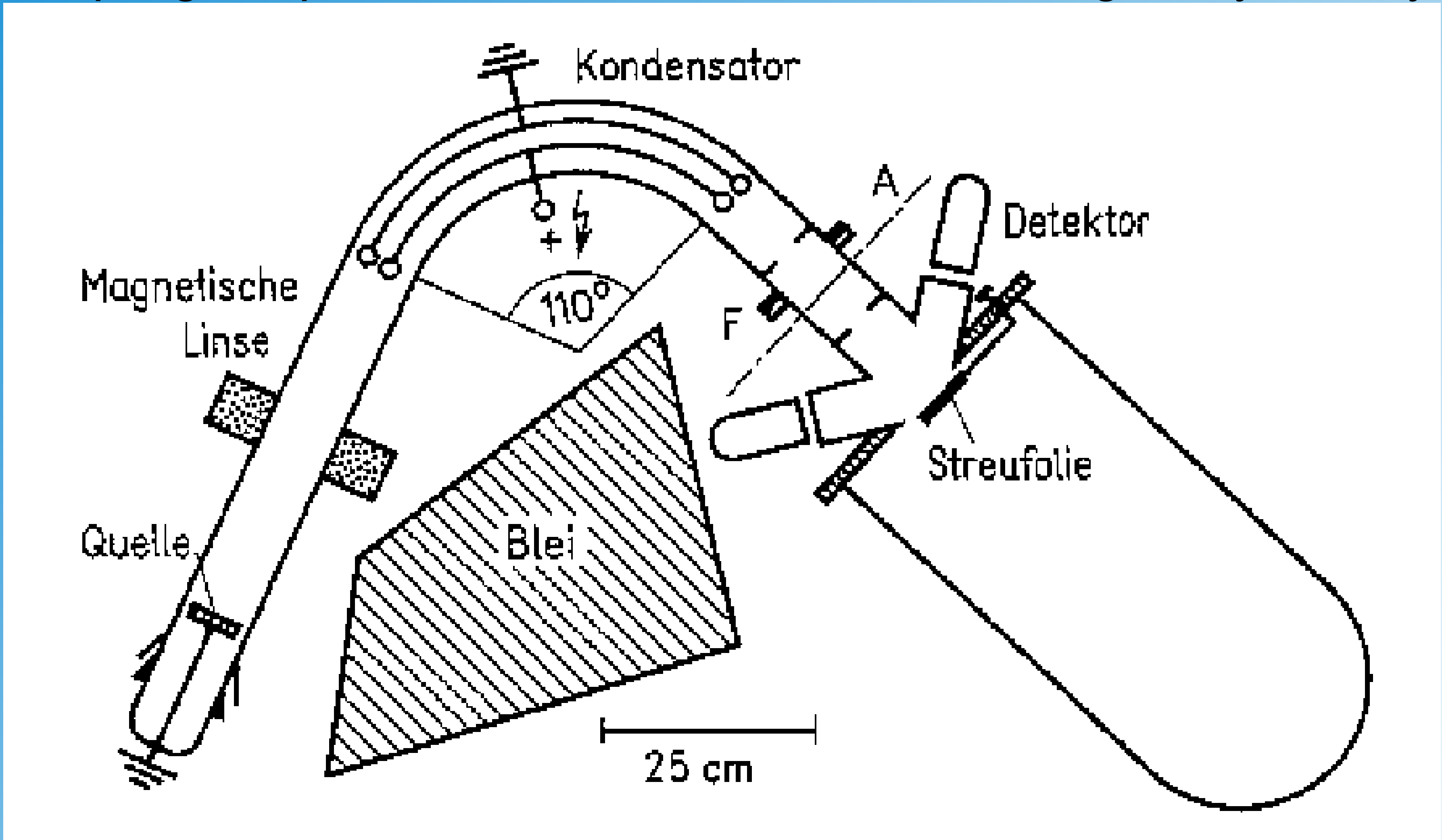
Test of Lorentz Structure in ^{60}Co

Gamov Teller transitions



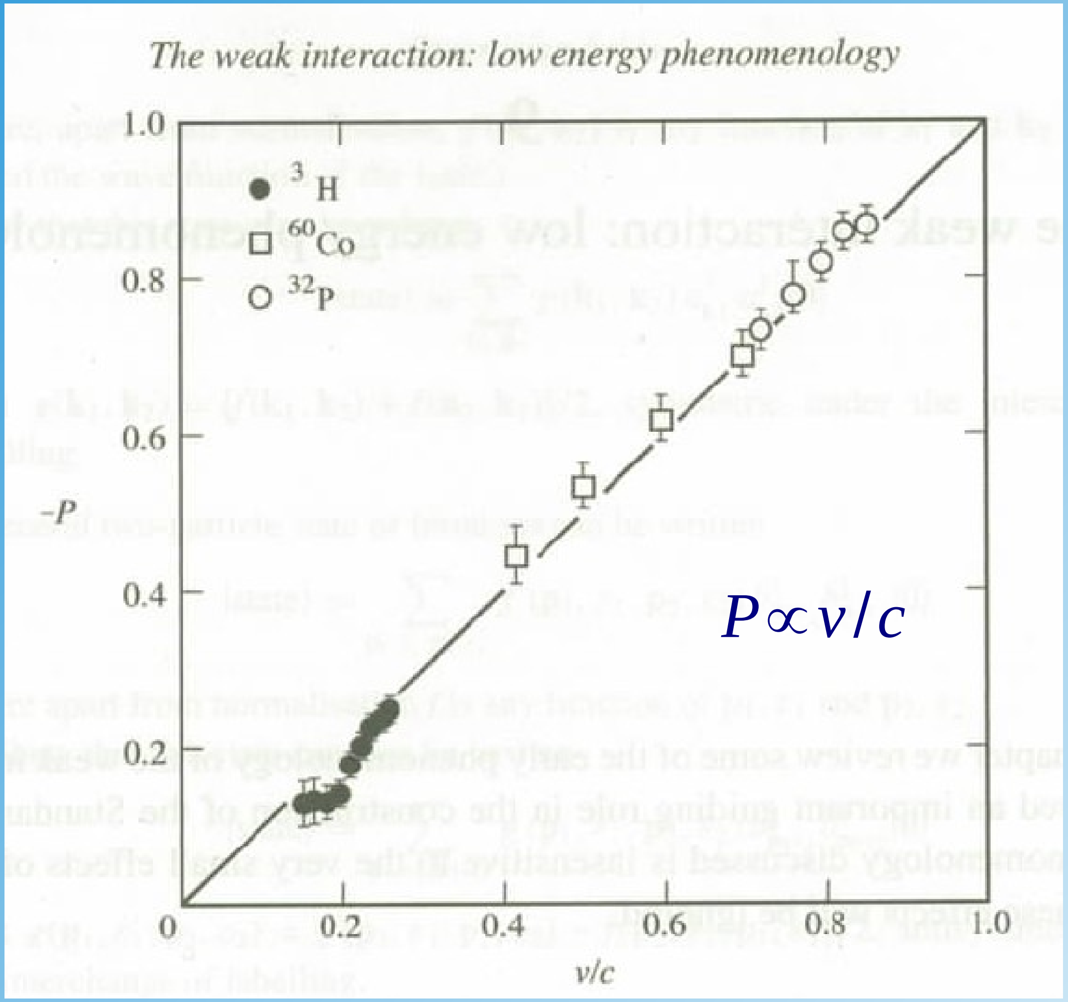
Mott Scattering for Polarisation Measurements

reaction: e N → e N is polarisation dependent:
coupling of spin and orbital momentum → left-right asymmetry



Polarisation in Beta Decays

polarisation: $P = \frac{N(+\frac{1}{2}) - N(-\frac{1}{2})}{N(+\frac{1}{2}) + N(-\frac{1}{2})}$ in different radioactive decays:



only left-handed particles!

Myon Decay

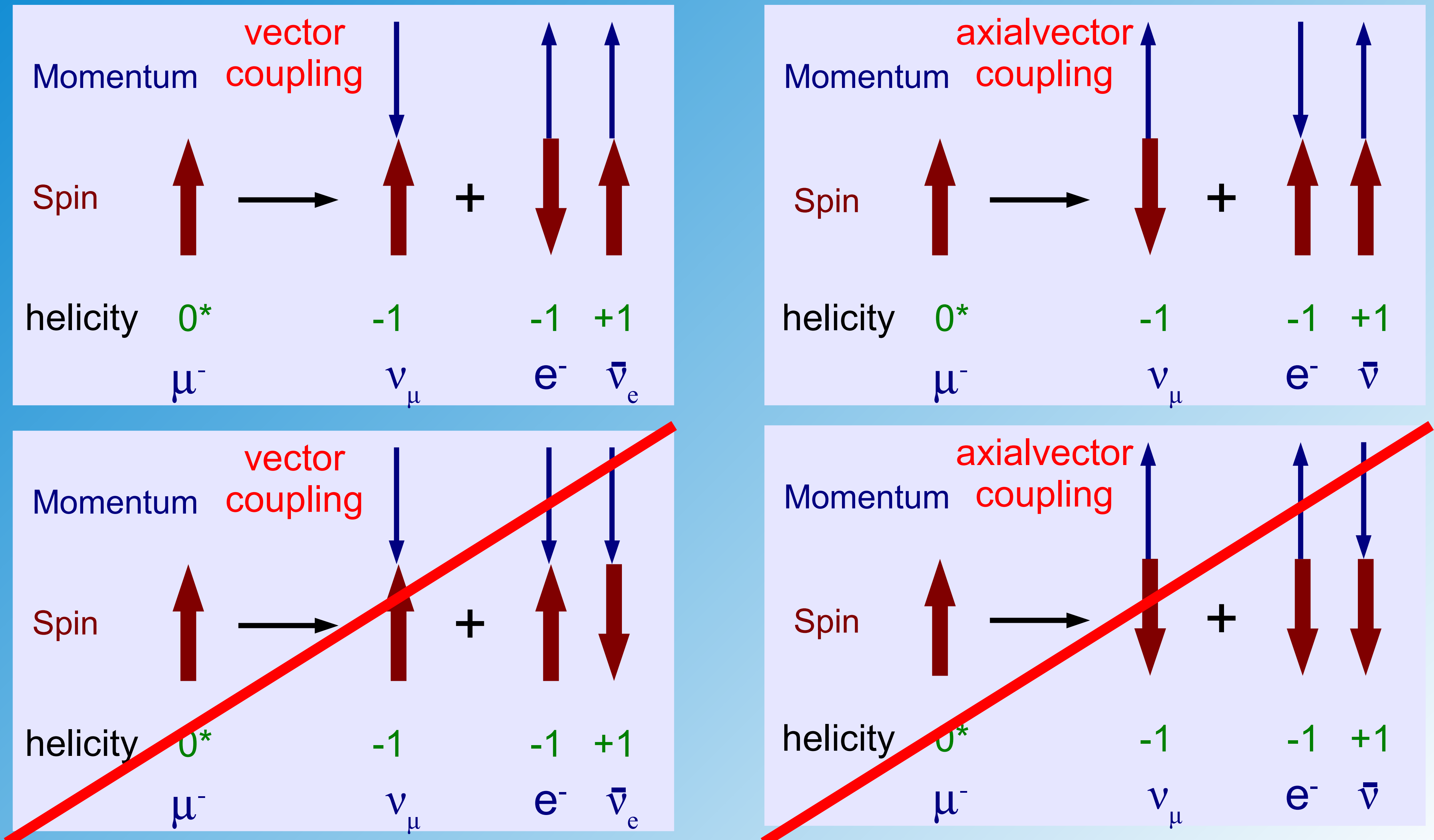
- classical 3-body decay $\mu \rightarrow e \bar{\nu}_e \nu_\mu$
- only left handed particles interaction (→ exercise)

$$L = \frac{G}{\sqrt{2}} \left(\bar{f} \; \Gamma f' \right) \left(\overline{f''} \; \tilde{\Gamma} f''' \right) \quad \text{with} \quad \Gamma_\mu = \gamma_\mu (1 - \gamma_5)$$

Lorentz Structure in Muon Decay

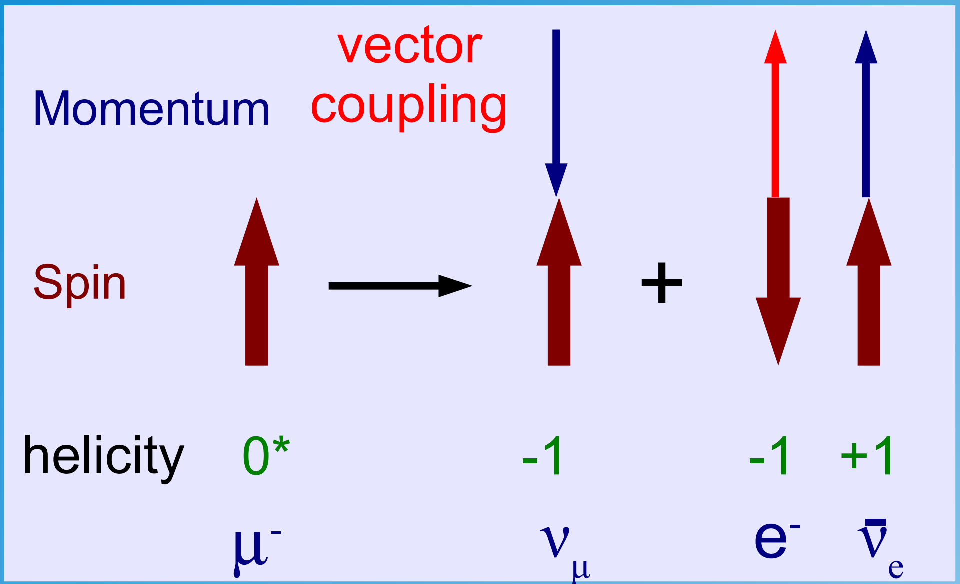
Fermi transition

Gamov Teller transition



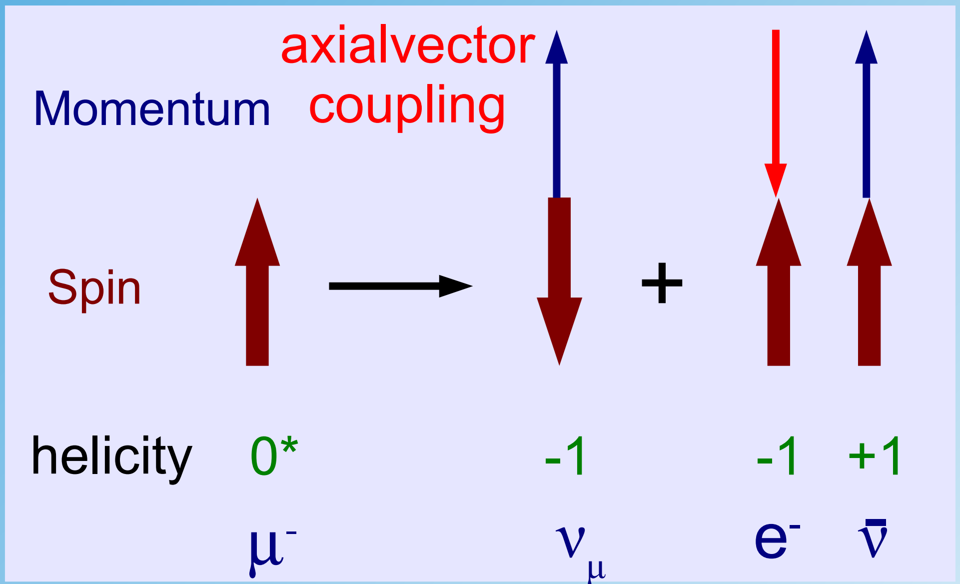
Lorentz Structure in Muon Decay

<u>Fermi transition</u>



myon neutrino has highest energy

<u>Gamov Teller transition</u>



electron has highest energy

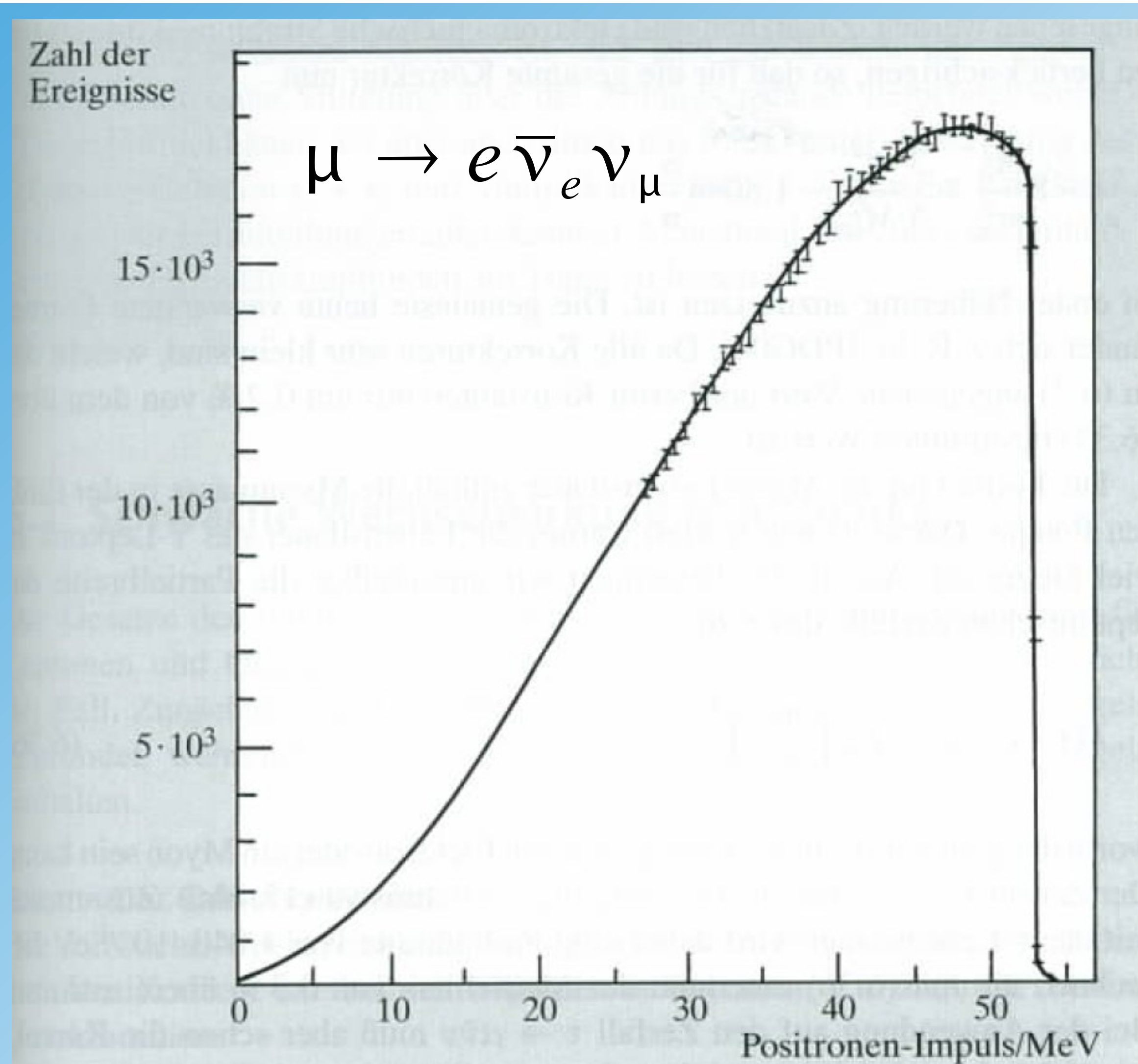
In average the electron is expected to have an energy of ~3/4 of half the muon mass (exact value is 7/10)

→ Michel spectrum

Michel-Spectrum

$$\frac{d^2\Gamma}{dx \ d\cos\vartheta} = \frac{G_F^2 m_\mu^5}{192\pi^3} \left[3 - 2x \pm P_\mu \cos\vartheta (2x-1)\right] \ x^2$$

$$x \equiv 2E_e/m_\mu$$



Very good agreement between measurement and SM prediction (V-A) theory

Michel-Spectrum beyond the SM

The muon decay is one of the best SM testing grounds:

Extended Michel spectrum formula:

$$\frac{d^2\Gamma}{dx\ d\cos\vartheta} \sim x^2 \cdot \Big\{ 3(1-x) + \frac{2\rho}{3}(4x-3) + 3\eta\ x_0(1-x)/x$$
$$\pm P_\mu \cdot \xi \cdot \cos\vartheta \left[1 - x + \frac{2\delta}{3}(4x-3) \right] \Big\} \,. \qquad (2)$$

$$x_0 = \frac{2 m_e m_\mu}{m_e^2 + m_\mu^2}$$

Inportant parameters: $\rho = \xi\delta = 3/4,\ \xi = 1,\ \eta = 0$

They can be related to 4-fermion couplings

$$L = \frac{G}{\sqrt{2}} (\bar{f}\ \Gamma f') (\overline{f''}\ \tilde{\Gamma} f''')$$

Experimental results:

$\lvert g^S_{RR}\rvert < 0.062$	$\lvert g^V_{RR}\rvert < 0.031$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.074$	$\lvert g^V_{LR}\rvert < 0.025$	$\lvert g^T_{LR}\rvert < 0.021$
$\lvert g^S_{RL}\rvert < 0.412$	$\lvert g^V_{RL}\rvert < 0.104$	$\lvert g^T_{RL}\rvert < 0.103$
$\lvert g^S_{LL}\rvert < 0.550$	$\lvert g^V_{LL}\rvert > 0.960$	$\lvert g^T_{LL}\rvert \equiv 0$
$\lvert g^S_{LR} + 6g^T_{LR}\rvert < 0.143$	$\lvert g^S_{RL} + 6g^T_{RL}\rvert < 0.418$	
$\lvert g^S_{LR} + 2g^T_{LR}\rvert < 0.108$	$\lvert g^S_{RL} + 2g^T_{RL}\rvert < 0.417$	
$\lvert g^S_{LR} - 2g^T_{LR}\rvert < 0.070$	$\lvert g^S_{RL} - 2g^T_{RL}\rvert < 0.418$	

Confirmation of V-A coupling

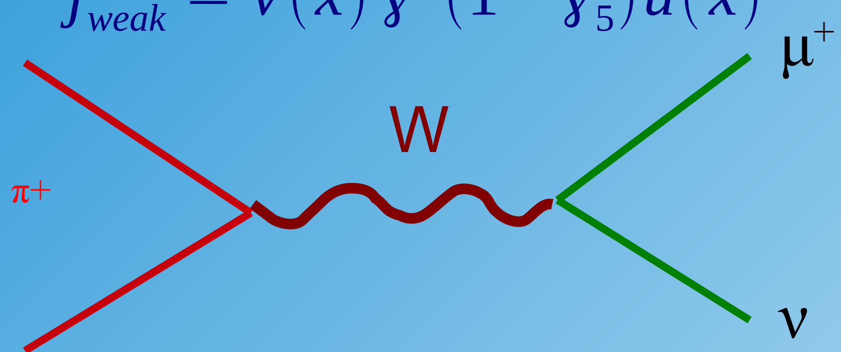
Test Charged Pion Branching Ratios

- dominant decay: $B(\pi^+ \to \mu^+ \nu) = 99.9877\ \%$
- suppressed decay: $B(\pi^+ \to e^+ \nu) = 1.23 \cdot 10^{-4}$

Similar to the neutral pion in QED the more obvious decay is suppressed!

V-A Currents:

$$j^{\mu}_{weak} = \bar{\nu}(x)\gamma^{\mu}(1-\gamma_5)u(x)$$



Polarisation of helicity state is given by fermion velocity:

$\langle\lambda\rangle = -\beta$ for left chiral states

- V-A currents conserve helicity.
- Resulting spin should be $J(\pi^+)=1$
- But pion is a Pseudo-scalar $J(\pi^0)=0$
  - ➔ helicity suppression

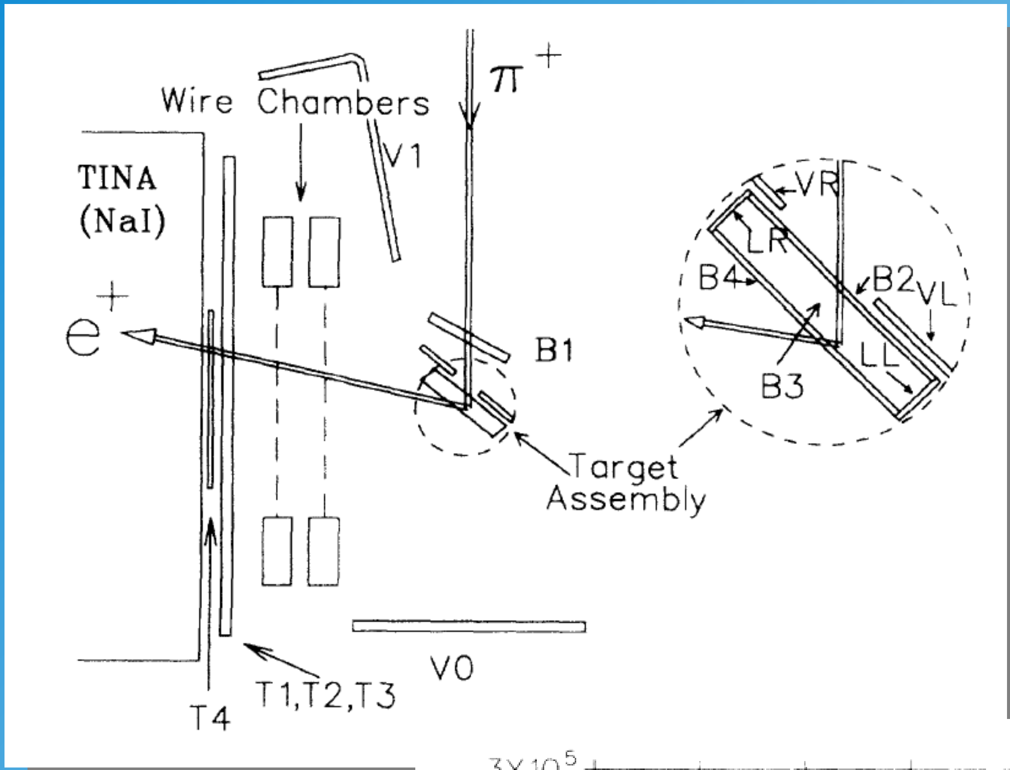
Decay width:

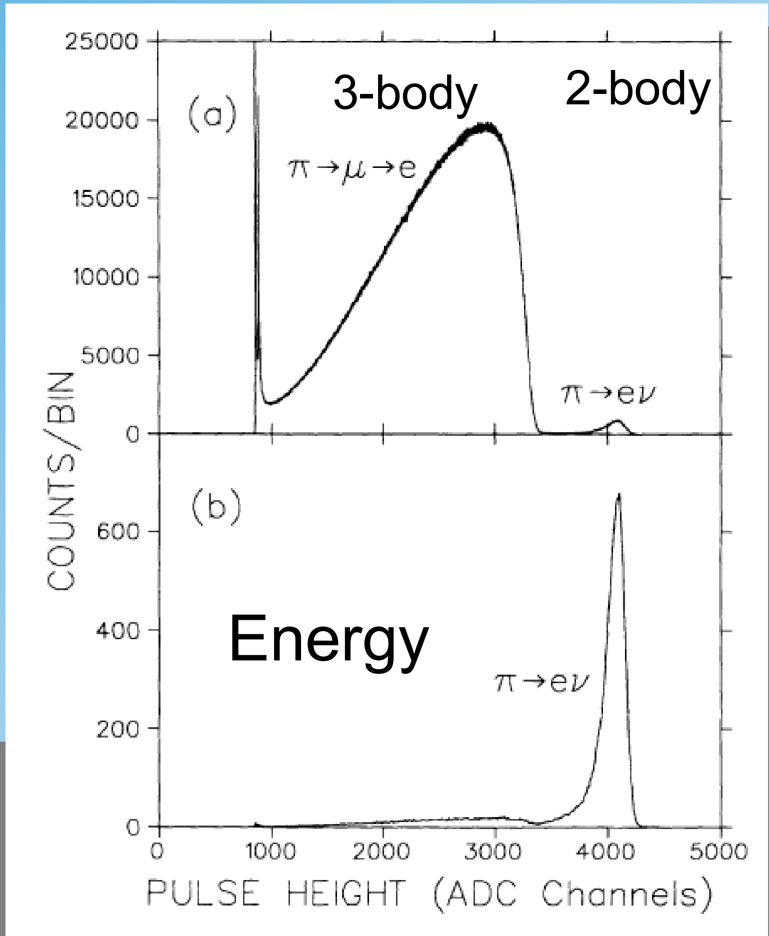
$$\Gamma(\pi^- \to \mu^-) = \frac{G_F^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)$$

$$\frac{\Gamma(\pi^+ \to e^+)}{\Gamma(\pi^+ \to \mu^+)} = \frac{m_e^2(1-m_e^2/m_\pi^2)}{m_\mu^2(1-m_\mu^2/m_\pi^2)} \sim 10^{-4}$$

Measurement of $\pi^+ \to e^+ \nu$

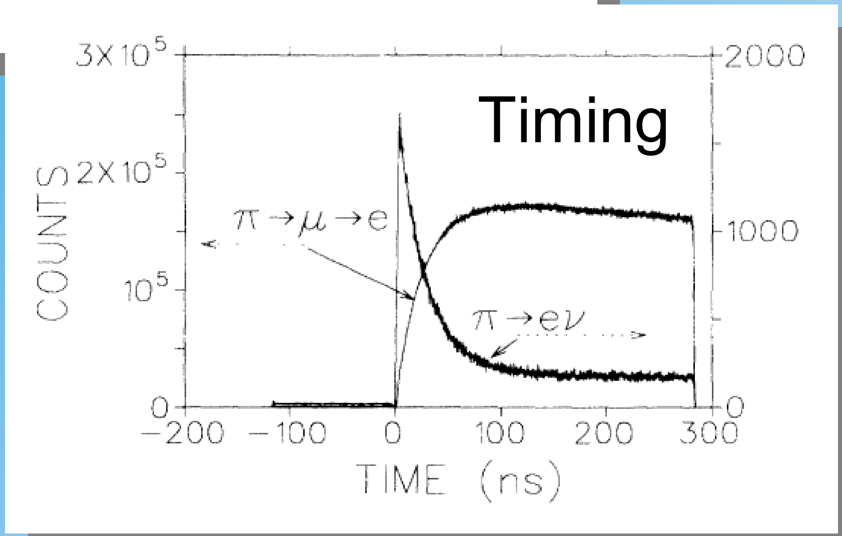
(Britton PRL 68, 20, 1992, 3000)





Pion lifetime 25 ns

Muon lifetime 2 μs



- Result:

$B(\pi^+ \to e^+ \nu) = 1.23 \cdot 10^{-4}$

Conclusion: no scalar coupling!

Gamma Matrices II

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(in other representations
g^5 is diagonal)