Exercises for Neutrino Physics: Theory and Experiment

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Sheet 9: Theory

1. **Optical Theorem [10 Points]**

In the context of partial wave expansion, the scattering amplitude at an angle θ for the process $a + b \to c + d$ is given by

$$f(\theta) = \frac{1}{2ki} \sum_l (2l+1)\left(\eta_l \exp\left[2i\delta_l\right] - 1\right) P_l\left(\cos\theta\right), \tag{1}$$

where P_l are the Legendre-polynomials, θ is the scattering angle, k is the wavenumber in the incident direction and δ_l and η_l are both real functions. δ_l denotes the phase difference and η_l was introduced to describe inelastic scattering. We have $\eta_l = 1$ for elastic and $\eta_l < 1$ for inelastic scattering. The optical theorem states that the cross section in a forward scattering process is given by

$$\sigma_{\text{tot}} = \frac{4\pi}{k} \text{Im}\left[f(0)\right] \tag{2}$$

(a) Show with the help of the optical theorem that

$$\sigma_{\text{tot}} = \frac{2\pi}{k^2} \sum_l (2l+1)\left(1 - \eta_l \cos\left(2\delta_l\right)\right). \tag{3}$$

(b) The differential cross section for elastic scattering is given by

$$\frac{\mathrm{d}\sigma_{\text{el}}}{\mathrm{d}\Omega} = |f(\theta)|^2. \tag{4}$$

From this, derive the following expression for the elastic scattering cross section

$$\sigma_{\text{el}} = \frac{\pi}{k^2} \sum_l (2l+1)\left|\eta_l \exp\left[2i\delta_l\right] - 1\right|^2. \tag{5}$$

(c) From a) and b) it follows that

$$\sigma_{\text{inel}} = \frac{\pi}{k^2} \sum_l (2l+1)\left(1 - \eta_l^2\right). \tag{6}$$

Show with this equation that for the reaction $\nu_\mu + e^- \to \mu^- + \nu_e$ we obtain the relation

$$\sigma\left(\nu_\mu + e^- \to \mu^- + \nu_e\right) \leq \frac{2\pi}{E_{\text{cm}}^2}, \tag{7}$$

where E_{cm} denotes the center-of-mass energy (k should be considered in the center-of-mass system). Note that this is an $l = 0$ scattering process and that a spin factor $1/(2s+1)$ should be taken into account.

(d) In Fermi theory the cross section is given by

$$\sigma = \frac{G_F^2 s}{\pi}, \tag{8}$$

where G_F is Fermi's constant and $\sqrt{s}$ denotes the invariant mass. Use Eqs. (7) and (8) to find the energy at which Fermi theory breaks down.

2. Number of lepton flavours [10 Points]

The total decay width of the Z boson is given by:

$$\Gamma_Z = \Gamma_e + \Gamma_\mu + \Gamma_\tau + \Gamma_{\text{had}} + \Gamma_{\text{inv}} \tag{9}$$

where Γ_{had} is the sum of all possible hadronic decays $\Gamma_{e,\mu,\tau}$ are the leptonic partial widths, and Γ is the partial decay width of the Z boson to invisibles (i.e. into final states not detectable within colliders).

(a) What decay channels in the Standard Model can contribute to the invisible decay width (at tree level)

(b) Assuming only neutrinos contribute to the invisible Z branching fraction, one can calculate the number of light neutrino generations using

$$N_\nu = \left(\frac{\Gamma_{\text{inv}}}{\Gamma_l}\right)_{\text{exp}} \left(\frac{\Gamma_{\text{l}}}{\Gamma_\nu}\right)_{\text{theory}} . \tag{10}$$

Calculate the theory prediction of $\left(\frac{\Gamma_{\text{l}}}{\Gamma_\nu}\right)_{\text{theory}}$ using the expression for the partial rate of the Z boson to fermions:

$$\Gamma_f = N_C^f \frac{\alpha m_Z}{12 \sin^2\theta_W \cos^2\theta_W} \left[(g_V^f)^2 + (g_A^f)^2\right] \tag{11}$$

Keep in mind that you are interested in the ratio for **one** neutrino type.

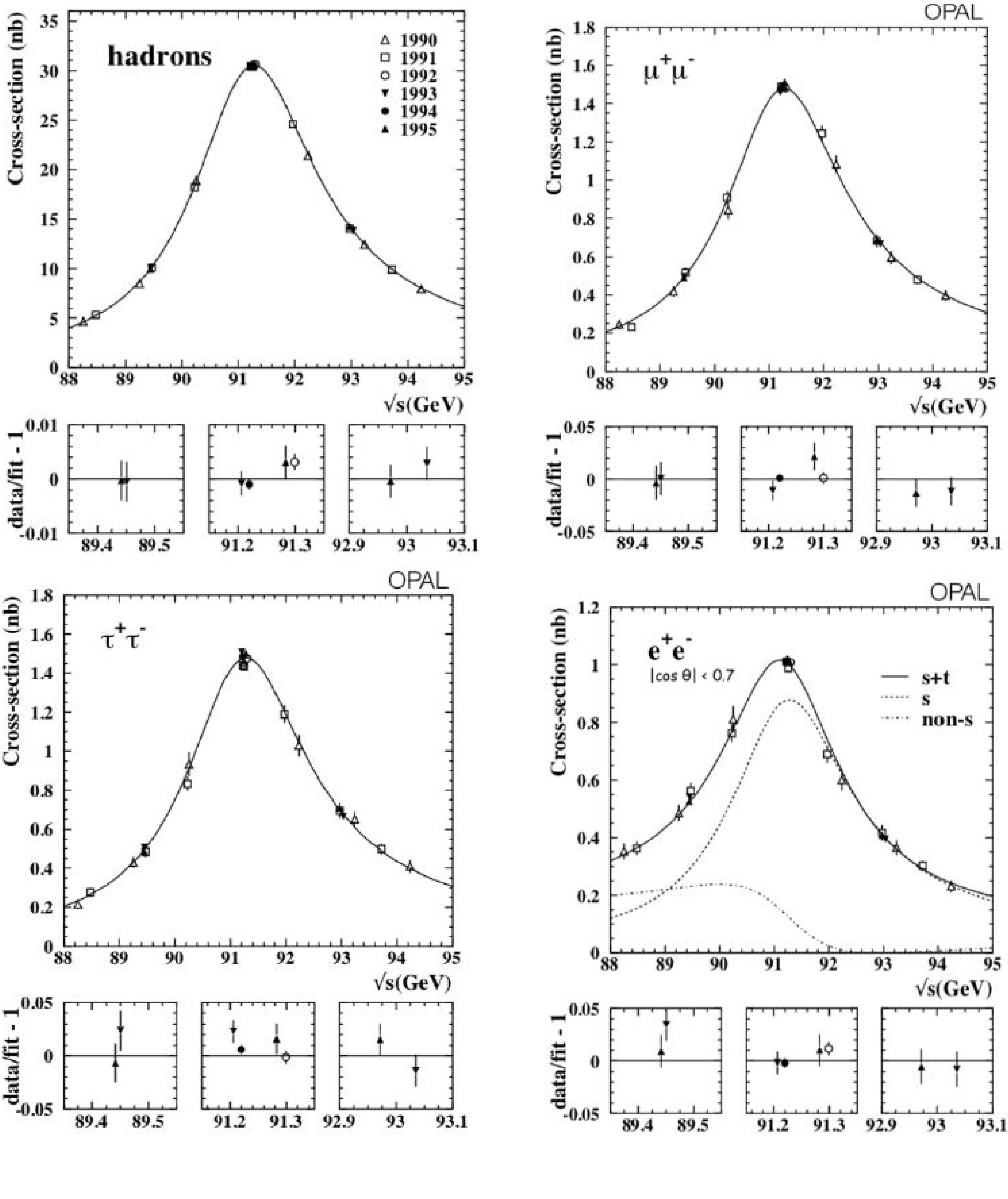
(c) the partial cross section at the peak of the distribution is given by

$$\sigma_{ff}^{\text{peak}} \simeq \frac{12\pi}{m_Z^2} \frac{\Gamma_e \Gamma_f}{\Gamma_Z^2} . \tag{12}$$

Using the plots provided below, read off Γ_Z and calculate the partial width to hadrons and leptons.

(d) Calculate N_ν.

(e) You have calculated the number of light neutrinos. What does *light* mean in this context? Is there any other way to introduce a fourth neutrino into the Standard Model?



hadrons
1990
1991
1992
1993
1994
1995
Cross-section (nb)
√s(GeV)
data/fit - 1
OPAL
μ+μ-
τ+τ-
e+e-
|cos θ| < 0.7
s+t
s
non-s