Exercises for Neutrino Physics: Theory and Experiment

Teresa Marrodán Undagoitia
Werner Rodejohann

Submission Date: 06. Jul. 2021 before 11:00

Tutors:
Janine Hempfling: janine.hempfling@mpi-hd.mpg.de
Oliver Scholer: scholer@mpi-hd.mpg.de

Sheet 12: Theory

1. **Majorana Neutrinos [10 Points]**

   The Lagrangian for the coupling of a fermion pair f with the Z-Boson is given by

$$\mathcal{L} = \frac{g}{2\cos\theta_W}\overline{f}\gamma^\mu(v_f - a_f\gamma_5)fZ_\mu \tag{1}$$

   For neutrinos we have $a_\nu = v_\nu = 1/2$

   (a) On sheet 9 we discussed the decay of the Z-boson. Calculate the decay width for $Z \to \overline{\nu}\nu$ in the Standard Model but keeping a possible neutrino mass in the expression.

   (b) Neutrinos could be Majorana particles, which obey the relation $\nu^C = \nu$. The superscript C denotes charge conjugation

$$\nu^C = C\overline{\nu}^T, \tag{2}$$

   with $C = i\gamma_2\gamma_0$ in the Dirac basis. Show the following properties:

$$-C = C^T = C^{-1} = -C^* = C^\dagger, \tag{3}$$

$$C^{-1}\gamma_\mu C = -\gamma_\mu^T, \tag{4}$$

$$C^{-1}\gamma_5 C = \gamma_5^T, \tag{5}$$

$$\overline{\Psi^C} = -\Psi^T C^{-1}, \tag{6}$$

$$(\Psi_L)^C = \left(\Psi^C\right)_R, \tag{7}$$

   where $\left(\Psi^C\right)_L = P_L\left(\Psi^C\right)$.

   (c) Show that for Majorana neutrinos the vector current $\overline{\nu}\gamma_\mu\nu$ vanishes. What happens with $\overline{\nu}\gamma_5\nu, \overline{\nu}\gamma_\mu\gamma_5\nu$ and $\overline{\nu}\left[\gamma_\mu,\gamma_\nu\right]\nu$?

   (d) Using the previous result calculate the decay width $Z \to \nu\nu$ for Majorana neutrinos and compare with a)
   (Hint 1: Note that due to the Majorana properties the final state particles are indistinguishable from each other resulting in a factor of $1/2$.
   Hint 2: Compared to the Dirac case the Majorana axial-vector contribution gains a factor of 2)

2. **Seesaw [10 Points]**

   The Higgs mechanism generates Dirac masses for the active neutrinos when right-handed sterile neutrinos N_R are introduced through the following term:

$$\mathcal{L}_{\text{Dirac}} = -\overline{\nu}_L M_D N_R + \text{h.c.} \tag{8}$$

where ($\nu_L = \nu_L^1, \nu_L^2, \nu_L^3)^T$ is the column vector of the active neutrinos and N_R the corresponding vector for the sterile neutrinos. THe matrix M_D is (in general) a complex 3×3 matrix. Additionally, the right-handed sterile neutrinos can have a Majorana mass with the Lagrangian density

$$\mathcal{L}_{\text{Majorana}} = -\frac{1}{2}\overline{(N_R)^C} M_R N_R + \text{h.c.} \tag{9}$$

where M_R is a symmetric 3×3 matrix and $\Psi^C = C\overline{\Psi}^T$ for a general Dirac spinor Ψ and the charge conjugation matrix $C = i\gamma^2\gamma^0$.
Let m_D and m_R denote the mass scale of M_D and M_R, respectively. Suppose the entries of M_R are much larger than the ones of M_D ($m_R \gg m_D$)

(a) Show that it is possible to rewrite the whole mass matrix in the flavour basis in the following way:

$$\mathcal{L}_{\text{mass}} \equiv \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Majorana}} = -\frac{1}{2}\overline{\Psi^C} M \Psi + \text{h.c.} \tag{10}$$

with

$$\Psi = \begin{pmatrix} (\nu_L)^C \\ N_R \end{pmatrix} \qquad \text{and} \qquad M \equiv \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \tag{11}$$

For the solution prove and use the identity

$$\overline{\nu_L} M_D N_R = \overline{(N_R)^C} M_D^T (\nu_L)^C \ . \tag{12}$$

(b) Using the (unitary) transformation $\Psi = U\xi$ with $U = \begin{pmatrix} 1 & \rho \\ -\rho^\dagger & 1 \end{pmatrix}$ (change of basis) it is possible to convert the 6×6 matrix M into a block diagonal form, i.e. that it takes on the following form

$$U^T M U \simeq \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}, \tag{13}$$

with symmetric 3×3 matrices M_1, M_2. The matrix ρ in the transformation matrix U is assumed to be proportional to the scale m_R^{-1} and terms of order m_R^{-2} (and smaller) can be neglected in the calculation.

Determine ρ, M_1 and M_2 from Eq.(13). What is the connection between the fields χ_1, χ_2 and the original fields ν_L, N_R? (Hint: You may assume that M_R is invertible.)

(c) Where does the name "seesaw" come from?