Neutrino forces and experimental probes

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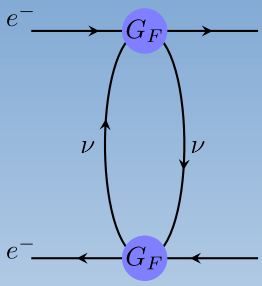
https://xunjiexu.github.io/

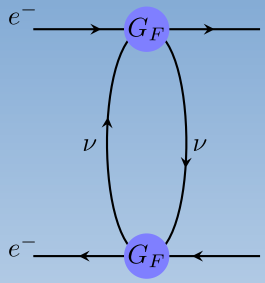
Talk based on 2112.03060, 2203.05455, 2209.07082, 2404.xxxx
In collaboration with Rupert Coy, Mijo Ghosh, Yuval Grossman, Walter Tangarife, Bingrong Yu

- exchange any $m = 0$ particles $\rightarrow$ long-range forces
  - graviton, photon
- what about ν?
  - has to be a pair



History

- 1930s: Bethe & Bacher, Gamow & Teller, ...
  - exchange ν pairs $\rightarrow$ long-range forces between nucleons
- 1960s: carefully computed by G. Feinberg and J. Sucher
- 1960s: Feynman Lectures, Feynman:"ν-forces=gravity?"
- 1990s: Fischbach:"ν-forces destroy neutron stars!"
  - refuted by Kiers & Tytgat, Smirnov & Vissani, ...



Effective potential:

$$V_{ee}(r) = \left(2\sin^2\theta_W + \frac{1}{2}\right)^2 \frac{G_F^2}{4\pi^3}\frac{1}{r^5}$$

- Why $1/r^5$?
  - dimensional analysis
- If $\nu \to$ scalar or $e^- \to$ scalar, $V \propto 1/r^3$
  - also dimensional analysis

Feynman's idea: neutrino forces = gravity. ... But how did Feynman get $1/r$?

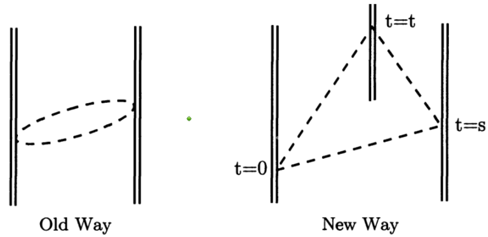
Feynman Lectures on Gravitation, Sec. 2

Two-body:

$$E = m_1 m_2 G'^2 \int \frac{i\,dt}{(t^2 - r^2 + i\epsilon)^2}. \qquad (2.4.1)$$

$$E = m_1 m_2 \frac{G'^2 \pi}{2} \frac{1}{r^3}, \qquad (2.4.2)$$

Figure 2.5



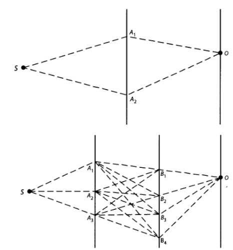
Three-body:

$$E = -G'^3 m_1 m_2 m_3 \pi^2 \frac{1}{(r_{12} + r_{23} + r_{13}) r_{12} r_{23} r_{13}}. \qquad (2.4.4)$$

Now the 3rd body = the entire universe:

$$E = -\frac{G'^3 m_1 m_2 \pi^2}{r_{12}} \int \frac{4\pi \rho(R) R^2 dR}{2R^3}, \qquad (2.4.5)$$

similar to Feyman's idea of path integral



Compare to gravity: $1/r^5$ vs $1/r$

- ν-force $>$ gravity if $r < 10^{-8}$ cm
- so either ... or ...

short-range probe ($r < 10^{-8}$ cm)

- atomic and nuclear spectroscopy
  Y. V. Stadnik, PRL'17
- atomic parity violation
  Ghosh, Grossman, Tangarife, PRD'19

long-range probe ($r > 10^{-8}$ cm)

- precision test of gravity:
  - $1/r^2$ law
  - weak equivalence principle

short-range probe ($r < 10^{-8}$ cm)

- atomic and nuclear spectroscopy
  Y. V. Stadnik, PRL'17
- atomic parity violation
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$\rightarrow$ how?

put $V(r) \propto G_F^2/r^5$ into

$$i\frac{\partial}{\partial t}\Psi = \left[-\frac{\nabla^2}{2m} + V(r)\right]\Psi$$

However, ...

The problematic $1/r^5$

$$\int \left\langle \psi \left| \frac{1}{r^5} \right| \psi \right\rangle d^3r \rightarrow \text{divergent at } \int_0$$

- In QM, $1/r^n$ with $n < 2$ (> 2) known as regular (singular) potential
- Landau and Lifshitz's argument: $E_k \approx \frac{k^2}{2m}$ with $k \sim \frac{1}{r}$ vs $V \propto -\left(\frac{1}{r}\right)^n$
  - so n must be less than 2.

$1/r^n$ with $n < 2$ (> 2) known as regular (singular) potential

REVIEWS OF MODERN PHYSICS VOLUME 43, NUMBER 1 JANUARY 1971

Singular Potentials

WILLIAM M. FRANK AND DAVID J. LAND
U.S. Naval Ordnance Laboratory, Silver Spring, Maryland 20910
RICHARD M. SPECTOR*
Physics Department, Wayne State University, Detroit, Michigan 48202

Why $1/r^n$ with $n > 2$ is problematic?

Landau and Lifshitz's argument (1960):

When the particle is approaching the center of the potential, the kinetic energy $E_k = k^2/(2m_\chi)$ with $k \sim r^{-1}$ increases as $1/r^2$ while V decreases as $-1/r^n$. Therefore, the total energy would not be bounded from below and the particle would keep falling to infinitely small r, corresponding to infinitely high energy.

Is $1/r^5$ always valid down to arbitrarily small r?

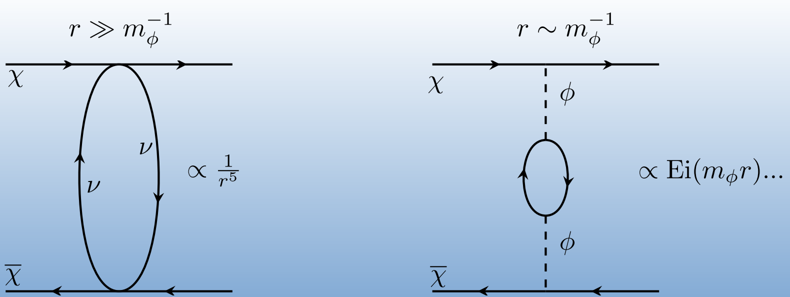
arXiv > hep-ph > arXiv:2112.03060v1

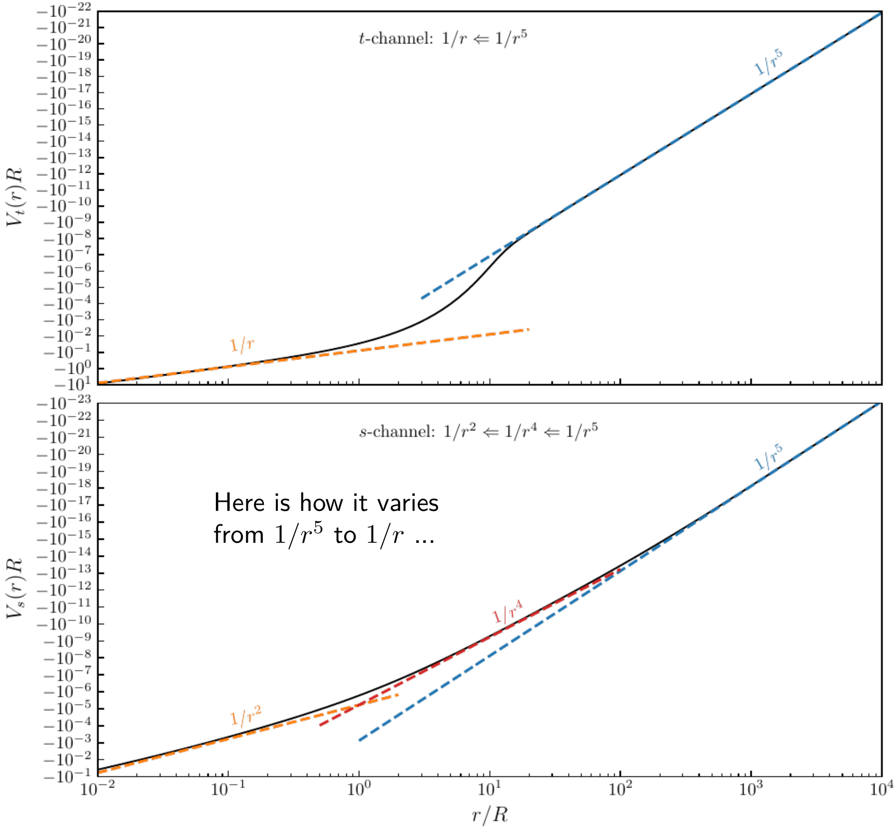
High Energy Physics - Phenomenology

On the short-range behavior of neutrino forces: from $1/r^5$ to $1/r^4$, $1/r^2$, and $1/r$

Xun-jie Xu, Bingrong Yu

The exchange of a pair of neutrinos between two objects, seperated by a distance r, leads to a long-range effective potential proportional to $1/r^5$, assuming massless neutrinos and four-fermion contact interactions. In this paper, we investigate how this known form of neutrino-mediated potentials might be altered if the





Here is how it varies from $1/r^5$ to $1/r$ ...

solving Schrödinger Eq.

short-range probe ($r < 10^{-8}$ cm)

- atomic and nuclear spectroscopy
  Y. V. Stadnik, PRL'17
- atomic parity violation
  Ghosh, Grossman, Tangarife, PRD'19

The problematic $1/r^5$

$$\int \left\langle \psi \left| \frac{1}{r^5} \right| \psi \right\rangle d^3r \to \text{divergent at } \int_0$$

Problem solved!

$1/r^5 \to 1/r$ or $1/r^2$

$$i\frac{\partial}{\partial t}\Psi = \left[-\frac{\nabla^2}{2m} + V(r)\right]\Psi$$

However, within 10^{-8} cm, we have

$$V_{\text{Coulomb}} \gg V_{\text{neutrino}}$$

Recall that Bohr radius
$= 0.53 \times 10^{-8}$ cm

How to observe the tiny effect of neutrino forces?

General idea: look for what is absent in electromag.

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Parity Violation (PV)

- EM and gravity all respect parity, but ν forces do not!
  - first proposed by Ghosh, Grossman, Tangarife, PRD'19
- The SM Z-mediated PV effect already observed in atoms (APV)
  - $^{133}\mathrm{Cs}$, $^{205}\mathrm{Tl}$, $^{208}\mathrm{Pb}$, $^{209}\mathrm{Bi}$, ...
  - probing the SM NC with $1 \sim 0.1\%$ precision
- ν forces $\Rightarrow$ "long-range" PV effect, unlike "short-range" Z-mediated PV.

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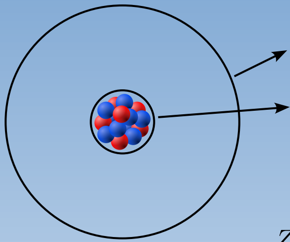
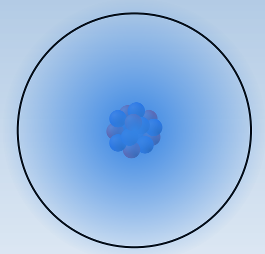


electron cloud radius $\approx R_{\rm Bohr}$

nucleus radius

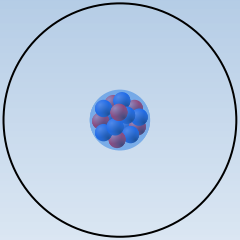
ν forces

long-range



Z forces

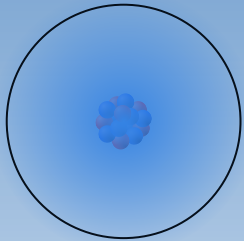
short-range



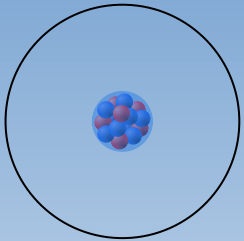
point-like nucleus: $V_Z \sim G_F \delta^3(\mathbf{r})$

finite-size nucleus: $V_Z \sim G_F n_N$

ν forces
long-range



Z forces
short-range



point-like nucleus: $V_Z \sim \boxed{G_F \delta^3(\mathbf{r})}$

$$\langle \Psi_f | V | \Psi_i \rangle = \int \Psi_f^*(\mathbf{r}) \boxed{V(\mathbf{r})} \Psi_i(\mathbf{r}) d^3\mathbf{r}$$

$$\langle \Psi_f | V_Z | \Psi_i \rangle \sim G_F \Psi_f^*(0) \Psi_i(0)$$

requiring the electron cloud density $\neq 0$ at $r = 0$

Indeed, one of the most studied case, transition between 6S and 7S states in cesium, is in this case

ν forces, however, do not require this!

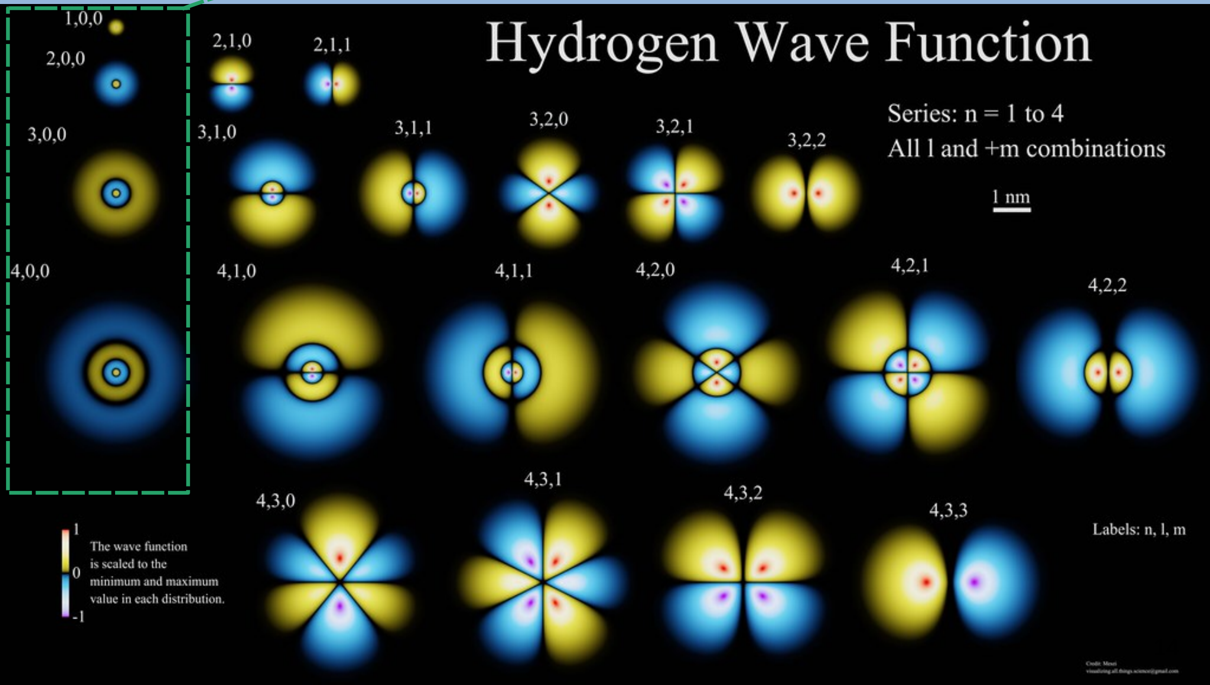
ν forces
long-range

$$\langle \Psi_f | V_\nu | \Psi_i \rangle \sim G_F^2 \int \Psi_f^*(\mathbf{r}) \frac{1}{r^5} \Psi_i(\mathbf{r}) d^3\mathbf{r}$$

Z forces
short-range

$$\langle \Psi_f | V_Z | \Psi_i \rangle \sim G_F \Psi_f^*(0) \Psi_i(0)$$

Only s wavefunctions $\neq 0$ at $r = 0$



PHYSICAL REVIEW LETTERS **120,** 223202 (2018)

Probing Long-Range Neutrino-Mediated Forces with Atomic and Nuclear Spectroscopy

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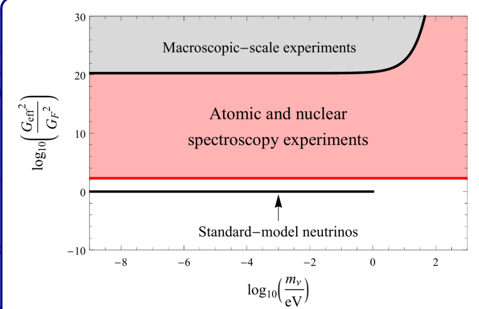
(Received 18 November 2017; published 1 June 201

The exchange of a pair of low-mass neutrinos between electrons, protons, a
"long-range" $1/r^5$ potential, which can be sought for in phenomena origina
subatomic length scales. We calculate the effects of neutrino-pair exchange o
energies in atoms and nuclei. In the case of atomic *s*-wave states, there is a la
induced energy shifts due to the lack of a centrifugal barrier and the highly singul
mediated potential. We derive limits on neutrino-mediated forces from measu
binding energy and transition energies in positronium, muonium, hydrogen, an
isotope-shift measurements in calcium ions. Our limits improve on existing
mediated forces from experiments that search for new macroscopic forces by
Future spectroscopy experiments have the potential to probe long-range forces me
pairs of standard-model neutrinos and other weakly charged particles.



looks promising ... but ...

long-range probe ($r > 10^{-8}$ cm)

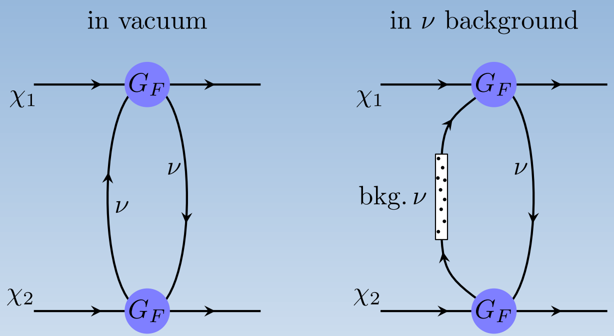
- precision test of gravity:
  - $1/r^2$ law
  - weak equivalence principle

looks impressive!
But how much do we need?
... think about $1/r$ vs $1/r^5$ ...

exp	$\delta V/V_{\text{gravity}}$	$\langle r \rangle$	Refs
Washington2007	3.2×10^{-16}	~ 6400 km	[45]
Washington1999	3.0×10^{-9}	~ 0.3 m	[46]
Irvine1985	0.7×10^{-4}	$2 - 5$ cm	[42]
Irvine1985	2.7×10^{-4}	$5 - 105$ cm	[42]
Wuhan2012	10^{-3}	~ 2 mm	[47]
Wuhan2020	3×10^{-2}	~ 0.1 mm	[44]
Washington2020	~ 1	52 μm	[43]
Future levitated optomechanics	$\sim 10^4$	1 μm	[48]

Is it possible to make $1/r^5 \to 1/r$ at long ranges?

yes, if there is a backgound ... †



propagator in finite-T/D

$$S_\nu(k) = (\not{k} + m_\nu)\left\{\frac{i}{k^2 - m_\nu^2 + i\epsilon} - 2\pi\delta\left(k^2 - m_\nu^2\right)\left[\Theta\left(k^0\right) n_\nu\left(\mathbf{k}\right) + ...\right]\right\}$$

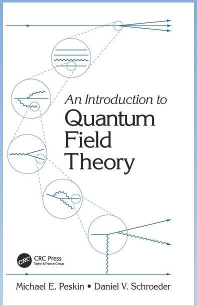
extra term $\propto$ number density

† Feynman used basically the same trick to make neutrino forces gravity-like $(1/r)$

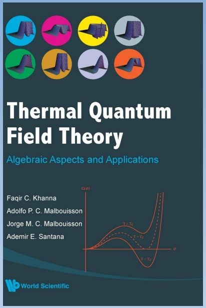
Finite-Temperature/Density correction

Zero-T QFT

propagator: $\dfrac{i}{p^2 - m^2}$



finite T $\longrightarrow$



page 137-138

Finite-T QFT

propagator: $\dfrac{i}{p^2 - m^2} - \underline{(2\pi)\delta\left(p^2 - m^2\right)\left[\Theta\left(p^0\right) n_+(\mathbf{p}) + \Theta\left(-p^0\right) n_-(\mathbf{p})\right]}$

extra term $\propto$ number density

Why can Finite T/D modify the propagator?

Answer: because of coherent scattering

very similar to the MSW effect of neutrinos

$$S_F(x-y) \equiv \langle 0|\psi(x)\overline{\psi(y)}|0\rangle \xrightarrow{\text{finite-T}} S_F(x-y) \equiv \langle \text{bkg}|\psi(x)\overline{\psi(y)}|\text{bkg}\rangle$$

$$\langle ①, ②, ③ \cdots | \int_{\mathbf{p}} \int_{\mathbf{k}} \cdots \left[a_{\mathbf{p}} \cdots + b_{\mathbf{p}}^\dagger \cdots\right] \left[a_{\mathbf{k}}^\dagger \cdots + b_{\mathbf{k}} \cdots\right] | ①, ②, ③ \cdots \rangle$$

contraction 1

contraction 2

coherent scattering

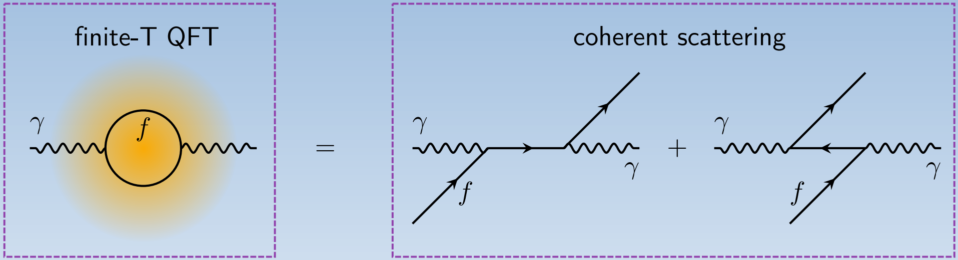
with ① or ② or ③ ...?
indistinguishable = coherency

$$\text{propagator} = \boxed{\frac{i}{p^2 - m^2}} - \boxed{(2\pi)\delta\left(p^2 - m^2\right)\left[\Theta\left(p^0\right) n_+(\mathbf{p}) + \Theta\left(-p^0\right) n_-(\mathbf{p})\right]}$$

extra term $\propto n_\pm$ (number density)

Interesting example: the finite-T photon mass

$$m_\gamma^2 = 4\pi\alpha \frac{Q_f^2 n_f}{m_f}$$

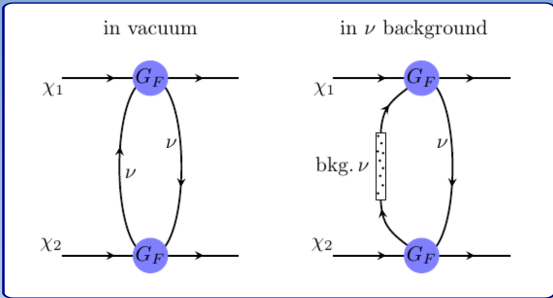


Loop-level result in finite-T QFT = Tree-level result in zero-T QFT

interested in the deatils? see backup slides

thanks to Evgeny for discussions on photon coherent scattering ...

Finite-Temperature/Density correction

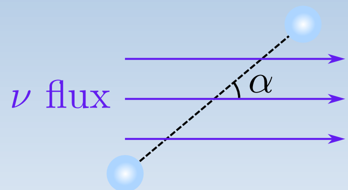


Effective potential

Vacuum: $$V_0 \approx \frac{G_F^2}{4\pi^3}\frac{1}{r^5}$$

With CνB: $$V \approx V_0 - \frac{8G_F^2}{\pi^3}\frac{T^4}{r\left(1+4r^2T^2\right)^2}$$

- Vacuum ν force: first calculated in 1960s
  - Feinberg, Sucher, Phys.Rev. (1968).
- including CνB: first calculated in 1993
  - Horowitz, Pantaleone PLB (1993).

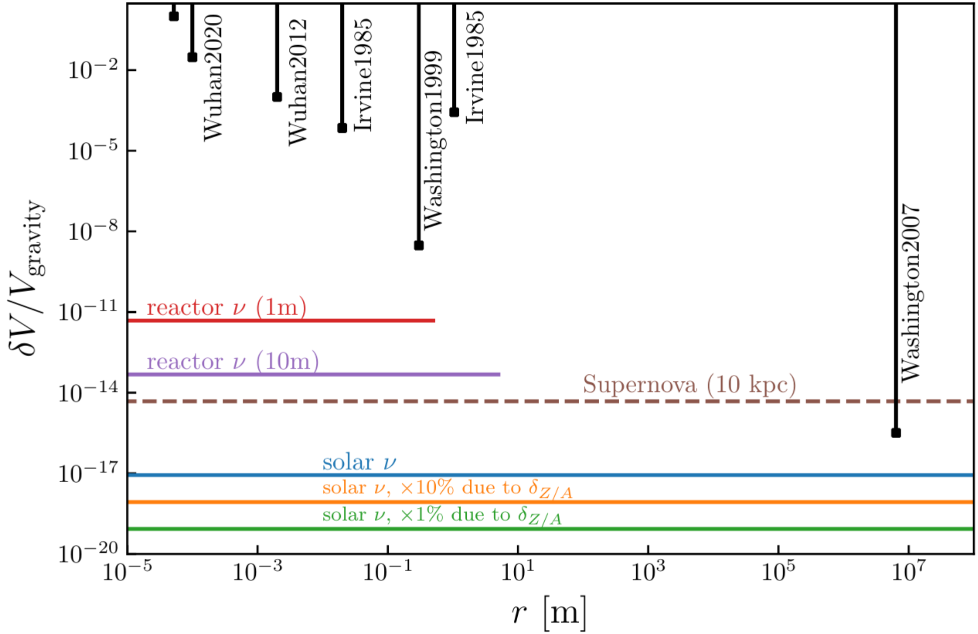


Ghosh, Grossman, Tangarife, Xu, Yu, JHEP'23 ⇒
$$V \approx -\frac{1}{\pi}G_F^2\Phi_0 E_\nu \frac{1}{r}\left\{\cos^2\left(\frac{\alpha}{2}\right)\cos\left[\left(1-\cos\alpha\right)E_\nu r\right] + \sin^2\left(\frac{\alpha}{2}\right)\cos\left[\left(1+\cos\alpha\right)E_\nu r\right]\right\}$$

average E_ν&α

Result

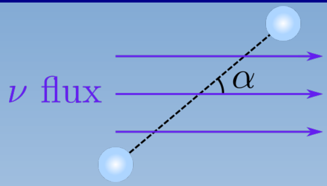
$$V_{\rm bkg}\left(r \gg E_\nu^{-1}, \alpha \ll 1\right) = -\frac{1}{\pi} G_F^2 \times \Phi_0 E_\nu \times \frac{1}{r} \times \left[1 + \mathcal{O}(\alpha^2)\right]$$



[Ghosh, Grossman, Tangarife, Xu, Yu, JHEP'23]

However ...

However ... smearing effect at finite α ...



- ... only able to integrate it analytically at $\alpha = 0$ or 90°
  - both $\rightarrow V \propto 1/r$, exciting!
- for $\forall\alpha$, ... difficult ..., but eventually

Result

$$V \approx -\frac{1}{\pi} G_F^2 \Phi_0 E_\nu \frac{1}{r} \left\{ \cos^2\left(\frac{\alpha}{2}\right) \cos\left[\left(1 - \cos\alpha\right) E_\nu r\right] + \sin^2\left(\frac{\alpha}{2}\right) \cos\left[\left(1 + \cos\alpha\right) E_\nu r\right] \right\}$$

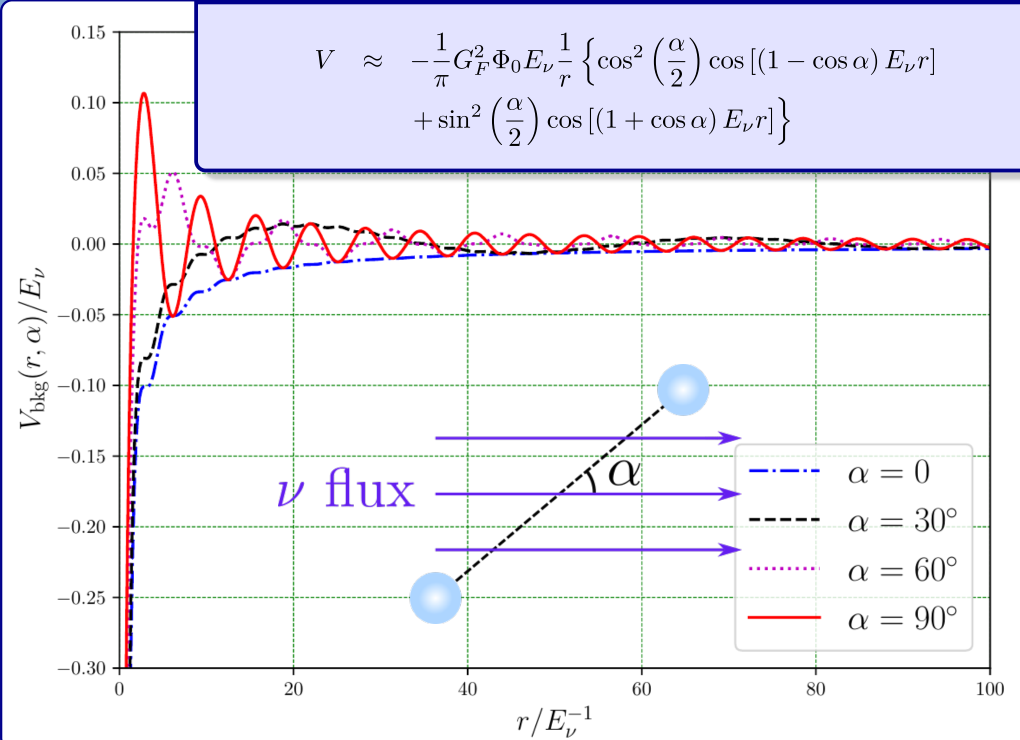
  - unfortunately smearing effect at finite α!
  - ... unless you design an exp with extremely suppressed α

condition to avoid smearing

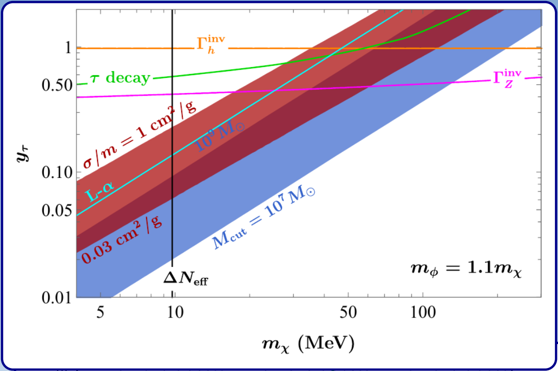
$$\alpha^2 \lesssim \frac{\pi}{\Delta(E_\nu r)}$$

Result

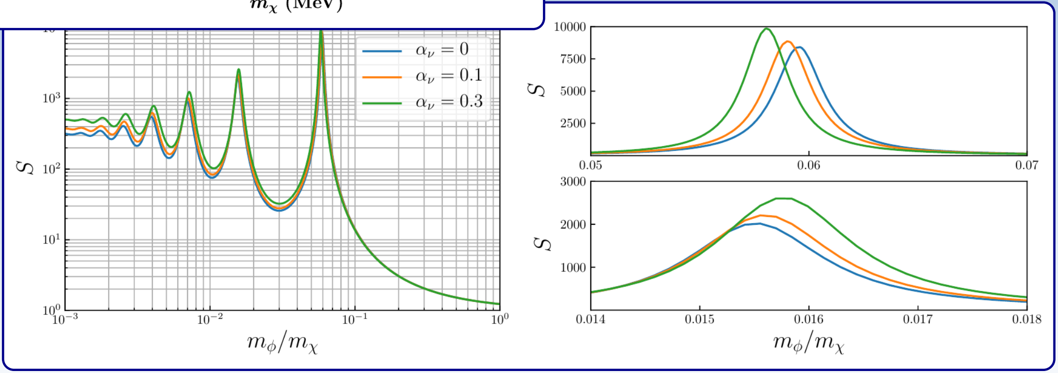
$$V \approx -\frac{1}{\pi} G_F^2 \Phi_0 E_\nu \frac{1}{r} \left\{ \cos^2\left(\frac{\alpha}{2}\right) \cos\left[\left(1-\cos\alpha\right) E_\nu r\right] + \sin^2\left(\frac{\alpha}{2}\right) \cos\left[\left(1+\cos\alpha\right) E_\nu r\right] \right\}$$

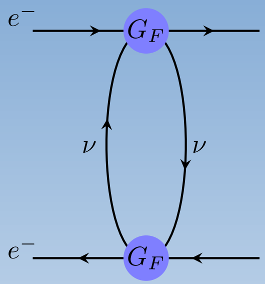


Application to Dark Matter



- ν-force $\rightarrow$ DM self-int.
  - N. Orlofsky, Yue Zhang, PRD'21
- the Sommerfeld enhancement
  - R. Coy, X. Xu, B. Yu, JHEP'22





Theory

- $1/r^5$
  - why? and what happens if $r \to 0$?
- very different in ν bkg.

Experimental probes

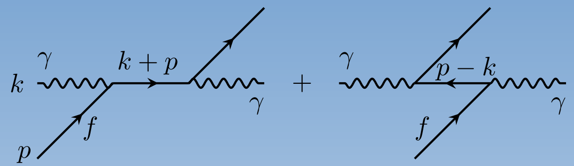
- parity violation at large scales
- atomic spectroscopy
- torsion balance experiments (with ν bkg.)

Can we detect neutrino forces? — No

... Future† ? Maybe ...

† atomic/muonic spectroscopy in ν bkg.; laser interferometer; BSM ν ...

backup slides



$$= \mathcal{M}_{\mu\nu}\epsilon^\mu\epsilon^\nu \; ,$$

$$\mathcal{M}^{\mu\nu} = (eQ_f)^2 \frac{4k.p\,(j^\nu k^\mu + j^\mu k^\nu - j\cdot k\, g^{\mu\nu}) - 2k^2\,(j^\nu p^\mu + j^\mu p^\nu)}{4(k\cdot p)^2 - (k^2)^2}$$

where $j_\mu \equiv \overline{u}\gamma_\mu u$

which implies

$$\mathcal{L}_{\rm eff} = \langle \mathcal{M}^{\mu\nu}|_{j\to J}\rangle A_\mu A_\nu$$

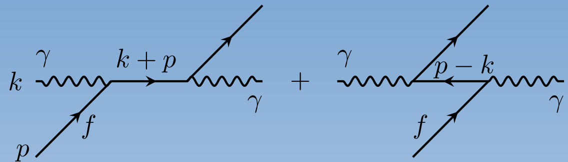
where $j_\mu \to J_\mu \equiv \overline{f}\gamma_\mu f$

non-relat. approx. $\to \langle J_\mu\rangle \approx (n_f,\ 0,\ 0,\ 0)$

$$(eQ_f)^2\left[-\frac{n_f}{m_f}g^{\mu\nu} - \frac{k^2}{2(k\cdot p)^2}\left(\langle J^\nu\rangle p^\mu + \langle J^\mu\rangle p^\nu\right)\right]$$

Recall that $\langle J_\mu\rangle$ is just the electric current. So the $\mu = 0$ component $=$ the number density

$$\partial_\mu J^\mu = 0 \to \nabla\vec{J} = dn/dt$$



$$= \mathcal{M}_{\mu\nu}\epsilon^{\mu}\epsilon^{\nu} \ ,$$

$$\mathcal{L}_{\text{eff}} = \langle \mathcal{M}^{\mu\nu}|_{j\to J}\rangle A_{\mu} A_{\nu}$$

non-relat. approx. $\rightarrow \langle J_{\mu}\rangle \approx (n_f,\ 0,\ 0,\ 0)$

$$(eQ_f)^2 \left[-\frac{n_f}{m_f} g^{\mu\nu} - \frac{k^2}{2(k\cdot p)^2} \left(\langle J^{\nu}\rangle p^{\mu} + \langle J^{\mu}\rangle p^{\nu} \right) \right]$$

transverse polarization

$$4\pi\alpha \frac{Q_f^2 n_f}{m_f}$$

longitudinal polarization

$$4\pi\alpha \frac{Q_f^2 n_f}{m_f} \left[1 - \frac{|\mathbf{k}|^2}{\omega^2} \right]$$