Neutrinos

massless in SM $\longleftrightarrow$ ν - oscillations $\Rightarrow$ physics beyond the SM

$\rightarrow$ Formalism to describe massive neutrinos

$\rightarrow$ How do we know that $m_\nu \neq 0$

$\rightarrow$ How do we investigate neutrinos ?

$\rightarrow$ Neutrino sources : natural and human woman-made sources

$\rightarrow$ How do we generate m_ν ?

$\rightarrow$ Consequences of m_ν in various places (T and E)

Today :

$\rightarrow$ masses and neutrinos in the SM

$\rightarrow$ what if $m_\nu \neq 0$: Neutrino oscillations

1) Neutrinos + Masses in the SM

The SM*: $SU(2)_L \times U(1)_Y \xrightarrow{\text{Higgs}} U(1)_{em}$ $\qquad P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$

ingredients: $L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \sim (2_L, -1)$ [doublet under $SU(2)_L$, hypercharge -1]

$e_R \sim (1_L, -2)$; $u_R \sim (1_L, 4/3)$; $d_R \sim (1_L, -2/3)$

$Q = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (2_L, 1/3)$

Higgs doublet: $\phi \sim (2_L, 1) \longrightarrow \begin{pmatrix} 0 \\ v+h \end{pmatrix} \sqrt{\frac{1}{2}}$ $\qquad v = \text{const}$ ~~V~~EV, h: Higgs particle

Particles couple to each other: ($\leftrightarrow$ Dirac equation: $\mathcal{L} = \overline{\psi}(\not{\partial} - m)\psi$)

rule of the game: $\not{\partial} = \gamma_\mu \partial^\mu \longrightarrow \gamma_\mu D^\mu = \gamma_\mu \left(\partial^\mu + i g \frac{\sigma_i}{2} W^{\mu, i} + \frac{i}{2} g' Y B^\mu \right)$

g: gauge coupling; σ_i: Pauli matrices; $W_\mu^{1,2,3}$; g': gauge coupling; Y: hypercharge; B^μ: gauge boson

$W_\mu^\pm = \sqrt{\frac{1}{2}} (W_\mu^1 \mp W_\mu^2)$

$A_\mu, Z_\mu = f(W_\mu^3, B_\mu)$

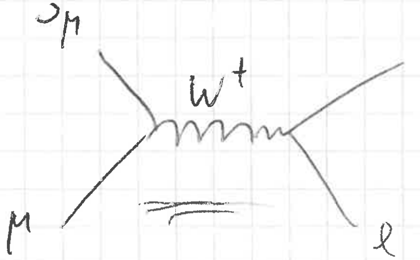
*) no QCD

<u>allowed terms:</u> Lorentz-invariant, dimension-4 $[W_\mu, \phi$ have dim 1, fermions $\frac{3}{2}]$
and gauge-invariant
$\hookrightarrow$ Hypercharges add to zero
#) $L \to \mathcal{U} L$, $Q \to \mathcal{U} Q$, $\phi \to \mathcal{U} Q$
where $\mathcal{U} = \exp\{\frac{i}{2} g\, \sigma_i\, \alpha_i(x)\}$

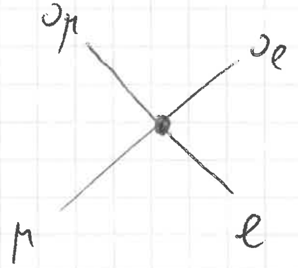
For instance $\bar{Q} \not{D} Q$ contains terms $\frac{i}{\sqrt{2}} g\, \bar{u}_L\, W_\mu^+ \gamma^\mu d_L$ charged current

$\bar{L} \not{D} L$ $\frac{i}{\sqrt{2}} g\, \bar{\nu}_e\, W_\mu^+ \gamma^\mu e_L$ "

in total: $\mathcal{L} = \frac{g}{2\sqrt{2}} \left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu + \bar{\nu}_e \gamma_\mu (1-\gamma_5) e \right] W^{\mu,+}$



$\xrightarrow{m_W \gg m_\mu}$



$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu \right] * \times \left[\bar{\nu}_e \gamma^\mu (1-\gamma_5) e \right]$$

$$G_F = \frac{g^2}{8 m_W^2}, \quad g = 0.65, \quad m_W = 80.3 \text{ GeV}$$

$L \to \mathcal{U} L, \; \phi \to \mathcal{U}\phi$

<u>Masses</u> $\mathcal{L} = -y_e \, \overline{L} \, \phi \, e_R \longrightarrow -\frac{y_e v}{\sqrt{2}} \, \overline{e_L} \, e_R \equiv -m_e \, \overline{e_L} \, e_R$ $\quad\Big/\quad \tilde{\phi} = i\sigma_2 \phi^*$

$\mathcal{L} = -y_\nu \, \overline{L} \, \tilde{\phi} \, \nu_R \longrightarrow -\frac{y_\nu v}{\sqrt{2}} \, \overline{\nu_L} \, \nu_R \equiv -m_\nu \, \overline{\nu_L} \, \nu_R$

$\boxed{\nu_R \sim (1_L, 0) \quad \text{introduced here: „total singlet"} \quad \Rightarrow \text{no } \nu_R \text{, no neutrino mass}}$

<u>Now</u>: $L'_1 = \begin{pmatrix} \nu'_e \\ e' \end{pmatrix}_L \; ; \; L'_2 = \begin{pmatrix} \nu'_\mu \\ \mu' \end{pmatrix}_L \; ; \; L'_3 = \begin{pmatrix} \nu'_\tau \\ \tau' \end{pmatrix}_L$ flavor states

$e'_{R,1} \equiv e'_R \; ; \; e'_{R,2} \equiv \mu'_R \; ; \; e'_{R,3} \equiv \tau'_R$ same for $\nu_{R,j}$ $j = 1,2,3$

$$-\mathcal{L}_Y = \overline{L'_i} \left[g^{(e)}_{ij} \, \phi \, e'_{R,j} + g^{(\nu)}_{ij} \, \tilde{\phi} \, \nu_{R,j} \right] \longrightarrow \overline{e'_{L,i}} \, \underbrace{\mathcal{M}^{(e)}_{ij}}_{U_e U_e^\dagger} \, \underbrace{e'_{R,j}}_{V_e V_e^\dagger} + \underbrace{\overline{\nu'_{L,i}}}_{U_\nu U_\nu^\dagger} \, \mathcal{M}^{(\nu)}_{ij} \, \underbrace{\nu'_{R,j}}_{V_\nu V_\nu^\dagger}$$

such that $U_e^\dagger \mathcal{M}^{(e)} V_e = \text{diag}$

physical states $\longrightarrow$ mass states

$\overline{e'_L} \, U_e = \overline{e_L} \; , \; \overline{\nu'_L} \, U_\nu = \overline{\nu_L}$

Also in $\mathcal{L}_{CC}$: $\overline{\nu'_L} \, e'_L \longrightarrow \overline{\nu'_L} \, U_\nu U_\nu^\dagger U_e U_e^\dagger e'_L \Rightarrow \overline{\nu_L} \, \overbrace{U_\nu^\dagger U_e}^{\text{mixing matrix!}} \, e_L$

$\boxed{U = U_\ell^\dagger U_\nu \text{ is Pontecorvo-Maki-Nakagawa-Sakata Matrix}}$

PMNS-matrix (analogon of CKM)

2) Neutrino Oscillations

$$\boxed{|\nu_\alpha\rangle = U^*_{\alpha i}\,|\nu_i\rangle}$$

flavor $\alpha = e, \mu, \tau$

mass $i = 1, 2, 3$

- at time $t=0$: produce flavorstate: $|\nu(0)\rangle = |\nu_\alpha\rangle = U^*_{\alpha j}\,|\nu_j\rangle$

  time evolution: $|\nu_j(t)\rangle = e^{-iE_jt}\,|\nu_j(t=0)\rangle$

- at time t, amplitude to find state ν_β:

$$A(\nu_\alpha \to \nu_\beta, t) = \langle\nu_\beta|\nu_\alpha(t)\rangle = U_{\beta i} U^*_{\alpha j}\, e^{-iE_jt}\,\overbrace{\langle\nu_i|\nu_j\rangle}^{\delta_{ij}}$$

$$= U^*_{\alpha i} U_{\beta i}\, e^{-iE_it}$$

- Probability: $P(\nu_\alpha \to \nu_\beta, t) = P_{\alpha\beta} = \left| A(\nu_\alpha \to \nu_\beta, t) \right|^2$

$$= \left| U_{\alpha i}^* \, U_{\beta i} \, e^{-iE_i t} \right|^2$$

$$= \sum_{i,j} \underbrace{U_{\alpha i}^* \, U_{\beta i} \, U_{\beta j}^* \, U_{\alpha j}}_{J_{\alpha\beta}^{ij}} \; \underbrace{\exp\{-i(E_i - E_j)t\}}_{\exp\{-i\Delta_{ij}\}}$$

$$\boxed{P_{\alpha\beta} = \ldots = \delta_{\alpha\beta} - 4 \sum_{j>i} \mathrm{Re}\{J_{\alpha\beta}^{ij}\} \sin^2 \frac{\Delta_{ij}}{2} + 2 \sum_{j>i} \mathrm{Im}\{J_{\alpha\beta}^{ij}\} \sin \Delta_{ij}}$$

$t = L$ baseline

with phase: $\frac{1}{2}\Delta_{ij} = \frac{1}{2}(E_i - E_j)t = \frac{1}{2}\left(\sqrt{p_i^2 - m_i^2} - \sqrt{p_j^2 - m_j^2}\right) L$

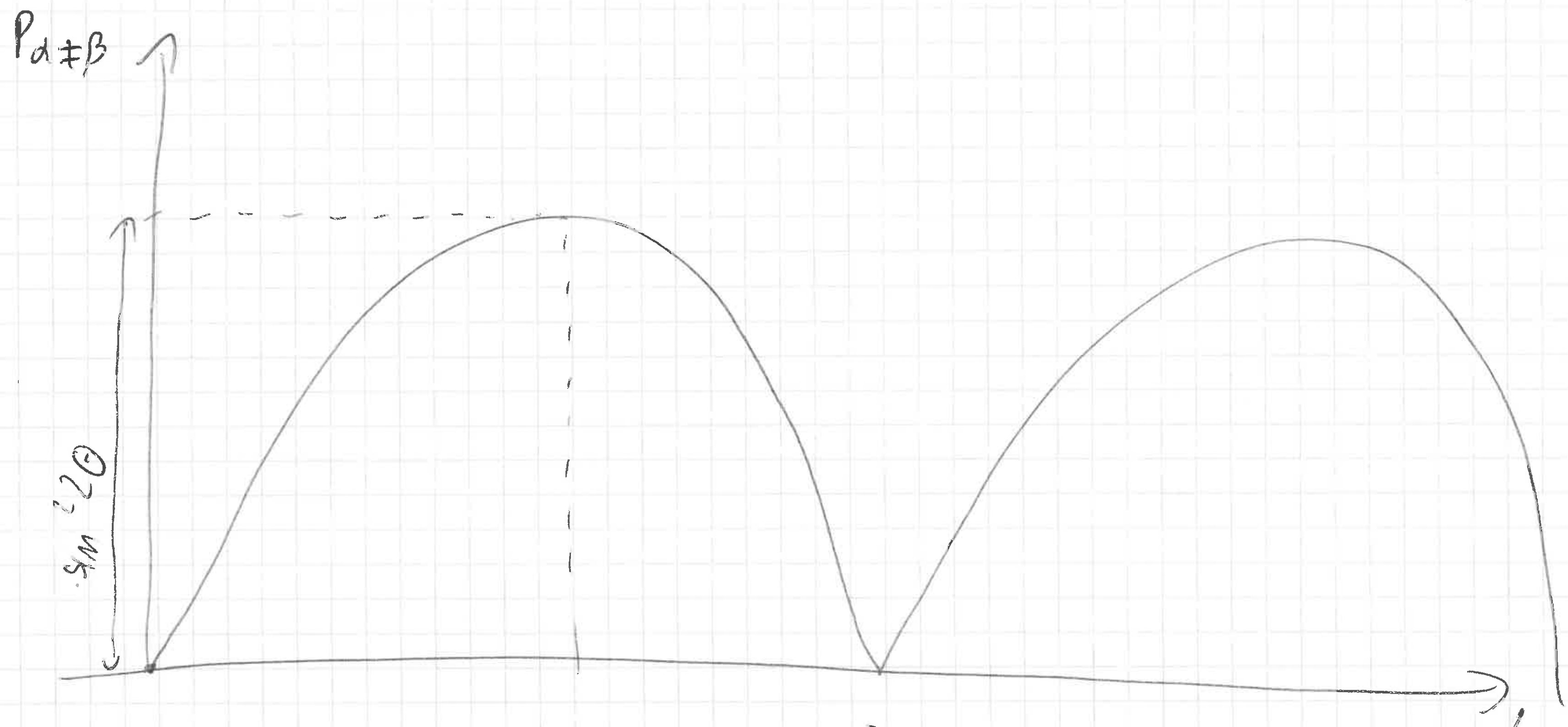
$$= \frac{m_i^2 - m_j^2}{4E} L \equiv \frac{\Delta m_{ij}^2}{4E} L$$

$$= 1.27 \left(\frac{\Delta m^2}{eV}\right)\left(\frac{L}{km}\right)\left(\frac{GeV}{E}\right)$$

2 Neutrino Case

$$\mathcal{U} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \qquad c = \cos\theta \quad s = \sin\theta$$

$$\Rightarrow P_{\alpha \neq \beta} = \sin^2 2\theta \ \sin^2 \frac{\Delta m^2}{4E} L = \frac{1}{2} \sin^2 2\theta \left(1 - \cos \frac{\Delta m^2 L}{2E} \right)$$



$L_{osc} = \frac{4\pi E}{\Delta m^2}$ oscillation length

e.g. $E = GeV$; $\Delta m^2 = 10^{-3} eV^2$: $L_{osc} \sim 10^3$ km