Exercises for Neutrino Physics: Theory and Experiment

Teresa Marrodán Undagoitia
Werner Rodejohann

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Tutors:
Janine Hempfling: janine.hempfling@mpi-hd.mpg.de
Oliver Scholer: scholer@mpi-hd.mpg.de

Sheet 14: Experiment

1. **Neutrinoless Double-Beta Decay [10 Points]**

   If the neutrino is its own antiparticle the (total) lepton number can not be a conserved quantity, which would be demonstrated by observing the neutrinoless double-beta decay.

   (a) Explain the mechanism leading to a neutrinoless double-beta decay involving light Majorana neutrinos. Draw the Feynman-diagram of the process.

   (b) In the double-beta decay problem the effective Majorana mass is given by $m_{\beta\beta} = |\sum U_{\mathrm{ei}}^2 m_{\mathrm{i}}|$. Expand this mass taking into account that U_{ei}^2 are complex numbers and express it finally in terms of only two phases (the so-called majorana phases), one mass state Δm_{21}^2 and one mass state Δm_{31}^2.

   (c) The two phases, appearing in $m_{\beta\beta}$, are unknown, so they can combine in different ways. Which are the maximum and minimum values of $m_{\beta\beta}$ depending on the majorana phases? Discuss the normal and inverted hierarchy cases and explain the differences in figure 1.

   (d) Using the usual parametrisation of the leptonic mixing matrix, express $m_{\beta\beta}$ as a function of the quantities $\Delta m_{\mathrm{sol}}^2, \Delta m_{\mathrm{atm}}^2, \theta_{12}, \theta_{13}, m_1$ and the two phases.

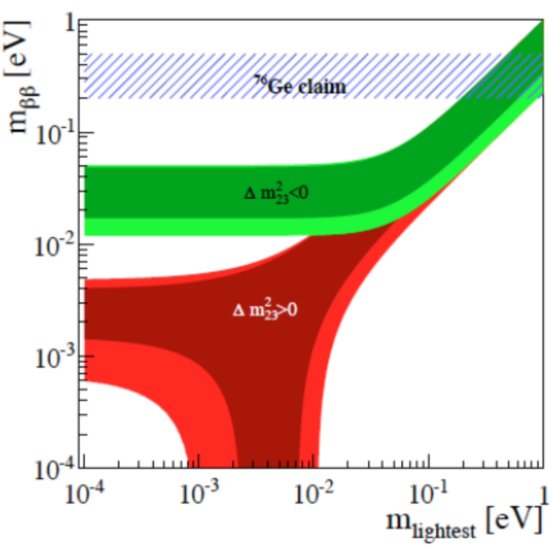

Figure 1: Effective double-beta mass $m_{\beta\beta}$ as function of the lightest neutrino mass in eV.

2. The GERDA Experiment [10 points]

In the case of massive Majorana neutrinos, the process of a neutrinoless double-beta decay ($0\nu\beta\beta$)of the form $X(Z,A) \rightarrow Y(Z+2,A) + 2e$ is allowed and could be observed by experiments.

The GERDA experiment is designed to search for $0\nu\beta\beta$ in$^{76}\text{Ge} \rightarrow^{76} \text{Se} + 2e$ by using high purity germanium detectors (a brief overview of the experiment can be found here: https://inspirehep.net/literature/1467802).

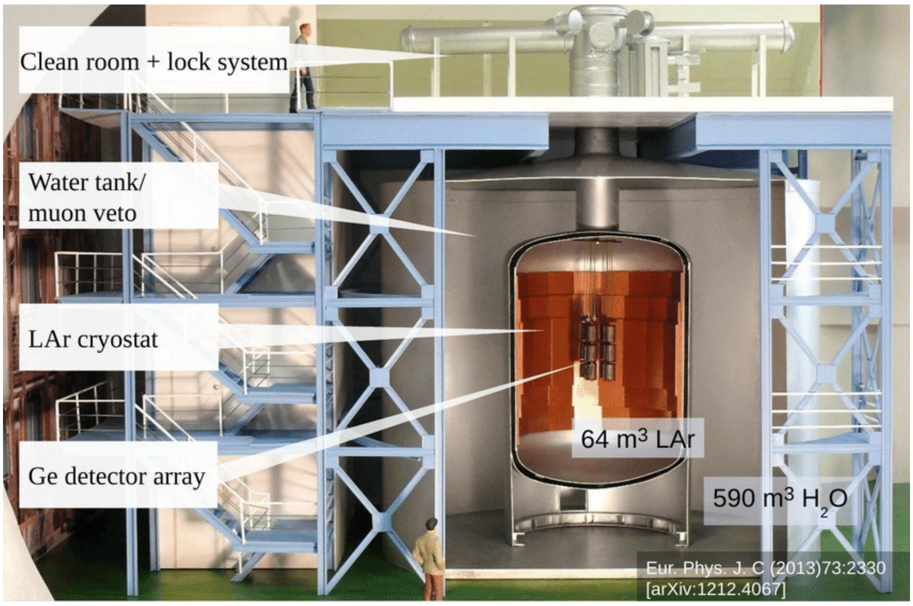

Figure 2: Setup of the GERDA experiment

(a) How can $0\nu\beta\beta$ events be recognized in an experiment?

(b) What is the natural abundance of ^{76}Ge? If one were to test for a half-life of 10^{25} years of the $0\nu\beta\beta$-decay, what is the mass m_{Ge} of Ge that needs to be monitored to have a chance to observe 5 events in one year?

(c) Explain the expression "zero background limit" in relation to $0\nu\beta\beta$ experiments.

(d) What is the dependence of the sensitivity of an experiment to test a given half-life on the exposure ($m_{\text{Ge}} \times t$) in case of zero background?

(e) Which background could compromise the "zero background limit" and how has the GERDA experiment been designed to remove the majority of the background contributions?