Exercises for Neutrino Physics: Theory and Experiment

Teresa Marrodán Undagoitia
Werner Rodejohann

Submission Date: 13. Jul. 2021 before 11:00

Tutors:
Janine Hempfling: janine.hempfling@mpi-hd.mpg.de
Oliver Scholer: scholer@mpi-hd.mpg.de

Sheet 13: Theory

1. **"Integrating out" heavy sterile Neutrinos [10 Points]**

   On the last sheet we discussed the seesaw type I mechanism which is generated by introducing heavy sterile neutrinos into the Standard Model

   $$\mathcal{L} = -\frac{1}{2} N_R^T C M_R N_R - \overline{\nu_L} M_D N_R \tag{1}$$

   At low energies we can describe physics in terms of an effective theory by "integrating out" the heavy N_R. At tree level this can be done by requiring the classical equations of motion to hold:

   $$\frac{\partial \mathcal{L}}{\partial N_R} = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu N_R)} \tag{2}$$

   Use this to derive the mass matrix M_ν of the light neutrinos ν_L by integrating out N_R.

2. **Neutrinoless Double Beta Decay [10 Points]**

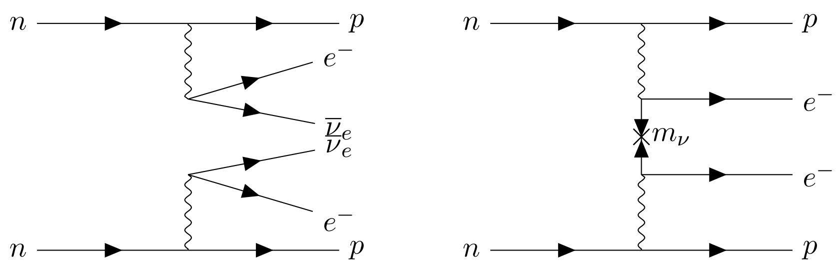

Figure 1: Feynman diagrams of the standard two neutrino (left) and neutrinoless (right) double-β-decays.

   If the active neutrinos are indeed Majorana fermions the so-called neutrinoless double beta decay ($0\nu\beta\beta$) can occur. In the usual framework the decay is mediated via the exchange of a Majorana neutrino (i.e. by closing/connecting the lines of the 2 outgoing neutrinos in the usual double beta decay, see Figure 1). Because this exchange involves a chirality flip the transition amplitude is proportional to the effective Majorana mass of the electron neutrino

   $$m_{\beta\beta} = \sum_i U_{ei}^2 m_i \; . \tag{3}$$

The half-life of the decay process can be written in terms of 3 parameters

$$T_{1/2}^{-1} = G_{0\nu\beta\beta}(A,Z)|\mathcal{M}_{0\nu\beta\beta}(A,Z)|^2 \left|\frac{m_{\beta\beta}}{m_e}\right|^2 \tag{4}$$

where $G_{0\nu\beta\beta}$ is the so-called phase-space factor (PSF) and $\mathcal{M}_{0\nu\beta\beta}$ is the so-called nuclear matrix element (NME). Both depend on the specific decaying isotope (A, Z)

(a) Plot (e.g. using python/pyplot) the inverse half-life $T_{1/2}^{-1}$ for the decay of $^{136}\mathrm{Xe}$ for both normal and inverted mass ordering in dependency of the lightest neutrinos mass in the range $10^{-4}\,\mathrm{eV} \leq m_{\min} \leq 1\,\mathrm{eV}$. Choose a logarithmic scale for both the x- and y-axis. Use

$$G_{0\nu\beta\beta}(^{136}\mathrm{Xe}) = 1.88 \times 10^{-14} y^{-1} \tag{5}$$

$$\mathcal{M}_{0\nu\beta\beta}(^{136}\mathrm{Xe}) = -7.25 \tag{6}$$

(Hint 1: You can find the current best fit values for neutrino oscillation data in the particle data group review at `https://pdg.lbl.gov/2021/tables/contents_tables.html`
Hint 2: For a reasonable estimate you should vary over unknown parameters)

(b) Calculate analytically the minimum $m_{\beta\beta}$ and the corresponding minimum $T_{1/2}$ in $^{136}\mathrm{Xe}$ for the inverted mass ordering.

(c) In the normal ordering you will see a funnel region in which oscillation data allows for a cancellation $m_{\beta\beta} = 0$. What could make this funnel go away?

(d) **Bonus [5 Points]** Explain the so-called *black-box theorem* by Schechter and Valle.