Lecture 2: Neutrinos in the SM

1) The Standard Model

Fermions, vector bosons, scalar bosons

Lagrange density $\mathcal{L} = \mathcal{L}(\psi, \partial_\mu \psi)$, dimension 4

Euler-Lagrange: $\boxed{\frac{\partial \mathcal{L}}{\partial \psi} = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}}$ $\Rightarrow$ equation of motion

Example: $\mathcal{L} = \bar{\psi}(\not{p} - m)\psi \longrightarrow (\not{p} - m)\psi = 0$ Dirac

Constraints on possible interactions obtained by choosing a certain particle content, and how these particles transform under a (gauge) symmetry

$$SU(2)_L \times U(1)_Y$$

$SU(2)_L$ $\downarrow$ only left-handed particles

$U(1)_Y$ $\downarrow$ hypercharge

A singlet under $SU(2)$: $\psi \rightarrow \psi' = \psi$

A doublet under $SU(2)$: $\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \rightarrow \begin{pmatrix}\psi_1'\\ \psi_2'\end{pmatrix} = U \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$

where $U = \exp\{-i\alpha_i \frac{\sigma_i}{2}\}$

α_i: infinitesimally

σ_i: Pauli matrices

analogously for $U(1)_Y$: $\psi \rightarrow \psi' = e^{i\beta \hat{Y}}\psi$ where $\hat{Y}\psi = Y_\psi \psi$ (hypercharge operator)

I need invariance under such transformations...

$$\Rightarrow \partial_\mu \longrightarrow D_\mu = \partial_\mu + ig \frac{\sigma_i}{2} W_\mu^i + \frac{i}{2} g' Y B_\mu$$

partial derivative — covariant derivative — gauge coupling — gauge boson — Y-charge

Our „sudoku" rules: Only terms that are „gauge-invariant", Lorentz-invariant and dimension-4

Dimension: [fermions] = 3/2 , [boson] = 1 , $[\partial_\mu] = 1$

gauge-invariant: Y drops out, Y-charges add to zero

We have: $Q = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (2_L, 1/3)$ Isospin: $I_3(u_L) = 1/2$, $I_3(d_L) = -1/2$

$u_R \sim (1_L, 4/3)$, $d_R = (1_L, -2/3)$ $I_3(u_R) = 0$

$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \sim (2_L, -1)$

$e_R \sim (1_L, -2)$

$\phi \sim (2_L, 1) \rightarrow \sqrt{\tfrac{1}{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$ Higgs-doublet

$v = const = 246$ GeV

h = Higgs-particle

Only energy scale of SM!

Examples : $-\mathcal{L} = y_d \, \bar{Q} \, \phi \, d_R$

$$= \underbrace{\frac{y_d}{\sqrt{2}} v}_{m_d} \, \bar{d}_L \, d_R + \underbrace{\frac{y_d}{2}}_{} \bar{d}_L \, h \, d_R$$

Higgs-d_L-d_R coupling $\propto m_d$

$-\mathcal{L} = y_u \, \bar{Q} \, \tilde{\phi} \, u_R$ with $\tilde{\phi} = i\sigma_2 \, \phi^* = \sqrt{\tfrac{1}{2}} \begin{pmatrix} v+h \\ 0 \end{pmatrix}$

$(\sigma_i^+ \, i\sigma_2 \, \sigma_i^* = i\sigma_2 \quad \forall i)$

$\mathcal{L} = (D_\mu \phi)^+ (D^\mu \phi) = \dots =$

$$\frac{1}{4} v^2 g^2 \, W_\mu^+ W^{\mu,-} + \frac{v^2}{8} (W_\mu^3, B_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^{\mu 3} \\ B^\mu \end{pmatrix}$$

with $W_\mu^\pm = \sqrt{\tfrac{1}{2}} \left(W_\mu^1 \mp i W_\mu^2 \right)$; $m_W^2 = \frac{1}{4} v^2 g^2$

and $A_\mu = c_W B_\mu + s_W W_\mu^3$ massless photon!

$Z_\mu = c_W W_\mu^3 - s_W B_\mu \quad \perp A_\mu$

$c_W = \cos\theta_W$, $s_W = \sin\theta_W$; $\tan\theta_W = g'/g$

$\frac{m_W^2}{m_Z^2} = \cos^2\theta_W$; $s_W^2 \simeq 0.231$

$m_W \simeq 80.4$ GeV

$m_Z \simeq 91.2$ GeV

$e = g \sin\theta_W$

$\boxed{Q = \frac{1}{2} Y + I_3}$ where e.g. $I_3(u_R) = 0$

$I_3(u_L) = \frac{1}{2}$, $I_3(d_L) = -\frac{1}{2}$

Fermions couple to gauge bosons:

Express $\mathcal{L} \sim \bar{\psi} \not{D} \psi$ in terms of $W^{\pm}, Z, A$

E.g. $\mathcal{L} \subset Z_\mu \bar{f} \gamma^\mu (v_f - a_f \gamma_5) f \frac{e}{2 c_w s_w}$

	u	d	ν	e
$2v_f$	$1-\frac{8}{3}s_w^2$	$-1+\frac{4}{3}s_w^2$	1	$-1+4s_w^2$
$2a_f$	1	-1	1	-1

$$\mathcal{L} \subset \frac{g}{2\sqrt{2}} \left[\bar{\nu}_e \gamma_\mu (1-\gamma_5) e + \bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu \right] W^{\mu,+}$$

$$+ m_W^2 W_\mu^+ W^{\mu,-}$$

because there are more than one generation...

At low energy: $\partial_\mu W^{\mu,+} = 0 \rightarrow \frac{\partial \mathcal{L}}{\partial W_\mu^+} = 0$

solve for W_μ^-, insert back in $\mathcal{L}$:

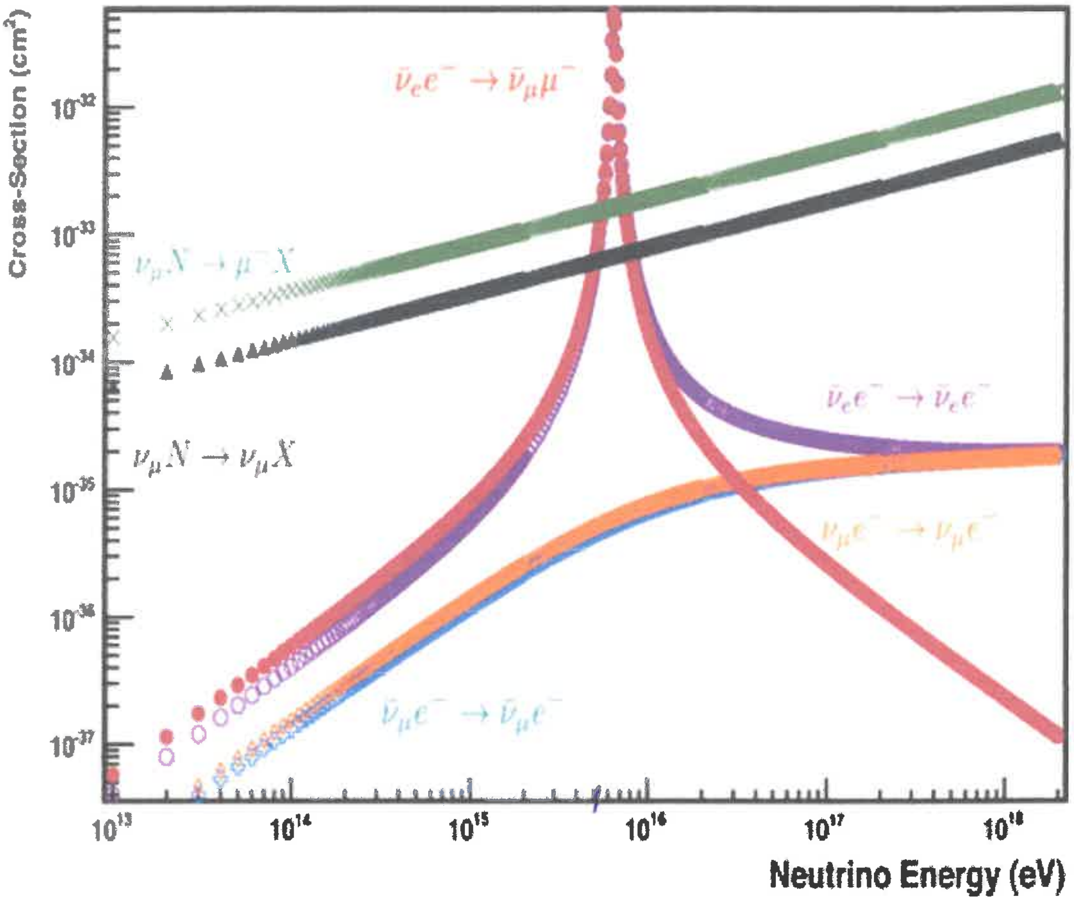
$$\mathcal{L}_{eff} = -\frac{g^2}{8 m_W^2} \left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu \right] \left[\bar{e} \gamma^\mu (1-\gamma_5) \nu_e \right]$$

Fermi-theory! $\frac{g^2}{8 m_W^2} = \frac{G_F}{\sqrt{2}}$ effective theory integrated out W_μ

β-Decay

$\boxed{\text{Fermi: } \sigma \propto G_F^2 s \nearrow \text{ with } E\text{s}\ldots}$

$\sigma \sim \frac{e}{m_W^2 - s}$ ok behavior



$s = 2 m_e E_\nu$: $E_\nu \simeq 6$ PeV

$\Rightarrow \sqrt{s} \simeq 80$ GeV ...

4a

Note: we have not introduced ν_R: it would have no isospin, hypercharge, electric charge: „total singlet"

$\Rightarrow$ While $-\mathcal{L} = y_e \bar{L} \Phi e_R \rightarrow m_e \bar{e}_L e_R$ is there,

$-\mathcal{L} = y_\nu \bar{L} \tilde{\Phi} \nu_R$ is absent, and no m_ν

$\Rightarrow$ Neutrino is massless

#) consistent with measurements at the time

#) nowadays not okay anymore

#) still, $m_\nu << m_e$ by at least 10^6... in each family...

#) related possibly to another mass term $M_R \bar{\nu}_R^c \nu_R$ ...

2) Flavor

Still unexplained fact that more than 1 generation exists

$Q_1' = \begin{pmatrix} u' \\ d' \end{pmatrix}_L$, $Q_2' = \begin{pmatrix} c' \\ s' \end{pmatrix}_L$, $Q_3' = \begin{pmatrix} t' \\ b' \end{pmatrix}_L$

$u_R', c_R', t_R' \equiv u_{iR}'$; $d_R', s_R', b_R' = d_{iR}'$

„Flavor states" (couple to W, Z)

$\Rightarrow$ most general $\mathcal{L}$

$$-\mathcal{L} = \sum_{i,j} \overline{Q_i'} \left[Y_{ij}^{(d)} \, \phi \, d_{j,R}' + Y_{ij}^{(u)} \, \tilde{\phi} \, u_{j,R}' \right]$$

$$\hookrightarrow \quad \overline{d_{i,L}'} \underbrace{\frac{Y_{ij}^{(d)} v}{\sqrt{2}}}_{M_{ij}^{(d)}} d_{j,R}' + \overline{u_{i,L}'} \underbrace{\frac{Y_{ij}^{(u)} v}{\sqrt{2}}}_{M_{ij}^{(u)}} u_{j,R}'$$

„Mass matrices" arbitrary complex

$$\equiv \overline{d_L'} \underbrace{\ }_{\mathcal{U}_d \mathcal{U}_d^+} M^{(d)} \underbrace{\ }_{V_d V_d^+} d_R' + \overline{u_L'} \underbrace{\ }_{\mathcal{U}_u \mathcal{U}_u^+} M^{(u)} \underbrace{\ }_{V_u V_u^+} u_R'$$

$$= \overline{d_L} \, M_{diag}^{d} \, d_R + \overline{u_L} \, M_{diag}^{u} \, u_R \qquad \text{diagonal masses!}$$

not done yet: $\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} W_\mu^+ \underbrace{\overline{u_L'} \, \mathcal{U}_u}_{\overline{u_L}} \mathcal{U}_u^+ \gamma_\mu \mathcal{U}_d \underbrace{\mathcal{U}_d^+ d_L'}_{d_L}$

$$= \frac{g}{\sqrt{2}} W_\mu \, \overline{u_L} \, V \, d_L$$

with $\boxed{V = \mathcal{U}_u^+ \mathcal{U}_d}$

CKM - matrix

(in neutral currents: no effect)

with no mass term for neutrinos, I can freely rotate e, μ, τ to no effect. $\Rightarrow$ The analogon of the CKM-matrix for leptons is trivial (1)

$$V = R_{23}(\Theta_{23})\, R_{13}(\Theta_{13}, \delta)\, R_{12}(\Theta_{12})$$

with $R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$

$$R_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & & c_{13} \end{pmatrix}$$

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

OR:

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$\lambda \simeq 0.22 \qquad A = 0.81$

$\varrho \simeq 0.12 \qquad \eta = 0.35$

hierarchical!