Determining Properties of Dark Matter Particles with Direct Detection Experiments as Model Independently as Possible

Chung-Lin Shan

Department of Physics, National Cheng Kung University

Max-Planck-Institut für Kernphysik, Heidelberg
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in collaboration with M. Drees, M. Kakizaki and Y. T. Chou
Introduction
   Direct Dark Matter detection

Model-independent data analyses
   Reconstruction of the WIMP velocity distribution
   Determination of the WIMP mass
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   Determinations of ratios of WIMP-nucleon cross sections

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   On the reconstruction of the WIMP velocity distribution

AMIDAS code and website

Summary
Direct Dark Matter detection
Direct Dark Matter detection

DM should have small, but non-zero interactions with ordinary matter.
Direct Dark Matter detection: elastic WIMP-nucleus scattering

- WIMPs could scatter elastically off target nuclei and produce nuclear recoils which deposit energy in the detector.

- The event rate depends on the WIMP density near the Earth, the WIMP-nucleus cross section, the WIMP mass and the velocity distribution of incident WIMPs.

- In typical SUSY models with neutralino WIMPs, the WIMP-nucleus cross section is about $10^{-1} \sim 10^{-6}$ pb, the optimistic expected event rate is then $\sim 10^{-3}$ events/kg-day, but could be $< 1$ event/ton-yr.

- The recoil energy spectrum is approximately exponential and most events would be with energies less than 50 keV.

- Typical background events due to cosmic rays and ambient radioactivity is much larger.
Direct Dark Matter detection: elastic WIMP-nucleus scattering

- Differential event rate for elastic WIMP-nucleus scattering

\[
\frac{dR}{dQ} = AF^2(Q) \int_{v_{\text{min}}}^{v_{\text{max}}} \left[ \frac{f_1(v)}{v} \right] dv
\]

Here

\[v_{\text{min}} = \alpha \sqrt{Q}\]

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy \(Q\) in the detector.

\[\mathcal{A} \equiv \frac{\rho_0 \sigma_0}{2m_\chi m_{N}}\]

\[\alpha \equiv \sqrt{\frac{m_N}{2m_{r,N}^2}}\]

\[m_{r,N} = \frac{m_\chi m_N}{m_\chi + m_N}\]

\(\rho_0\): WIMP density near the Earth

\(\sigma_0\): total cross section ignoring the form factor suppression

\(F(Q)\): elastic nuclear form factor

\(f_1(v)\): one-dimensional velocity distribution of halo WIMPs
Direct Dark Matter detection: elastic WIMP-nucleus scattering

- Differential event rate for elastic WIMP-nucleus scattering

\[
\frac{dR}{dQ} = A F^2(Q) \int_{v_{\text{min}}}^{v_{\text{max}}} f_1(v) \frac{v}{v} dv
\]

Here

\[ v_{\text{min}} = \alpha \sqrt{Q} \]

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy \( Q \) in the detector.

\[ A \equiv \frac{\rho_0 \sigma_0}{2 m_\chi m_r^2} \quad \quad \alpha \equiv \sqrt{\frac{m_N}{2 m_r^2}} \quad \quad m_{r,N} = \frac{m_\chi m_N}{m_\chi + m_N} \]

\( \rho_0 \): WIMP density near the Earth
\( \sigma_0 \): total cross section ignoring the form factor suppression
\( F(Q) \): elastic nuclear form factor
\( f_1(v) \): one-dimensional velocity distribution of halo WIMPs
Direct Dark Matter detection: elastic WIMP-nucleus scattering

- **Spin-independent (SI) WIMP-nucleus cross section**

\[
\begin{align*}
\sigma_0^{SI} &= \left( \frac{4}{\pi} \right) m_r N \left[ Z f_p + (A - Z) f_n \right]^2 \\
\sigma_{\chi p}^{SI} &= \left( \frac{4}{\pi} \right) m_{\chi p} |f_p|^2
\end{align*}
\]

- Exclusion limits on the (predicted) SI WIMP-nucleon cross section

\[\text{http://dmtools.berkeley.edu/limitplots/}\]
Direct Dark Matter detection: elastic WIMP-nucleus scattering

- **Spin-dependent (SD) WIMP-nucleus cross section**

\[
\sigma_{SD}^0 = \left( \frac{32}{\pi} \right) G_F^2 m_r N \left( \frac{J+1}{J} \right) \left[ \langle S_p \rangle_{ap} + \langle S_n \rangle_{an} \right]^2
\]

\[
\sigma_{SD}^{\chi p/n} = \left( \frac{32}{\pi} \right) G_F^2 m_r \langle \frac{a_p}{a_n} \rangle^2
\]

- **Exclusion limits on the SD WIMP-proton cross section**

[C. L. Shan, NCKU Physics MPIK Heidelberg, Feb 15, 2010]
Direct Dark Matter detection: elastic WIMP-nucleus scattering

- **Spin-dependent (SD) WIMP-nucleus cross section**

\[
\sigma_{0}^{SD} = \left( \frac{32}{\pi} \right) G_F^2 m_r^2 \left( \frac{J + 1}{J} \right) \left[ \langle S_p \rangle a_p + \langle S_n \rangle a_n \right]^2
\]

\[
\sigma_{\chi p/n}^{SD} = \left( \frac{32}{\pi} \right) G_F^2 m_r^{2} \left( \frac{3}{4} \right) a_p^2 / n
\]

- Total nuclear spin, expectation values of the proton/neutron group spin
  - \(a_p, a_n\): SD effective WIMP-proton/neutron couplings

- **Exclusion limits on the SD WIMP-neutron cross section**

[http://dmtools.berkeley.edu/limitplots/]
Direct Dark Matter detection: elastic WIMP-nucleus scattering

- Spin-dependent (SD) WIMP-nucleus cross section
  \[ \sigma_{0,SD}^{SD} = \left( \frac{32}{\pi} \right) G_F^2 m_r N \left( \frac{J+1}{J} \right) \left[ \langle S_p \rangle a_p + \langle S_n \rangle a_n \right]^2 \]
  \[ \sigma_{\chi p/n,SD}^{SD} = \left( \frac{32}{\pi} \right) G_F^2 m_r p/n \cdot \left( \frac{3}{4} \right) a_p^2/p/n \]
  
  \(J, \langle S_p \rangle, \langle S_n \rangle\): total nuclear spin, expectation values of the proton/neutron group spin
  \(a_p, a_n\): SD effective WIMP-proton/neutron couplings

- Exclusion limits on the \(a_p\) and \(a_n\) couplings

[V. N. Lebedenko et al., PRL 103, 151302 (2009)]
Model-independent data analyses
Motivation

- Differential event rate for elastic WIMP-nucleus scattering

\[ \frac{dR}{dQ} = AF^2(Q) \int_{v_{\text{min}}}^{v_{\text{max}}} \left[ \frac{f_1(v)}{v} \right] dv \]

Here

\[ v_{\text{min}} = \alpha \sqrt{Q} \]

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy \( Q \) in the detector.

\[ A \equiv \frac{\rho_0 \sigma_0}{2m_\chi m_{r,N}^2} \quad \alpha \equiv \sqrt{\frac{m_N}{2m_{r,N}^2}} \quad m_{r,N} = \frac{m_\chi m_N}{m_\chi + m_N} \]

- \( \rho_0 \): WIMP density near the Earth
- \( \sigma_0 \): total cross section ignoring the form factor suppression
- \( F(Q) \): elastic nuclear form factor
- \( f_1(v) \): one-dimensional velocity distribution of halo WIMPs
Reconstruction of the WIMP velocity distribution

- Normalized one-dimensional WIMP velocity distribution function

\[
f_1(v) = N \left\{-2Q \cdot \frac{d}{dQ} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] \right\}_{Q = v^2/\alpha^2}
\]

\[
N = \frac{2}{\alpha} \left\{ \int_0^\infty \frac{1}{\sqrt{Q}} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ \right\}^{-1}
\]

- Moments of the velocity distribution function

\[
\langle v^n \rangle = N(Q_{\text{thre}}) \left( \frac{\alpha^{n+1}}{2} \right) \left[ \frac{2Q_{\text{thre}}^{(n+1)/2}}{F^2(Q_{\text{thre}})} \left( \frac{dR}{dQ} \right) \right]_{Q = Q_{\text{thre}}} + (n + 1) I_n(Q_{\text{thre}})
\]

\[
N(Q_{\text{thre}}) = \frac{2}{\alpha} \left[ \frac{2Q_{\text{thre}}^{1/2}}{F^2(Q_{\text{thre}})} \left( \frac{dR}{dQ} \right)_{Q = Q_{\text{thre}}} + I_0(Q_{\text{thre}}) \right]^{-1}
\]

\[
I_n(Q_{\text{thre}}) = \int_{Q_{\text{thre}}}^\infty Q^{(n-1)/2} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ
\]

[M. Drees and CLS, JCAP 0706, 011]
Reconstruction of the WIMP velocity distribution

- **Ansatz:** reconstructing the measured recoil spectrum in the \( n \)th \( Q \)-bin

\[
\left( \frac{dR}{dQ} \right)_\text{expt, } Q \approx Q_n \equiv r_n e^{k_n(Q - Q_{s,n})}
\]

- Logarithmic slope and shifted point in the \( n \)th \( Q \)-bin

\[
Q - Q_n \bigg|_n \equiv \frac{1}{N_n} \sum_{i=1}^{N_n} (Q_{n,i} - Q_n) = \left( \frac{b_n}{2} \right) \coth \left( \frac{k_n b_n}{2} \right) - \frac{1}{k_n}
\]

\[
Q_{s,n} = Q_n + \frac{1}{k_n} \ln \left[ \frac{\sinh(k_n b_n/2)}{k_n b_n/2} \right]
\]

- Reconstructing the one-dimensional WIMP velocity distribution

\[
f_1(v_{s,n}) = \mathcal{N} \left[ \frac{2 Q_{s,n} r_n}{F^2(Q_{s,n})} \right] \left[ \frac{d}{dQ} \ln F^2(Q) \bigg|_{Q = Q_{s,n}} - k_n \right]
\]

\[
\mathcal{N} = \frac{2}{\alpha} \left[ \sum_a \frac{1}{\sqrt{Q_a} F^2(Q_a)} \right]^{-1}
\]

\[
v_{s,n} = \alpha \sqrt{Q_{s,n}}
\]

\[\text{[M. Drees and CLS, JCAP 0706, 011]}\]
Reconstruction of the WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s, n)$
  - $^{76}\text{Ge}$, 500 events, 5 bins, up to 3 bins per window

\[ \chi^2/\text{dof} = 0.73 \]

[M. Drees and CLS, JCAP 0706, 011]
Determining the WIMP mass

- Estimating the moments of the WIMP velocity distribution

$$\langle v^n \rangle = \alpha^n \left[ \frac{2Q_{\min}^{1/2}r_{\min}}{F^2(Q_{\min})} + l_0 \right]^{-1} \left[ \frac{2Q_{\min}^{(n+1)/2}r_{\min}}{F^2(Q_{\min})} + (n+1)l_n \right]$$

$$l_n = \sum_a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)}$$

$$r_{\min} = \left( \frac{dR}{dQ} \right)_{\text{expt}, Q=Q_{\min}} = r_1 e^{k_1(Q_{\min} - Q_s, 1)}$$

- Determining the WIMP mass

$$m_\chi \left| \langle v^n \rangle \right| = \frac{\sqrt{m_\chi m_Y} - m_\chi \mathcal{R}_n}{\mathcal{R}_n - \sqrt{m_\chi / m_Y}}$$

$$\mathcal{R}_n = \left[ \frac{2Q_{\min,X}^{(n+1)/2}r_{\min,X}/F_X^2(Q_{\min,X}) + (n+1)l_n,X}{2Q_{\min,X}^{1/2}r_{\min,X}/F_X^2(Q_{\min,X}) + l_0,X} \right]^{1/n} \begin{pmatrix} X \rightarrow Y \end{pmatrix}^{-1} (n \neq 0)$$

- With the assumption of a dominant SI WIMP-nucleus interaction

$$m_\chi \left| \sigma \right| = \frac{(m_\chi / m_Y)^{5/2} m_Y - m_\chi \mathcal{R}_\sigma}{\mathcal{R}_\sigma - (m_\chi / m_Y)^{5/2}}$$

$$\mathcal{R}_\sigma = \frac{\mathcal{E}_Y}{\mathcal{E}_X} \left[ \frac{2Q_{\min,X}^{1/2}r_{\min,X}/F_X^2(Q_{\min,X}) + l_0,X}{2Q_{\min,Y}^{1/2}r_{\min,Y}/F_Y^2(Q_{\min,Y}) + l_0,Y} \right]$$

[CLS and M. Drees, arXiv:0710.4296]
Determining Properties of DM Particles with Direct Detection Experiments as Model Independently as Possible

Model-independent data analyses

Determination of the WIMP mass

- $\chi^2$-fitting

$$\chi^2(m_\chi) = \sum_{i,j} (f_{i,X} - f_{i,Y}) C_{ij}^{-1} (f_{j,X} - f_{j,Y})$$

where

$$f_{i,X} = \alpha_X^i \left[ \frac{2 Q_{\text{min},X}^{(i+1)/2} r_{\text{min},X} / F_X^2 (Q_{\text{min},X}) + (i + 1) l_{i,X}}{2 Q_{\text{min},X}^{1/2} r_{\text{min},X} / F_X^2 (Q_{\text{min},X}) + l_{0,X}} \right] \left( \frac{1}{300 \text{ km/s}} \right)^i$$

$$f_{n_{\text{max}}+1,X} = \mathcal{E}_X \left[ \frac{A_X^2}{2 Q_{\text{min},X}^{1/2} r_{\text{min},X} / F_X^2 (Q_{\text{min},X}) + l_{0,X}} \right] \left( \frac{\sqrt{m_X}}{m_X + m_\chi} \right)$$

$$C_{ij} = \text{cov} (f_{i,X}, f_{j,X}) + \text{cov} (f_{i,Y}, f_{j,Y})$$

- Algorithmic $Q_{\text{max}}$ matching

$$Q_{\text{max},Y} = \left( \frac{\alpha_X}{\alpha_Y} \right)^2 Q_{\text{max},X} \quad \left( \nu_{\text{cut}} = \alpha \sqrt{Q_{\text{max}}} \right)$$

[M. Drees and CLS, JCAP 0806, 012]
Determining Properties of DM Particles with Direct Detection Experiments as Model Independently as Possible

- Model-independent data analyses
- Determination of the WIMP mass

Determining of the WIMP mass

- Reconstructed $m_{\chi, \text{rec}}$
  \[ (^{28}\text{Si} + ^{76}\text{Ge}, \ Q_{\text{max}} < 100 \text{ keV}, \ 2 \times 50 \text{ events}) \]

![Graph showing the relationship between $m_{\chi, \text{rec}}$ and $m_{\chi, \text{in}}$.](attachment:graph.png)

[M. Drees and CLS, JCAP 0806, 012]
Determining Properties of DM Particles with Direct Detection Experiments as Model Independently as Possible

- Model-independent data analyses
- Estimation of the SI WIMP-nucleon coupling

### Estimation of the SI WIMP-nucleon coupling

1. **Estimating the SI WIMP-nucleon coupling**
   
   $$ |f_p|^2 = \frac{1}{\rho_0} \left[ \frac{\pi}{4\sqrt{2}} \left( \frac{1}{E_Z A_Z^2 \sqrt{m_Z}} \right) \right] \left[ \frac{2 Q_{\text{min},Z}^{1/2} r_{\text{min},Z}}{F_Z^2(Q_{\text{min},Z})} + l_0, Z \right] (m_\chi + m_Z) $$


2. **$|f_p|^2_{\text{rec}} (^{76}\text{Ge} (+^{28}\text{Si} + ^{76}\text{Ge}), Q_{\text{max}} < 100 \text{ keV, } \sigma^{\text{SI}}_{\chi p} = 10^{-8} \text{ pb, } 1(3) \times 50 \text{ events})$**

   [M. Drees and CLS, in progress]
Determining Properties of DM Particles with Direct Detection Experiments as Model Independently as Possible

- Model-independent data analyses

Estimation of the SI WIMP-nucleon coupling

- Estimating the SI WIMP-nucleon coupling

\[
|f_p|^2 = \frac{1}{\rho_0} \left[ \frac{\pi}{4\sqrt{2}} \left( \frac{1}{\epsilon_Z A_Z^2 \sqrt{m_Z}} \right) \right] \left[ \frac{2Q_{\text{min},Z}^{1/2} r_{\text{min},Z}}{F_Z^2(Q_{\text{min},Z})} + l_{0,Z} \right] (m_\chi + m_Z)
\]


- \( |f_p|^2 \) vs. \( m_\chi, \) recoiling \( ^{76}\text{Ge} (^{28}\text{Si} + ^{76}\text{Ge}) \), \( Q_{\text{max}} < 100 \text{ keV} \), \( \sigma_{\chi p}^{\text{SI}} = 10^{-8} \text{ pb} \), 1(3) \times 50 \text{ events}]

[M. Drees and CLS, in progress]

C. L. Shan, NCKU Physics

MPIK Heidelberg, Feb 15, 2010
Determining Properties of DM Particles with Direct Detection Experiments as Model Independently as Possible

Model-independent data analyses

Determinations of ratios of WIMP-nucleon cross sections

- Determining the ratio of two SD WIMP-nucleon couplings
  \[
  \left( \frac{a_n}{a_p} \right)_{SD, \pm, n} = - \frac{\langle S_p \rangle_X \pm \langle S_p \rangle_Y R_{J,n}}{\langle S_n \rangle_X \pm \langle S_n \rangle_Y R_{J,n}}
  \]
  
  \[ R_{J,n} \equiv \left[ \left( \frac{J_X}{J_X + 1} \right) \left( \frac{J_Y + 1}{J_Y} \right) \frac{R_{\sigma}}{R_n} \right]^{1/2} \quad (n \neq 0) \]
  
  [M. Drees and CLS, arXiv:0903.3300]

- \( \left( \frac{a_n}{a_p} \right)_{SD, \text{rec}, n} \) (5 – 100 keV, \( ^{73}\text{Ge} + ^{37}\text{Cl} \), 2 \times 50 events, \( m_\chi = 100 \text{ GeV} \) or \( a_n/a_p = 0.7 \))

[M. Drees and CLS, arXiv:0903.3300; in progress]
Determinations of ratios of WIMP-nucleon cross sections

○ Differential rate for the combination of the SI and SD cross sections

\[
\left( \frac{dR}{dQ} \right)_{\text{expt}, Q=Q_{\text{min}}} = \mathcal{E} \left( \frac{\rho_0 \sigma_0^{\text{SI}}}{2m_\chi m_{r,N}} \right) F_{\text{SI}}^{r^2} (Q_{\text{min}}) \cdot \frac{1}{\alpha} \left[ \frac{2r_{\text{min}} / F_{\text{SI}}^{r^2} (Q_{\text{min}})}{2Q_{\text{min}}^{1/2} r_{\text{min}} / F_{\text{SI}}^{r^2} (Q_{\text{min}}) + l_0} \right]
\]

\[ F_{\text{SI}}^{r^2} (Q) = F_{\text{SI}}^2 (Q) + \left( \frac{\sigma_{\chi p}^{\text{SD}}}{\sigma_{\chi p}^{\text{SI}}} \right) C_p F_{\text{SD}}^2 (Q) \quad C_p = \frac{4}{3} \left( \frac{J + 1}{J} \right) \left[ \langle S_p \rangle + \frac{a_n / a_p}{A} \langle S_n \rangle \right]^2 \]

○ Determining the ratio of two WIMP-proton cross sections

\[
\frac{\sigma_{\chi p}^{\text{SD}}}{\sigma_{\chi p}^{\text{SI}}} = \frac{F_{\text{SI}, Y}^2 (Q_{\text{min}}, Y) \mathcal{R}_{m,XY} - F_{\text{SI}, X}^2 (Q_{\text{min}}, X)}{C_p, X F_{\text{SD}, X}^2 (Q_{\text{min}}, X) - C_p, Y F_{\text{SD}, Y}^2 (Q_{\text{min}}, Y) \mathcal{R}_{m,XY}}
\]

\[ \mathcal{R}_{m,XY} = \left( \frac{r_{\text{min}} X}{\mathcal{E}_X} \right) \left( \frac{\mathcal{E}_Y}{r_{\text{min}}, Y} \right) \left( \frac{m_Y}{m_X} \right) ^2 \]

○ Determining the ratio of two SD WIMP-nucleon couplings

\[
\left( \frac{a_n}{a_p} \right)^{\text{SI+SD}} = \left( c_p, X s_{n/p, X} - c_p, Y s_{n/p, Y} \right) \pm \sqrt{c_p, X c_p, Y} \left[ s_{n/p, X} - s_{n/p, Y} \right] \frac{c_p, X s_{n/p, X}^2 - c_p, Y s_{n/p, Y}^2}{c_p, X s_{n/p, X}^2 - c_p, Y s_{n/p, Y}^2}
\]

\[ c_p, X = \frac{4}{3} \left( \frac{J_X + 1}{J_X} \right) \left[ \frac{\langle S_p \rangle X}{A_X} \right] ^2 \left[ F_{\text{SI}, Z}^2 (Q_{\text{min}}, Z) \mathcal{R}_{m,YZ} - F_{\text{SI}, Y}^2 (Q_{\text{min}}, Y) \right] F_{\text{SD}, X}^2 (Q_{\text{min}}, X) \]

[M. Drees and CLS, arXiv:0903.3300]
Determinations of ratios of WIMP-nucleon cross sections

Reconstructed \((a_n/a_p)_{rec}^{SI+SD}\) vs \((a_n/a_p)_{rec,1}^{SD}\)

\((^{19}F + ^{127}I + ^{28}Si, Q_{min} > 5 \text{ keV}, Q_{max} < 100 \text{ keV}, 3 \times 50 \text{ events}, \sigma_{\chi p}^{SI} = 10^{-8} / 10^{-10} \text{ pb}, a_p = 0.1, m_\chi = 100 \text{ GeV})\)

[Reconstructed plots showing the ratio of cross sections for different nuclei]
Determinations of ratios of WIMP-nucleon cross sections

Reconstructed \( \left( \frac{\sigma_{SD}^{\chi p}}{\sigma_{SI}^{\chi p}} \right)_{rec} \) and \( \left( \frac{\sigma_{SD}^{\chi n}}{\sigma_{SI}^{\chi p}} \right)_{rec} \)

\(^{19}\text{F} + ^{127}\text{I} + ^{28}\text{Si} \text{ vs. } ^{76}\text{Ge} + ^{23}\text{Na}/^{131}\text{Xe}, Q_{\text{min}} > 5 \text{ keV}, Q_{\text{max}} < 100 \text{ keV}, \sigma_{\chi p}^{SI} = 10^{-8} \text{ pb}, a_p = 0.1, m_{\chi} = 100 \text{ GeV}, 3/2 \times 50 \text{ events} \)

\[ [\text{M. Drees, M. Kakizaki and CLS, in progress}] \]
Effects of residue background events
Measured recoil spectrum

- Background spectrum
  - Target-dependent exponential background spectrum
    \[ \left( \frac{dR}{dQ} \right)_{bg,ex} = \exp \left( - \frac{Q}{A^{0.6}} \right) \]
  - Constant background spectrum

- Background window
  - Entire experimental possible energy range (0 – 100 keV)
  - Low energy range (0 – 50 keV)
  - High energy range (50 – 100 keV)

- (Naively) simulate
  - only a few residue background events
  - induced by two or more different sources
Measured recoil spectrum

- Measured recoil spectrum

\((^{76}\text{Ge}, 0 – 100 \text{ keV}, \text{exponential bg } 0 – 100 \text{ keV}, 500 \text{ events}, 20\% \text{ bg}, m_\chi = 10 \text{ GeV})\)

Measured recoil spectrum

- Measured recoil spectrum

\[
\text{\(^{76}\text{Ge}, 0 - 100 \text{ keV, exponential bg 0 - 100 keV, 500 events, 20\% bg, } m_{\chi} = 25 \text{ GeV}}\]

Measured recoil spectrum

- Measured recoil spectrum

\[^{76}\text{Ge}, \, 0 \, – \, 100 \, \text{keV}, \, \text{exponential bg} \, 0 \, – \, 100 \, \text{keV}, \, 500 \, \text{events}, \, 20\% \, \text{bg}, \, m_\chi = 50 \, \text{GeV}\]

\[\text{[Y. T. Chou and CLS, arXiv:1003.xxxx]}\]
Measured recoil spectrum

- Measured recoil spectrum

\((^{76}\text{Ge}, \, 0 – 100 \, \text{keV}, \, \text{exponential bg} \, 0 – 100 \, \text{keV}, \, 500 \, \text{events}, \, 20\% \, \text{bg}, \, m_\chi = 100 \, \text{GeV})\)

Measured recoil spectrum

- Measured recoil spectrum

\((^{76}\text{Ge}, 0 – 100 \text{ keV, exponential bg 0 – 100 keV, 500 events, 20% bg, } m_\chi = 250 \text{ GeV})\)

Measured recoil spectrum

- Measured recoil spectrum

\((^{76}\text{Ge}, 0-100 \text{ keV}, \text{exponential bg } 0-100 \text{ keV}, 500 \text{ events}, 20\% \text{ bg}, m_\chi = 500 \text{ GeV})\)

On the determination of the WIMP mass

- **Reconstructed** $m_{\chi,\text{rec}}$

  \((^{28}\text{Si} + ^{76}\text{Ge}, 0 – 100 \text{ keV}, \text{exponential bg } 0 – 100 \text{ keV}, 2 \times 50 \text{ events})\)

![Graph showing the determination of the WIMP mass](image)

On the determination of the WIMP mass

- Reconstructed $m_{\chi, \text{rec}}$

$(^{28}\text{Si} + ^{76}\text{Ge}, \text{0} - \text{100 keV}, \text{exponential bg 0} - \text{100 keV}, 2 \times 500 \text{ events})$

On the reconstruction of the WIMP velocity distribution

- Kinematic maximal cut-off of the recoil energy

\[ Q_{\text{max,kin}} = \frac{v_{\text{esc}}^2}{\alpha^2} \]

- Reconstruction of the one-dimensional WIMP velocity distribution

\[
\begin{align*}
  f_1(v_{s,n}) &= \mathcal{N} \left[ \frac{2Q_{s,n}r_n}{F^2(Q_{s,n})} \right] \left[ \frac{d}{dQ} \ln F^2(Q) \right]_{Q=Q_{s,n}} - k_n \\
  \mathcal{N} &= \frac{2}{\alpha} \left[ \sum_a \frac{1}{\sqrt{Q_a F^2(Q_a)}} \right]^{-1} \\
  v_{s,n} &= \alpha \sqrt{Q_{s,n}} \\
  Q_{s,n} &= Q_n + \frac{1}{k_n} \ln \left[ \frac{\sinh(k_n b_n/2)}{k_n b_n/2} \right] \\
  \alpha &\equiv \sqrt{\frac{m_N}{2m_{r,N}^2}} = \frac{1}{\sqrt{2m_N}} \left( 1 + \frac{m_N}{m_\chi} \right)
\end{align*}
\]

[M. Drees and CLS, JCAP 0706, 011]
On the reconstruction of the WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s, n)$

$^{76}\text{Ge}$, $0 - 100$ keV, exponential bg $0 - 100$ keV, 500 events, $m_\chi = 10$ GeV

[Cite: arXiv:1003.xxxx]

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On the reconstruction of the WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s,n)$

($^{76}$Ge, 0 – 100 keV, exponential bg 0 – 100 keV, 500 events, $m_\chi = 25$ GeV)

[CLS, arXiv:1003.xxxx]
Determining Properties of DM Particles with Direct Detection Experiments as Model Independently as Possible

Effects of residue background events

On the reconstruction of the WIMP velocity distribution

Reconstructed $f_{1,\text{rec}}(v_s, n)$

$(^{76}\text{Ge}, 0 – 100 \text{ keV}, \text{exponential bg } 0 – 100 \text{ keV}, 500 \text{ events}, m_\chi = 50 \text{ GeV})$

On the reconstruction of the WIMP velocity distribution

[CLS, arXiv:1003.xxxx]
On the reconstruction of the WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s,n)$
  
  $(^{76}\text{Ge}, 0 - 100\text{ keV}, \text{exponential bg } 0 - 100\text{ keV}, 500\text{ events}, m_\chi = 100\text{ GeV})$

[CLS, arXiv:1003.xxxx]
On the reconstruction of the WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v, n)$

\((^{76}\text{Ge}, 0 - 100 \text{ keV}, \text{exponential bg } 0 - 100 \text{ keV}, 500 \text{ events, } m_\chi = 250 \text{ GeV})\)
On the reconstruction of the WIMP velocity distribution

○ Reconstructed $f_{1,\text{rec}}(v_s, n)$

$(^{76}\text{Ge}, 0 – 100 \text{ keV}, \text{exponential bg } 0 – 100 \text{ keV, 500 events, } m_\chi = 500 \text{ GeV})$
On the reconstruction of the WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s,n)$ with reconstructed $m_{\chi,\text{rec}}$

$(^{76}\text{Ge} + ^{28}\text{Si} + ^{76}\text{Ge}, 0-100 \text{ keV}, \text{exponential bg}, 3 \times 500 \text{ events}, m_\chi = 10 \text{ GeV})$
On the reconstruction of the WIMP velocity distribution

Reconstructed $f_{1,\text{rec}}(v_{s,n})$ with reconstructed $m_{\chi,\text{rec}}$

($^{76}\text{Ge} + ^{28}\text{Si} + ^{76}\text{Ge}$, 0 – 100 keV, exponential bg, 3 × 500 events, $m_\chi = 25$ GeV)

[CLS, arXiv:1003.xxxx]
On the reconstruction of the WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s,n)$ with reconstructed $m_{\chi,\text{rec}}$

$^{76}\text{Ge} + ^{28}\text{Si} + ^{76}\text{Ge}$, $0 - 100$ keV, exponential bg, $3 \times 500$ events, $m_\chi = 50$ GeV

[CLS, arXiv:1003.xxxx]
On the reconstruction of the WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s,n)$ with reconstructed $m_{\chi,\text{rec}}$

($^{76}\text{Ge} + ^{28}\text{Si} + ^{76}\text{Ge}, 0 – 100$ keV, exponential bg, $3 \times 500$ events, $m_\chi = 100$ GeV)

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On the reconstruction of the WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s, n)$ with reconstructed $m_{\chi,\text{rec}}$

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[CLS, arXiv:1003.xxxx]
Determining Properties of DM Particles with Direct Detection Experiments as Model Independently as Possible

AMIDAS code and website

AMIDAS code and website
AMIDAS code and website

- A Model-Independent Data Analysis System for direct Dark Matter detection experiments
  - DAMNED Dark Matter Web Tool (ILIAS Project)
    - http://pisrv0.pit.physik.uni-tuebingen.de/darkmatter/amidas/
    - [CLS, arXiv:0909.1459, 0910.1971]
  - Online interactive simulation/data analysis system
  - Full Monte Carlo simulations
  - Theoretical estimations
  - Real/user-uploaded data analyses

Further improvements/ideas

- More well-motivated velocity distributions and form factors
- More options for target materials
- Users’ personal setup uploading, storing and reloading
- Generating events with directional information
Summary
Once two or more experiments with different target nuclei observe positive WIMP signals, we could estimate

- WIMP mass $m_\chi$
- SI WIMP-proton coupling $|f_p|^2$
- ratio between the SD WIMP-nucleon couplings, $a_n/a_p$
- ratios between the SD and SI WIMP-nucleon cross sections, $\sigma_{SD\chi p}/n/\sigma_{SI\chi p}$

These analyses are independent of the velocity distribution, the local density, and the mass/couplings on nucleons of halo WIMPs (none of them is yet known).

For a WIMP mass of 100 GeV, these quantities could be estimated with statistical errors of $10 - 40\%$ with only $O(50)$ events from one experiment.
Summary

❖ These information will help us to
  ➢ constrain the parameter space
  ➢ distinguish the (neutralino) LSP from the (first KK hypercharge) LKP
  [G. Bertone et al., PRL 99, 151301 (2007); V. Barger et al., PRD 78, 056007 (2008); G. Belanger et al., PRD 79, 015008 (2009); R. C. Cotta et al., NJP 11, 105026 (2009)]
  ➢ identify the particle produced at colliders to be indeed halo WIMP
  ➢ predict the WIMP annihilation cross section $\langle \sigma_{\text{anni}} \nu \rangle$
  ➢ ......

❖ Furthermore, we could
  ➢ determine the local WIMP density $\rho_0$
  ➢ predict the indirect detection event rate $d\Phi/dE$
  ➢ test our understanding of the early Universe
  ➢ ......
Summary

With an exponential-like residue background spectrum:

- The reconstructed WIMP mass could be over-/underestimated, if WIMPs are lighter/heavier than $\sim 50/200$ GeV.

- Data sets with $\sim 10\% - 20\%$ residue background events could still be used for determining the WIMP mass.

- Background contribution in high/low energy ranges would shift the reconstructed WIMP velocity distribution to higher/lower velocities.

- Over-/underestimated WIMP mass (more strongly) would shift the reconstructed WIMP velocity distribution to lower/higher velocities.

- Data sets with $\sim 5\% - 10\%$ residue background events could still be used for reconstructing the WIMP velocity distribution.
Summary

Studies of effects of residue background events on the estimation of SI WIMP-nucleon coupling and on the determinations of ratios of WIMP-nucleon cross sections are currently under investigation.

Thank you very much for your attention

[http://myweb.ncku.edu.tw/~clshan/Publications/Talks/]